# Some classes of Türker equivalent graphs

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**Abstract**

Two graphs $G$ and $H$ are Türker equivalent if they have the same set of Türker angles. In this paper some Türker equivalent family of graphs are obtained.

## 1 Introduction

Let $G$ be a graph with $n$ vertices, $m$ edges and adjacency matrix $A$. The eigenvalues of A are the eigenvalues of G and form the spectrum of G denoted by $spec(G)$ [1]. The energy of $G$, denoted by $\mathcal{E}(G)$ is then defined as the sum of absolute value of its eigenvalues. The properties of $\mathcal{E}(G)$ are discussed in detail in [2, 3, 4, 5, 6, 7]. In chemistry, the energy of a graph is well studied since it can be used to approximate the total $\pi-$ electron energy of a molecule.

In order to express the fine molecular-structure-dependent difference in behavior of the total $\pi$ electron energy of isomeric alternate hydrocarbons Lemi Türker in [8] introduced the concept of angle of total $\pi$ electron energy $\theta$ defined as

$$\cos\theta = \frac{\mathcal{E}}{2\sqrt{mn}}$$

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and two other related angles $\alpha$ and $\beta$ connected by $\alpha + \beta = \theta$. These quantities are referred to as the Türker angles. This notion was extended to all graphs by I.Gutman [9].

The Türker angle $\theta$ has proven to be a useful novel concept in the theory of total $\pi-$ electron energy and it has found numerous applications. The fundamental properties of $\theta$, $\alpha$ and $\beta$ are discussed in [8, 9, 10, 11].

Recall from [9],

$$\cos\alpha = \frac{n + \mathcal{E}}{\sqrt{n}\sqrt{n + 2\mathcal{E} + 2m}} \tag{1}$$

$$\cos\beta = \frac{\mathcal{E} + 2m}{\sqrt{n + 2\mathcal{E} + 2m}\sqrt{2m}} \tag{2}$$

Set $Y = \sqrt{2mn - \mathcal{E}^2}$. Using the trigonometric identity $\tan x = \frac{\sqrt{1-\cos^2 x}}{\cos x}$ we get

$$\tan\alpha = \frac{Y}{n + \mathcal{E}}; \tan\beta = \frac{Y}{2m + \mathcal{E}} \text{ and } \tan\theta = \frac{Y}{\mathcal{E}} \tag{3}$$

Now, we study the nature of these angles in some family of graphs.

We use the following lemmas and definitions in this paper.

**Lemma 1.** *[1] Let $G$ be graph with spec$(G) = \{\lambda_i\}$, $i = 1$ to $n$ and $H$ be a graph with spec$(H) = \{\mu_j\}$, $j = 1$ to $n'$. Then the spectrum of the cartesian product, $G \times H$ of $G$ and $H$ is given by spec$(G \times H) = \{\lambda_i + \mu_j\}$, $i = 1$ to $n$, $j = 1$ to $n'$.*

**Lemma 2.** *[1] Let $A$ and $B$ be two matrices with spec$(A) = \{\lambda_i\}$, $i = 1$ to $m$ and spec$(B) = \{\mu_j\}$, $j = 1$ to $n$. Let $C = A \bigotimes B$, the tensor product of $A$ and $B$. Then spec$(C) = \{\lambda_i\, \mu_j\}$, $i = 1$ to $m$ and $j = 1$ to $n$.*

**Lemma 3.** *[6] Let $G$ be an $r$ regular graph on $n$ vertices, $r \geq 3$. Then its second iterated line graph $L^2(G)$ has $\frac{nr(r-1)}{2}$ vertices, $\frac{nr(r-1)(2r-3)}{2}$ edges and energy $2nr(r-2)$.*

