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EVALUATION OF THERMODYNAMIC PARAMETERS OF THE ATMOSPHERE BY A FORTRAN PROGRAM

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Abstract—Thermodynamic parameters of the atmosphere form part of the input to numerical forecasting models. Usually these parameters are evaluated from a thermodynamic diagram. Here, a technique is developed to evaluate these parameters quickly and accurately using a Fortran program. This technique is tested with four sets of randomly selected data and the results are in agreement with the results from the conventional method. This technique is superior to the conventional method in three respects: more accuracy, less computation time, and evaluation of additional parameters. The computation time for all the parameters on a PC AT 286 machine is 11 sec. This software, with appropriate modifications, can be used for verifying various lines on a thermodynamic diagram. Copyright © 1996 Elsevier Science Ltd

Key Words: Thermodynamic parameters, LCL, CCL, LFC, Wet bulb temperature, Equivalent temperature, Saturated adiabat, Precipitable water, Geopotential thickness.

INTRODUCTION

Knowledge of atmospheric thermodynamic parameters is essential for meteorological forecasting. They are used to study the possibility for the formation of clouds and their vertical extent, convective activity of the atmosphere, amount of water vapor contained in an atmospheric column of unit cross section, and geopotential thickness between two pressure levels. The definitions of these parameters are available in standard meteorology text books (Hess, 1959; Wallace and Hobbs, 1977). The thermodynamic parameters of the atmosphere are evaluated by plotting vertical dry bulb and dew point temperature profiles on a thermodynamic diagram (McIntosh and Thom, 1973; Saucier, 1989). Many thermodynamic diagrams are employed for this purpose, viz: tephigram, emagram, skew *T*-log *p* diagram, Stuve diagram, etc. Since these have applications in numerical weather prediction models, a technique using a Fortran program to evaluate these parameters quickly and more accurately compared to the conventional method is presented here. An extensive discussion on the empirical relationships of Goff and Gratch (1946) and Tetens (1930) between saturation vapor pressure for water and temperature on the basis of accuracy and computation time was made by Murray (1967). After studying the relationships with different sets of data, Tetens' formula was used in the program. This Fortran program can be run on a PC with 256 KB memory. The parameters which can be evaluated by this technique are:

(1) Lifting Condensation Level (*LCL*),
(2) Convective Condensation Level (*CCL*),
(3) condensation temperature (T_c),
(4) equivalent potential temperature(Θ_e),
(5) equivalent temperature at the surface (T_e),
(6) level of Free Convection (*LFC*),
(7) height of cloud top,
(8) the specific humidity, geopotential height and dry bulb temperature of surface air parcel at various levels on the saturated adiabat,
(9) wet bulb temperature at the surface (T_w),
(10) wet bulb potential temperature (Θ_w),
(11) geopotential thickness, and
(12) water vapor content of the atmosphere.

DATA REQUIREMENTS AND METHODS

The inputs to the program are vertical profiles of dry bulb temperature and dew point temperature, and the height of the station above mean sea level.

Lifting condensation level

An air parcel lifted from the surface rises dry adiabatically, retaining the water vapor content until it is saturated. Hence the potential temperature and water vapor content of the air parcel are invariant up to the *LCL*. From Poisson's equation (Hess, 1959), potential temperature is given by

$$\Theta = T_1(1000/p_1)^{(R/cp)} \tag{1}$$

where T_1 is the temperature of the surface air in degrees Kelvin, p_1 is the surface pressure in mb, R is the specific gas constant of dry air, and cp is the specific heat of air at constant pressure, so that the temperature of the parcel at the *LCL*, *TLCL* is given by

$$TLCL = T_1(PLCL/p_1)^{R/cp} \tag{2}$$

where *PLCL* is pressure at the *LCL* in mb.

The specific humidity of surface air (q_1) can be determined from the relationship

$$q = 0.622e/(p - 0.378e) \tag{3}$$

where e is the actual vapor pressure for water in mb and p is the surface pressure. The vapor pressure, e in the program corresponding to the temperature, T is evaluated using the Tetens formula (Eq. 4) as given by the relationship

$$e = 6.11\exp[A(T - 273.16)/(T - B)] \tag{4}$$

where T is the dew point temperature of the parcel at the surface in degrees Kelvin and the constants are set as follows: $A = 21.87$ and $B = 7.66$ when $T < 263$, $A = 17.26$ and $B = 35.86$ when $T \geq 263$.

