Enhancement of Calcifications in Mammograms using Volterra Series based Quadratic Filter

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***Abstract*—The paper summarizes the design and implementation of a quadratic edge detection filter, based on Volterra series, for enhancing calcifications in mammograms. The proposed filter can account for much of the polynomial nonlinearities inherent in the input mammogram image and can replace the conventional edge detectors like Laplacian, gaussian etc. The filter gives rise to improved visualization and early detection of microcalcifications, which if left undetected, can lead to breast cancer. The performance of the filter is analyzed and found superior to conventional spatial edge detectors.**

***Index Terms*—Breast cancer, microcalcifications, Mammogram, quadratic filter, singular value decomposition, Volterra series.**

I. Introduction

Breast cancer is the second leading cause of cancer deaths in women. Though cure at an advanced stage is difficult, early detection and treatment can cure breast cancer completely. The screening and diagnosis are based on noninvasive imaging of breast that can help visualizing the microcalcifications leading to breast carcinoma. The primary imaging technique for visualizing pathological disorders in breast is digital mammography. Often mammograms tend to be of low contrast and noisy and miss around 10% of cancerous lesions which can lead to mortality at a later stage. In young women, the breast tissue is dense and even digital mammography can miss cancerous lesions [1]. It is imperative that contrast enhancement is a critical step before radiological analysis.

Conventional edge detection filters like Laplace, Sobel, [2] Laplacian of Gaussian(LoG) filter [3] are employed to detect lesions. Wavelet based multiresolution analysis is used as a tool for detection of microcalcifications [4]. Histogram method for local contrast enhancement [5] has been reported for the detection of calcifications. Multilevel thresholding based segmentation is used for detecting masses in mammograms [6]. Various image enhancement techniques for the detection of calcifications [7] is also reported. These methods often suffer from poor edge resolution especially in presence of noise. Under such conditions, polynomial filters [8] are observed to perform better in detecting edges with high enough resolution. By nature, images are formed by nonlinear processes and human vision is inherently nonlinear. So image processing and analysis by polynomial methods become a natural alternative. It has been observed that much of the nonlinearities can be modeled by the quadratic term alone. Although the idea of modeling nonlinearities by power series was proposed a century back by Vito Volterra, [9] the practical applications were hampered by the large computational complexity. Recently, with increase in computational resources, interest is renewed in Volterra systems for signal and image processing. The proposed work is in implementing a quadratic Volterra filter [10] for enhancing calcifications present in noisy mammograms which remains invisible in conventional image enhancement schemes. [11]

II. Discrete Volterra Series

Although the theory of linear systems is very advanced and useful, most of the real life and practical systems are nonlinear. Mild polynomial nonlinearities can be modeled by Volterra power series. An N^{th} order Volterra filter [9], [10] with input vector $x[n]$ and output vector $y[n]$ is realized by

$$y[n] = h_0 + \sum_{r=1}^{\infty}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\cdots\sum_{n_r=1}^{N} h_r[n_1, n_2, \ldots n_r]. \\ x[n-n_1]x[n-n_2]\cdots x[n-n_r] \quad (1)$$

where r indicates the order of nonlinearity, with $r = 1$ implying a linear system, $r = 2$ implying a quadratic system and so forth. $h_r[n_1, n_2, \ldots n_r]$ is the r^{th} order Volterra kernel, identification [12], [13] of which is one of the key issues in polynomial signal processing. h_0 is the constant offset at the output when no input is present. The complexity of the kernel can be considerably reduced by assuming homogeneity. Also the output $y[n]$ is linear [14] with respect to the Volterra filter weights. Often, in practical systems, much of the nonlinearity is comprised of the quadratic components. It is thus proposed that a two dimensional quadratic filter can model and process inherent nonlinearites in medical images.