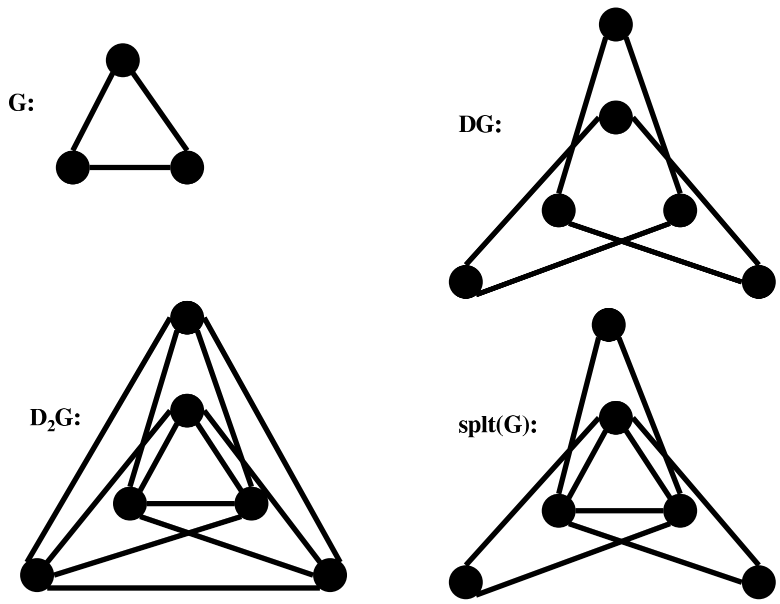
**Definition 1.** *[4] Let $G$ be a graph on $V = \{v_1, v_2, ........., v_n\}$. Take a copy of $G$ on $U = \{u_1, u_2, ...., u_n\}$ corresponding to $V = \{v_i\}$. Then make $u_i$ adjacent to vertices in $N(v_i)$ for each $i$, $i = 1$ to $n$. The resultant graph is called the double graph of $G$ denoted by $D_2(G)$.*

**Definition 2.** *[12] Let $G$ be a graph on $n$ vertices labelled as $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Then take another set $U = \{u_1, u_2, \ldots, u_n\}$ of $n$ vertices corresponding to $V = \{v_i\}$. Now define a graph*

*H with $V(H) = V \bigcup U$ and edge set of H consisting only of those edges joining $u_i$ to neighbors of $v_i$ in G for each i $i = 1$ to n. The resultant graph H is called the identity duplication graph of G denoted by DG .*

**Definition 3.** *[13] Let G be a graph on $V = \{v_1, v_2, ........., v_n\}$. Take a set $U = \{u_1, u_2, ...., u_n\}$ of n vertices corresponding to $V = \{v_i\}$. Then make $u_i$ adjacent to vertices in $N(v_i)$ for each i, $i = 1$ to n. The resultant graph is called the splitting graph of G denoted by splt(G).*

Illustration:




**Lemma 4.** *[4] Let G be a graph.Then $\mathcal{E}[D_2(G)] = \mathcal{E}[D(G)] = 2\mathcal{E}(G)$.*

**Lemma 5.** *Let G be a graph. Then $\mathcal{E}[splt(G)] = \sqrt{5}\mathcal{E}(G)$.*

*Proof.* By definition of splitting graph of $G$, the adjacency matrix of

$$splt(G) = \begin{bmatrix} A & A \\ A & 0 \end{bmatrix} = A \bigotimes \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

Then the theorem follows, since the eigenvalues of $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ are $\frac{1 \pm \sqrt{5}}{2}$. ∎

## 2 Some classes of Türker equivalent graphs

**Definition 4.** *Two graphs $G$ and $H$ are Türker equivalent if they have the same set of values for the Türker angles.*

It is known [9] that isomorphic graphs are Türker equivalent. In this section we obtain non-isomorphic Türker equivalent graphs.

**Theorem 1.** *Let $\mathcal{G} = \{G/G$ is an $r-$ regular graph, $r \geq 3\}$. Let $\mathcal{F}_k = \{L^k(G),\ k \geq 2/G \in \mathcal{G}\}$. Then the family $\mathcal{F}_k$ is Türker equivalent for each $k$.*