The specific humidity of the surface parcel gradually decreases beyond *LCL* due to condensation. The level at which the actual specific humidity of the parcel begins to decrease is the *LCL*. Using this criterion, *LCL* is now determined by an iterative solution of Equation (3) for a specific humidity of the atmosphere the same as the surface specific humidity. A pressure level (*PLCL*) above the *LCL* is chosen to begin the iteration. The result of iteration is pressure level of the *LCL* (*PLCL*) in mb is determined. The geopotential height of the *LCL* from mean sea level can also be determined. The dry adiabatic lapse rate, $-dT/dz = g/cp$. Assuming linear variation of T below *LCL*, the dry static energy equation is

$$g\,z_1 + cp\,T_1 = g\,z_2 + cp\,T_2 \tag{5}$$

where z_1 and z_2 are heights above mean sea level of the station and *LCL* respectively, T_1 and T_2 are temperatures of the parcel at the surface and *LCL* respectively, g is the acceleration due to gravity, and cp is specific heat for air at constant pressure. The dry static energy of the surface air parcel remains the same up to *LCL*. The geopotential height corresponding to *LCL* (z_2) can be evaluated by solving this equation.

Convective condensation level

This is the level at which the saturated specific humidity of the environmental air is equivalent to the actual specific humidity of the surface air. The actual specific humidity of the surface air is already determined by Equation (3). The saturation specific humidity at different levels can be found from temperature and pressure values by Equations (3) and (4). An iterative solution of Equations (3) and (4) of actual surface specific humidity for the saturated specific humidity along the dry bulb temperature profile evaluates the *CCL*. The geopotential height of the level can be determined also. The procedure will be explained later.

Condensation temperature

This is the temperature of air at the Convective Condensation Level when it is brought to the surface dry adiabatically and is computed from the value of pressure and temperature at the *CCL* by Equation (1).

Equivalent potential temperature, Θ_e and equivalent temperature at the surface, T_e

The equivalent potential temperature is obtained for Θ

$$C_p d\Theta/\Theta + C_w w_s dT/T + d(w_s L/T) = 0 \tag{6}$$

when Equation (6) is integrated from the lifting condensation level to a great height (Holton, 1992). As a simplification the middle term of Equation (6) usually is neglected giving the following approximate equation for Θ_e:

$$\Theta_e = \Theta\exp\{Lq/(cpT)\} \tag{7}$$

However, Bolton (1980) found that the error due to this approximation is more than 3°C and he suggested a formula (Eq. 8) for evaluating Θ_e to an accuracy of 0.0018°C.

$$\Theta_e = \Theta_{DL}\exp\{(3.036/T_L - 0.00178)$$
$$(1 + 0.448 \times 10^{-3}w)w\} \tag{8}$$

where T_L is dry bulb temperature at *LCL*, Θ_{DL} is potential temperature at *LCL*, and

$$\Theta_{DL} = T\{1000/(p - e)\}^{0.2854}(T/T_L)^{0.00028w}.$$

T, p, e, and w are dry bulb temperature, pressure, vapor pressure, and mixing ratio expressed in g kg^{-1} of the surface air, respectively.

In the program, Equation (8) used to evaluate Θ_e and T_e was obtained by applying Θ_e in Equation (1)

$$T_e = \Theta_e(p_1/1000)^{(R/cp)} \tag{9}$$

where p_1 is the surface pressure.

Level of free convection

The computation of this level is based on the principle that at this level equivalent potential temperature of the surface air and environmental air are the same. The equivalent potential temperature is obtained as explained in the previous section. An iterative solution of Equation (6) of Θ_e at the surface for the same value of Θ_e along the environmental profile evaluates the *LFC*. The computational procedure for the geopotential height of the level will be discussed later. If the surface Θ_e is smaller than the value of Θ_e throughout the dry bulb temperature profile, then it is a situation of no *LFC*.