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III. Two Dimensional Discrete Quadratic Volterra System

The two dimensional quadratic system with input $x[n_1, n_2]$ and output $y[n_1, n_2]$ is governed by the equation

$$y[n_1, n_2] = \sum_{m_{11}=0}^{N_1-1} \sum_{m_{12}=0}^{N_2-1} \sum_{m_{21}=0}^{N_1-1} \sum_{m_{22}=0}^{N_2-1} h_1[m_{11}, m_{12}, m_{21}, m_{22}] \times x[n_1 - m_{11}, n_2 - m_{12}] x[n_1 - m_{21}, n_2 - m_{22}] \quad (2)$$

Equation (2) can be represented in the matrix form as

$$y[n_1, n_2] = \mathbf{X}^T[n_1, n_2]\mathbf{H_2}\mathbf{X}[n_1, n_2] \quad (3)$$

The quadratic kernel $\mathbf{H_2}$ has $N_1N_2 \times N_1N_2$ elements and each element consists of N_2^2 submatrices $\mathbf{H}(i, j)$ with $N_1 \times N_2$ elements given as

$$\begin{bmatrix} \mathbf{H}(0,0) & \mathbf{H}(0,1) & \cdots & \mathbf{H}(0, N_2-1) \\ \mathbf{H}(1,0) & \mathbf{H}(1,1) & \cdots & \mathbf{H}(1, N_2-1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}(N_2-1, 0) & \mathbf{H}(N_2-1, 1) & \cdots & \mathbf{H}(N_2-1, N_2-1) \end{bmatrix}$$

where each submatrix $\mathbf{H}(i, j)$ is given by

$$\mathbf{H}(i,j) = \begin{bmatrix} h(0, i, 0, j) & \cdots & h(0, i, N_1-1, j) \\ h(1, i, 0, j) & \cdots & h(1, i, N_1-1, j) \\ \vdots & \vdots & \vdots \\ h(N_1-1, i, 0, j) & \cdots & h(N_1-1, i, N_1-1, j) \end{bmatrix} \quad (4)$$

The principal issues in Volterra systems are the identification of the kernel H_2 [12], [13], [14] and its computationally efficient implementation. Unlike in linear filtering there are no general design methods for finding H_2. Design of two dimensional kernels for specific applications can be done using methods like optimization, bi-impulse response method [15] etc. The current work uses optimization of mean square error using Powell method. The second step is in realizing the kernel with minimum computational complexity [16], [17]. The equation (3) can be viewed as a filtering operation on the kronecker product of $\mathbf{X}[n_1, n_2]$ with itself by the filter kernel H_2 . A feasible implementation can be done with appropriate decomposition of H_2 like LU or SVD decomposition.

IV. Methodology

The scheme of work is as depicted in Fig. 1. In the first phase of work, the quadratic kernel H_2 required is designed using optimization method. As the direct implementation of H_2 is not computationally feasible, an approximate realzation using singular value decomposition is performed in the second phase. These steps are outlined in Sec. V. The filter with the structure shown in Fig. 3 is tested with known images. Once the performance is found satisfactory, the filter is applied to mammogram images. The results are summarized in Sec. VII

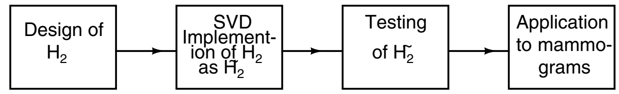


Fig. 1: Scheme of work

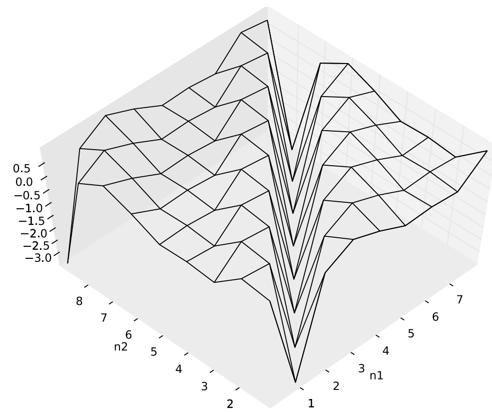


Fig. 2: Wireframe plot of the quadratic kernel

V. Design and Implementation of Quadratic Mammogram Enhancement Filter

It is proposed that a quadratic filter can enhance the edges better than Laplacian or LoG filter. The principal issue in employing a quadratic filter is the identification of its kernel H_2. Powell optimization [18] is used for obtaining H_2 as this algorithm has a fast rate of convergence. A blunt synthetic edge added with noise denoted as $x_{test}[n_1, n_2]$ of 9×9 dimension is simulated. A desired sharp synthetic edge $x_{ref}[n_1, n_2]$ of identical dimension is also simulated. Assume that the output of the quadratic filter $y_{test}[n_1, n_2] = \mathbf{X}_{test}^T[n_1, n_2]\mathbf{H_2}\mathbf{X}_{test}[n_1, n_2]$. The output of the filter for the input $x_{ref}[n_1, n_2]$ is $y_{ref}[n_1, n_2] = \mathbf{X}_{ref}^T[n_1, n_2]\mathbf{H_2}\mathbf{X}_{ref}[n_1, n_2]$. Then the sharpness of the edges at the output of the filter with respect to the reference image at the output is mathematically modeled as below.