*Proof.* Let $G$ be an $r-$ regular graph on $n$ vertices, $r \geq 3$. Then by Lemma 2 and Eq.3, for the family $L^2(G)$ we have the following,

$$\begin{aligned}
Y &= nr(r-1)\sqrt{\frac{2r-3}{2} - 4\left(\frac{r-2}{r-1}\right)^2} \\
\tan\theta &= \frac{(r-1)\sqrt{\frac{2r-3}{2} - 4\left(\frac{r-2}{r-1}\right)^2}}{2\,(r-2)} \\
\tan\alpha &= \frac{2(r-1)}{5r-9}\sqrt{\frac{2r-3}{2} - 4\left(\frac{r-2}{r-1}\right)^2} \\
\tan\beta &= \frac{2(r-1)}{2r^2-r-5}\sqrt{\frac{2r-3}{2} - 4\left(\frac{r-2}{r-1}\right)^2}.
\end{aligned}$$

Here tan$\theta$, tan$\alpha$ and tan$\beta$ are independent of $n$, the number of vertices of $G$ and depend only on $r$, regularity of $G$. Since $L^k(G) = L^2(H)$ for some regular graph $H$, this can be extended to the family $L^k(G)$, for $k \geq 3$. ■

**Theorem 2.** *Let $G$ be any graph. Let $\mathcal{D} = \bigcup_k D^kG$ where $D^kG$ is defined iteratively by $D^0G = G$ and $D^kG = D(D^{k-1}G), k \geq 2$. Then $\mathcal{D}$ is a Türker equivalent family of graphs.*

*Proof.* Let $G$ be an $(n, m)$ graph with energy $\mathcal{E}$ and Türker angles $\alpha, \beta$ and $\theta$. Then by [4], $DG$, the duplicate graph of $G$ is a $(2n, 2m)$ graph with energy $2\mathcal{E}$.

Let $\theta'$, $\alpha'$ and $\beta'$ be the Türker angles of $DG$. Then from Eq. 3 we have the following,

$$\begin{aligned}
\tan\alpha' &= \frac{\sqrt{2\times 2m\times 2n-(2\mathcal{E})^2}}{2n+2\mathcal{E}} = \frac{\sqrt{2mn-\mathcal{E}^2}}{n+\mathcal{E}} = \tan\alpha \\
\tan\beta' &= \frac{\sqrt{2\times 2m\times 2n-(2\mathcal{E})^2}}{2\times 2m+2\mathcal{E}} = \frac{\sqrt{2mn-\mathcal{E}^2}}{2m+\mathcal{E}} = \tan\beta \\
\tan\theta' &= \frac{\sqrt{2\times 2m\times 2n-(2\mathcal{E})^2}}{2\mathcal{E}} = \frac{\sqrt{2mn-\mathcal{E}^2}}{\mathcal{E}} = \tan\theta
\end{aligned}$$

Thus the theorem follows. ■

**Theorem 3.** *Let* $\mathcal{F}_k = \left\{L^k(G)/G \text{ is an } r- \text{ regular graph},\ r\geq 3,\ k\geq 2\right\}$ *and* $\mathcal{H}_k = \{splt(F_k) \text{ where } F_k\in\mathcal{F}_k\}$. *Then the family* $\mathcal{H}_k$ *is Türker equivalent for each* $k$.

*Proof.* Let $G$ be an $(n,m)$ graph and $k=2$. Then by [13], $splt(G)$ is a $(2n,3m)$ graph. Then

$$\begin{aligned}
N &= \left|V\left[splt\left\{L^2(G)\right\}\right]\right| = 2\times\left|V\left[L^2(G)\right]\right| \\
&= nr(r-1) \\
M &= \left|Edge\left[splt\left\{L^2(G)\right\}\right]\right| = 3\times\left|Edge\left\{L^2(G)\right\}\right| \\
&= 3\times\frac{nr(r-1)(2r-3)}{2} \\
\mathcal{E} &= Energy\left[splt\left\{L^2(G)\right\}\right] = \sqrt{5}\times Energy\left\{L^2(G)\right\} \\
&= 2\sqrt{5}nr(r-2) \text{ by Lemmas 3 and 5.}
\end{aligned}$$

Also $Y = \sqrt{2MN-\mathcal{E}^2} = \sqrt{3n^2r^2(r-1)^2(2r-3)-20n^2r^2(r-2)^2}$. Thus the Türker angles are given as follows.

$$\begin{aligned}
\tan\theta &= \frac{Y}{\mathcal{E}} = \frac{\sqrt{3(r-1)^2(2r-3)-20(r-2)^2}}{2\sqrt{5}(r-2)}. \\
\tan\alpha &= \frac{Y}{N+\mathcal{E}} = \frac{\sqrt{3(r-1)^2(2r-3)-20(r-2)^2}}{(r-1)+2\sqrt{5}(r-2)}. \\
\tan\beta &= \frac{Y}{2M+\mathcal{E}} = \frac{\sqrt{3(r-1)^2(2r-3)-20(r-2)^2}}{3(r-1)(2r-3)+2\sqrt{5}(r-2)}.
\end{aligned}$$

Since $L^k(G) = L^2[H]$ for some regular graph $H$, the theorem follows. ■

**Theorem 4.** *Let* $\mathcal{T}_k = \left\{ D_2 \left[ L^k(G) \right] / G \text{ is an } r- \text{ regular graph, } r \geqslant 3,\ k \geq 2 \right\}$. *Then the family* $\mathcal{T}_k$ *is Türker equivalent for each* $k$.

*Proof.* Let $G$ be an $(n, m)$ graph and $k = 2$. Then by [4], $D_2(G)$ is a $(2n, 4m)$ graph. Assume that $G$ is $r \geq 3$ regular. Then

$$\begin{aligned} N &= \left| V \left[ D_2 \left\{ L^2(G) \right\} \right] \right| = 2 \times \left| V \left[ L^2(G) \right] \right| = nr(r-1) \\ M &= \left| Edge \left[ D_2 \left\{ L^2(G) \right\} \right] \right| = 4 \times \left| Edge \left\{ L^2(G) \right\} \right| \\ &= 2nr(r-1)(2r-3) \\ \mathcal{E} &= Energy \left[ D_2 \left\{ L^2(G) \right\} \right] = 2 \times Energy \left\{ L^2(G) \right\} \\ &= 4nr(r-2) \text{ by Lemmas 3 and 4.} \end{aligned}$$

Also $Y = \sqrt{2MN - \mathcal{E}^2} = 2nr\sqrt{(r-1)^2(2r-3) - 4(r-2)^2}$. Thus the Türker angles are as follows.

$$\begin{aligned} \tan\theta &= \frac{Y}{\mathcal{E}} = \frac{\sqrt{(r-1)^2(2r-3) - 4(r-2)^2}}{2(r-2)}. \\ \tan\alpha &= \frac{Y}{N + \mathcal{E}} = \frac{2\sqrt{(r-1)^2(2r-3) - 4(r-2)^2}}{5r-9}. \\ \tan\beta &= \frac{Y}{2M + \mathcal{E}} = \frac{\sqrt{(r-1)^2(2r-3) - 4(r-2)^2}}{2\left[(r-1)(2r-3) + (r-2)\right]}. \end{aligned}$$

Since $L^k(G) = L^2[H]$ for some regular graph $H$, the theorem follows. ■

The following theorems provide some more Türker equivalent graphs, the proof of which are on similar lines.

**Theorem 5.** *Let* $\mathcal{G} = \{G/G \text{ is an } r- \text{ regular graph}\}$ *and* $\mathcal{H} = \{H/H \text{ is an } r'- \text{ regular graph }\}$ *where* $r,\ r' \geqslant 4$. *Then the family* $L^p(\mathcal{G}) \times L^q(\mathcal{H})$ *is Türker equivalent for each* $p \geq 2$ *and* $q \geq 2$.

**Theorem 6.** *Let* $\mathcal{G} = \{G/G \text{ is an } r\text{− regular graph, } r \geq 4\}$, $\mathcal{F}_k = \{L^k(G), k \geq 2/G \in \mathcal{G}\}$ *and* $\mathcal{R}_k = \{R = F_1 \bigotimes F_2\ /F_1 \text{ and } F_2 \in \mathcal{F}_k\}$. *Then* $\mathcal{R}_k$ *is Türker equivalent for each* $k$.