A. Crispness of Edges

The sharpness of edges in the filtered outputs $y_{test}[n_1, n_2]$ and $y_{ref}[n_1, n_2]$ is decided by a numerical figure of merit [19] of the filter is given as

$$\kappa = \frac{1}{N_1N_2} \sum_{n_1} \sum_{n_2} \frac{|\sigma^2_{l[n_1,n_2]_{test}} - \sigma^2_{l[n_1,n_2]_{ref}}|}{\sigma^2_{l[n_1,n_2]_{ref}} \mu_{l[n_1,n_2]_{ref}}} \quad (5)$$

$\sigma^2_{l[n_1,n_2]_{test}}$ is the localized variance (here a 3×3 pixel window is used to match the size of the filter mask) of the test image and $\sigma^2_{l[n_1,n_2]_{ref}}$ is that of the reference image. The localized mean of the reference image is $\mu_{l[n_1,n_2]_{ref}}$. The parameter κ should be zero when the edges at the output are unaffected and it increases monotonically as degradation of edges increases. A low value of this parameter is a direct indicator of the sharpness of the periphery of the calcification. Now κ is minimized using Powell method and the solution for H_2 for

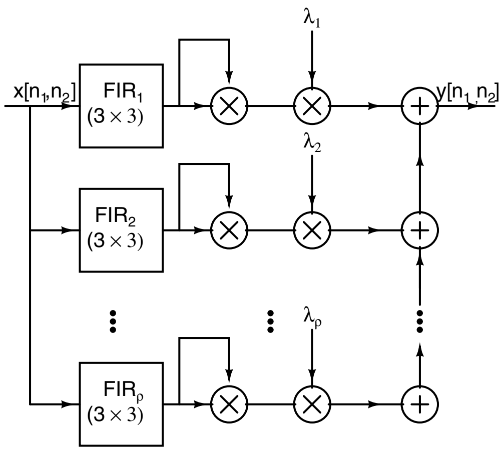


Fig. 3: Realization of H_2 by singular value decomposition

TABLE I: Table of singular values and singular vectors of H_2

λ_i	S_1	S_2	S_3	S_4
3.2783	-0.3333	-0.3333	-0.3333	-0.3333
2.9284	-0.0570	0.4337	-0.3768	-0.0570
2.9284	0.4680	-0.1846	-0.2833	0.4680
2.8494	-0.0388	0.2723	0.4559	0.4262
2.8494	-0.4698	-0.3848	-0.1199	0.2013
2.8467	0.09977	-0.2513	0.3726	-0.4489
2.8467	-0.4607	0.3988	-0.2888	0.1440
2.7422	0.4690	0.0344	-0.4570	-0.1931
2.7422	-0.0478	-0.4702	-0.1155	0.4300

S_5	S_6	S_7	S_8	S_9
-0.3333	-0.3333	-0.3333	-0.3333	-0.3333
0.4337	-0.3768	-0.0570	0.4337	-0.3768
-0.1846	-0.2833	0.4680	-0.1846	-0.2833
0.1971	-0.1243	-0.3875	-0.4694	-0.3317
0.4282	0.4547	0.2685	-0.0434	-0.3350
0.4711	-0.4364	0.3491	-0.2197	0.0638
0.0182	-0.1783	0.3168	-0.4171	0.4671
0.3900	0.3286	-0.2759	-0.4244	0.1285
0.2649	-0.3381	-0.38225	0.2053	0.4536

minimum κ is ascertained. The kernel H_2 is plotted as in Fig. 2 and is indicative of edge detection characteristics. A direct implementation as in (3) is computationally complex. Instead SVD decomposition [20] is performed on H_2 to yield an approximation $\tilde{H}_2$ as

$$\tilde{H}_2 = \sum_{i=1}^{\rho} \lambda_i S_i S_i^T \tag{6}$$

where λ_i are the singular values and each S_i is a 9×1 eigen vector. The singular values and singular vectors of H_2 are tabulated in Table I. The value of ρ is selected in such a manner that the Frobenius norm $||H_2 - \tilde{H}_2||$ is minimum. Each S_i can be resized as a 3×3 FIR image filter that is equivalent to $H(i, j)$ in equation (4). The outputs of FIR filters are squared and a weighted sum with λ_i values yields the filter output. The structure of the filter is as in Fig. 3.

VI. Experiment

The filter kernel $\tilde{H}_2$ designed in the last section is simulated in Python with the help of scipy and pylab modules. The noisy mammograms are imported into Python using the image processing toolbox and subjected to filtering by $\tilde{H}_2$. The filtered images are compared with those processed by spatial edge detectors like Laplacian, Canny, Laplacian of gaussian(LoG) and Prewitt filters in terms of visual quality and edge crispness and improvement in SNR and PSNR. The results of the experiments are in Sec. VII.

VII. Results and Discussion

The quadratic filter implemented as in Fig. 3 is tested with mammogram inputs. The results are as in Fig. 4. The input mammogram in Fig. 4a contains the noise contributed by the soft breast tissue that obscures the whiter hard regions of calcification. The enhancement of calcifications by Gabor filter as shown in Fig. 4b is but marginal. The breast tissue still remains visible and the exact edges of calcifications remain obscure. Gabor filter suffers from poor noise invulnerability. Laplacian of Gaussian (LoG) filter yields the output as shown in Fig. 4e. Although LoG is more invulnerable to additive noise it still cannot obscure the background tissue. The Prewitt filter whose output is as in Fig. 4d does not enhance the image at all. A Canny edge detector with a thresold of 50 yields the output as in Fig. 4c. Here the calcifications are not discernible. Fig. 4f shows the output of the quadratic filter. Here it is seen that the background tissue is not very visible. Besides, the calcifications are enhanced with clearly defined peripheries. A zoomed view shown in Fig. 6 indicates the sharp edges of calcifications.

A. Improvement in signal to noise ratio and peak signal to noise ratio

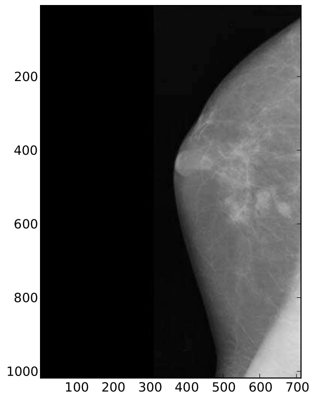
The improvement in signal to noise ratio and peak signal to noise ratio are computed as per the experimental setup in Fig. 5. Identical mammogram images are filtered by quadratic, LoG, Prewitt and Laplacian filters. The input image is taken as the reference image. The SNR is expressed as

$$SNR = 10 \log_{10} \left[\frac{\sum_{n_1}\sum_{n_2} r^2_{[n_1,n_2]}}{\sum_{n_1}\sum_{n_2}[r^2_{[n_1,n_2]} - t^2_{[n_1,n_2]}]} \right] \tag{7}$$

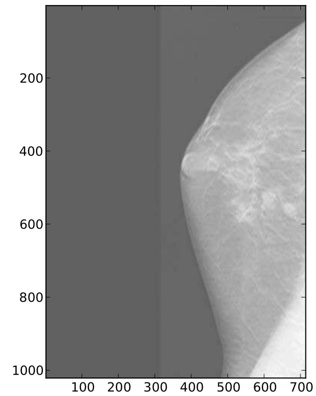
The peak value of the SNR is expressed as

$$PSNR = 10 \log_{10} \left[\frac{\max\left(r^2_{[n_1,n_2]}\right)}{\frac{1}{N_1 N_2}\sum_{n_1}\sum_{n_2}[r^2_{[n_1,n_2]} - t^2_{[n_1,n_2]}]} \right] \tag{8}$$

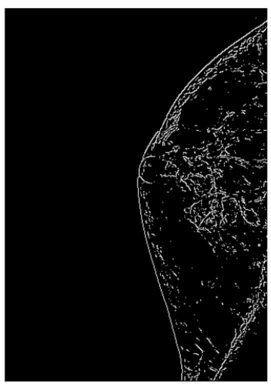
where r denotes the reference image and t denotes the test image. $N_1 N_2$ is the size of the image. The SNR and PSNR values for various filters are tabulated in Table II. It can be seen that the quadratic filter offers $\approx$ 10 db improvement over the LoG filter. Laplacian, Prewitt and Canny do offer much improvement in SNR.