**Theorem 7.** *Let* $G$ *be an* $r-$ *regular graph,* $r \geq 3$. *Then the family* $\{L^k(G) \bigotimes K_p\}$ *is Türker equivalent for each* $p$ *and each* $k \geq 2$.

**Theorem 8.** *Let $G$ be an $r-$ regular graph, $r \geq 4$. Then the family $\{L^k(G) \times C_p\}$ is Türker equivalent for each $p \geq 3$ and $k \geq 2$.*

# 3 Some operations on a graph

In this section we define some operations on a graph $G$ with $V(G) = \{v_1, v_2, ..., v_n\}$.

**Operation 1.** *Introduce two copies of $G$ on $U = \{u_i\}$ and $W = \{w_i\}$ corresponding to $V = \{v_i\}$. Make $u_i$ and $w_i$ adjacent to the vertices in $N(v_i)$ for each $i$, $i = 1$ to $n$ . Then remove the edges of $G$ only.*

**Operation 2.** *Introduce two copies of $G$ on $U = \{u_i\}$ and $W = \{w_i\}$ corresponding to $V = \{v_i\}$. Make $u_i$ adjacent to the vertices in $N(v_i)$ and $N(w_i)$ and make $w_i$ adjacent to the vertices in $N(v_i)$ and $N(u_i)$ for each $i$, $i = 1$ to $n$. Then remove the edges of $G$ only.*

**Operation 3.** *Introduce two copies of $G$ on $U = \{u_i\}$ and $W = \{w_i\}$ corresponding to $V = \{v_i\}$. Make $u_i$ adjacent to the vertices in $N(v_i)$ and $N(w_i)$ and make $w_i$ adjacent to the vertices in $N(v_i)$ and $N(u_i)$ for each $i$, $i = 1$ to $n$. Then remove the edges of $G$ on vertex sets $V$ and $W$.*

**Operation 4.** *Introduce two copies of $G$ on $U = \{u_i\}$ and $W = \{w_i\}$ corresponding to $V = \{v_i\}$. Make $u_i$ and $w_i$ adjacent to the vertices in $N(v_i)$ for each $i$, $i = 1$ to $n$.*

The graph obtained from $G$ using operation $i$ is denoted by $H_i$, $i = 1, 2, 3$ and 4.

**Theorem 9.** *Let $G$ be a graph on $n$ vertices with spectrum $\{\lambda_1, \lambda_2, ......., \lambda_n\}$ and $H_i$, $i = 1, 2, 3$ and 4 be the graphs obtained as above. Then*

1. $\mathcal{E}(H_1) = 4\mathcal{E}(G)$
2. $\mathcal{E}(H_2) = 2\sqrt{3}\mathcal{E}(G)$
3. $\mathcal{E}(H_3) = [2\sqrt{2} + 1]\mathcal{E}(G)$
4. $\mathcal{E}(H_4) = [2\sqrt{2} + 1]\mathcal{E}(G)$

*Proof.* The table 1 gives the adjacency matrix, its tensor partition and the eigenvalues of $H_i$, $i = 1, 2, 3$ and 4.

**Table 1**

| Operation | Adjacency Matrix | Eigenvalues |
|---|---|---|
| 1 | $\begin{bmatrix} 0 & A & A \\ A & A & 0 \\ A & 0 & A \end{bmatrix} = A \bigotimes \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ | $\{2\lambda_i, \lambda_i, -\lambda_i\}$ |
| 2 | $\begin{bmatrix} 0 & A & A \\ A & A & A \\ A & A & A \end{bmatrix} = A \bigotimes \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\left\{\left(1 \pm \sqrt{3}\right) \lambda_i, 0\right\}$ |
| 3 | $\begin{bmatrix} 0 & A & A \\ A & A & A \\ A & A & 0 \end{bmatrix} = A \bigotimes \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ | $\left\{\left(1 \pm \sqrt{2}\right) \lambda_i, -\lambda_i\right\}$ |
| 4 | $\begin{bmatrix} A & A & A \\ A & A & 0 \\ A & 0 & A \end{bmatrix} = A \bigotimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ | $\left\{\left(1 \pm \sqrt{2}\right) \lambda_i, \lambda_i\right\}$ |

Column 3 of Table 1 gives the eigenvalues of $H_i$, $i = 1, 2, 3$ and 4 and hence the theorem follows. ■

**Note:** $H_3 = H_4$ when $G$ is bipartite.

**Theorem 10.** *Let $\mathcal{G}$ be the collection of all $r-$ regular graphs, $r \geq 3$ and $\mathcal{F}_k = \{L^k(G), k \geq 2/G \in \mathcal{G}\}$. Let $\mathcal{F}_{ki} = \{F_{ki}/F_k \in \mathcal{F}_k\}$, $i = 1, 2, 3$ and 4 as defined by the above operations. Then each family $\mathcal{F}_{ki}$, $i = 1, 2, 3, 4$ and $k \geq 2$ is Türker equivalent.*

*Proof.* Let $G$ be an $r-$ regular graph on $n$ vertices, $r \geq 3$ and $k = 2$. Then by Lemma 3 and from the above operations we have the order, size and energy of $F_{2i}$ for $i = 1, 2, 3$ and 4 are as given in table 2.

**Table 2**

| i | Order of $F_{2i}$ | Size of $F_{2i}$ | $\mathcal{E}(F_{2i})$ |
|---|---|---|---|
| 1 | $\frac{3nr(r-1)(2r-3)}{2}$ | $3nr(r-1)$ | $8nr(r-2)$ |
| 2 | $\frac{3nr(r-1)(2r-3)}{2}$ | $4nr(r-1)$ | $4\sqrt{3}nr(r-2)$ |
| 3 | $\frac{3nr(r-1)(2r-3)}{2}$ | $\frac{7nr(r-1)}{2}$ | $2\left(2\sqrt{2}+1\right)nr(r-2)$ |
| 4 | $\frac{3nr(r-1)(2r-3)}{2}$ | $\frac{7nr(r-1)}{2}$ | $2\left(2\sqrt{2}+1\right)nr(r-2)$ |

Now for each $i$, the Table 3 gives the three Türker angles.

**Table 3**

| i | $\tan\theta$ | $\tan\alpha$ | $\tan\beta$ |
|---|---|---|---|
| 1 | $\frac{\sqrt{18r^3-127r^2+328r-283}}{8(r-2)}$ | $\frac{2\sqrt{18r^3-127r^2+328r-283}}{6r^2+r-23}$ | $\frac{\sqrt{18r^3-127r^2+328r-283}}{2(7r-11)}$ |
| 2 | $\frac{\sqrt{18r^3-127r^2+328r-283}}{4\sqrt{3}(r-2)}$ | $\frac{2\sqrt{18r^3-127r^2+328r-283}}{6r^2+r(8\sqrt{3}-15)-(16\sqrt{3}-9)}$ | $\frac{\sqrt{18r^3-127r^2+328r-283}}{4\left[\left(2+\sqrt{3}\right)r-2\left(1+\sqrt{3}\right)\right]}$ |
| 3 | $\frac{\sqrt{6r^3-33r^2+72r-57}}{\left[1+2\sqrt{2}\right](r-2)}$ | $\frac{4\sqrt{6r^3-33r^2+72r-57}}{\left[6r^2+r\left(8\sqrt{2}-11\right)-\left(16\sqrt{2}-1\right)\right]}$ | $\frac{2\sqrt{6r^3-33r^2+72r-57}}{\left[r\left(4\sqrt{2}+9\right)-\left(8\sqrt{2}+11\right)\right]}$ |
| 4 | $\frac{\sqrt{6r^3-33r^2+72r-57}}{\left[1+2\sqrt{2}\right](r-2)}$ | $\frac{4\sqrt{6r^3-33r^2+72r-57}}{\left[6r^2+r\left(8\sqrt{2}-11\right)-\left(16\sqrt{2}-1\right)\right]}$ | $\frac{2\sqrt{6r^3-33r^2+72r-57}}{\left[r\left(4\sqrt{2}+9\right)-\left(8\sqrt{2}+11\right)\right]}$ |

Since $L^k(G) = L^2[H]$ for some regular graph $H$ for $k \geq 3$, the theorem follows from table 3. ■

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