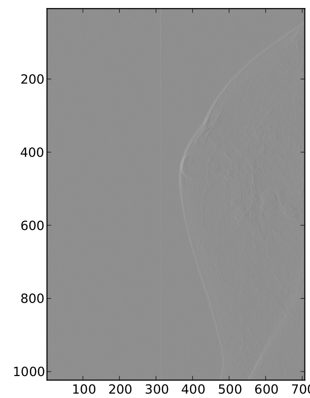
(a) Input mammogram



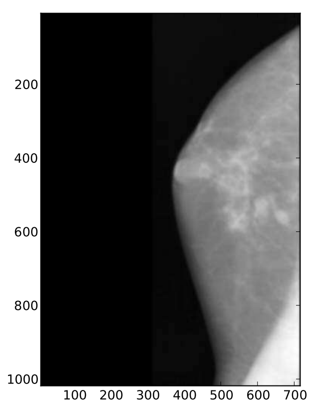
(b) Output of Gabor filter



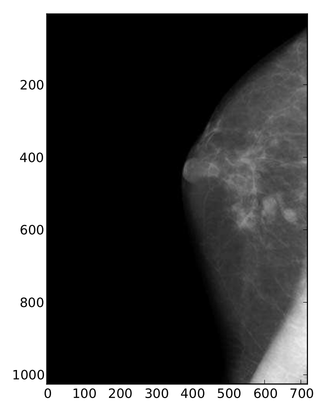
(c) Output of Canny edge detection filter



(d) Output of Prewitt filter



(e) Output of LoG filter



(f) Output of quadratic filter

Fig. 4: Outputs of various filters for the given input mammogram

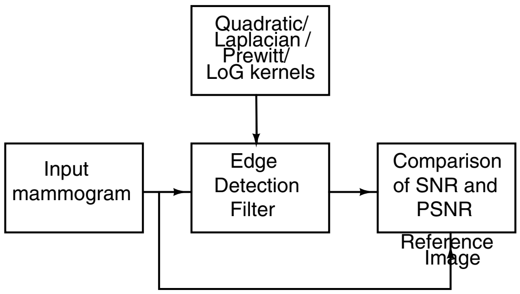


Fig. 5: Experimental set up for measuring SNR and PSNR

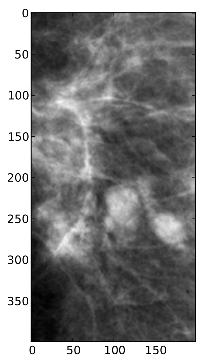


Fig. 6: Zoomed image of calcifications rendered by the quadratic filter

Table III shows the time taken for performing the computation on 1024×1024 size mammogram image. It is observed that Gaussian filter has the least computational complexity but it does not enhance the calcifications as a quadratic filter does. Similar is the case with a Gabor filter. Laplacian filter has the largest time of computation. Quadratic filter has a computation time of 2.4 second with calcifications enhanced.The approximate realization based on singular value decomposition reduces the complexity of implementation.

VIII. CONCLUSION

A quadratic filter based on Volterra power series is designed based on the optimization of the edge crispness function. The filter kernel so developed is implemented by singular value decomposition. The filter is tested for its edge enhancement characteristics with mammograms and is observed to enhance the calcifications, obscuring the background noise due to the breast tissue. The periphery of calcifications is more pronounced than that rendered by conventional edge detection filters. Also, the quadratic filter is observed to have an optimum computational complexity.

TABLE II: Comparison of SNR and PSNR for various filters

Filter	SNR(db)	PSNR(db)
Quadratic	19.14	25.4
LoG	10.63	16.9
Laplacian	0.76	2.02
Prewitt	0.11	3.67

TABLE III: Time of computation in seconds for various image filters

Image Size (Pixels)	Laplacian	LoG	Gabor	Quadratic (SVD)
1024×1024	5.38	0.09	0.24	2.40

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