Height of cloud top

This level is determined as the level at which the equivalent potential temperature of the surface air and environmental air are again the same above *LFC*. If this is a situation of no *LFC*, the question of top of cloud does not arise. The level in terms of pressure and geopotential height are computed.

Humidity, temperature, and height at different levels on the saturated adiabat

These parameters on a saturated adiabat can be evaluated using the moist static energy equation and the hydrostatic energy equation. On a saturated adiabat, moist static energy remains the same (Wallace and Hobbs, 1977).

$$\text{Moist static energy} = c_pT + gz + qL \qquad (10)$$

where c_p is specific heat of air at constant pressure, T is dry bulb temperature, g is acceleration due to gravity, z is geopotential height, q is specific humidity, and, L is latent heat of condensation of water vapor.

It may be noted here that the latent heat of condensation of water vapor varies with temperature. For accuracy, the variation of L with temperature was incorporated in the program by the equation (Gill, 1982)

$$L = 2.5008 \times 10^6 - 2.3 \times 10^3 t \qquad (11)$$

where L is in J kg^{-1} and t is in °C.

From the hydrostatic equation,

$$dp = \delta g dz \qquad (12)$$

that is $dz = dp/(\delta g)$, but from the gas equation, density, $\delta = p/(RT)$, that is $dz = dp\ RT\ /\ (gp)$ for dry air, modifying Equation (12) to account for water content in the layer using virtual temperature. This is given by the geopotential thickness,

$$dz = dp\ RT(1 + 1.609q)/(gp) \qquad (13)$$

where dp is the pressure difference of the layer, R is specific gas constant for dry air, T is the mean dry bulb temperature of the air in the layer, q is the mean specific humidity of the air in the layer, g is the acceleration due to gravity, and p is the mean pressure of the layer.

The moist static energy of the surface air can be computed based on the values at *LCL*. To start the computation of the parameters on the saturated adiabat at the new level, assume a temperature at this level which is slightly less than that at *LCL*. Specific humidity, q, can be obtained by Equations (3) and (4) from the pressure and temperature. The thickness between this level and *LCL* can be found by Equation (13) and the geopotential height of the new level is determined by adding this to the geopotential height of *LCL*. The moist static energy value at the new level can be obtained from the temperature, geopotential height and specific humidity by Equation (9). If this moist static energy value is not equal to that obtained at *LCL*, the temperature is modified. By iteration, a solution for temperature, height and humidity satisfying Equation (9) at the new level can be determined.

Wet bulb potential temperature, Θ_w and wet bulb temperature at the surface, T_w

Wet bulb potential temperature is the temperature of surface air parcel at 1000 mb level when it is brought to saturation adiabatically from *LCL*, and wet bulb potential temperature is the temperature on the saturated adiabat at the surface pressure. These temperatures can be evaluated at the respective pressures as explained in the previous section.

Geopotential thickness

The thickness between mean sea level and required pressure level is computed by dividing the atmosphere into different layer segments of convenient thickness. The mean values of dry bulb temperature and dew point temperature for the layer segments are obtained by interpolating the profiles linearly. The mean values of dew point temperature and pressure are applied in Equations (3) and (4) to get the specific humidity for the layer segment. From these values the thickness of the layer are obtained from Equation (13). The thickness of individual layers are then added up to obtain the required thickness.

Water vapor content of the atmosphere

The amount of water vapor contained in an atmospheric column of unit cross sectional area can be found by dividing the atmosphere into several segments.

From the definition of specific humidity, 1 kg of moist air contains q kg of water vapor. Using Equation (9), $dp = \delta\ g\ dz$ where δ is density of air, therefore

$$dp = gdzdM/dV$$

but

$$dV = 1 \times dz$$

since we consider an atmospheric column of unit

Table 1. Input data. Pressure, dry bulb temperature and dew point temperature at different levels

I	$P(I)$ (mb)	$T(I)$ (°C)	$TD(I)$ (°C)
1	1010.0	30.0	20.0
2	950.0	27.0	19.0
3	900.0	23.0	16.0
4	800.0	18.0	12.5
5	700.0	11.0	6.0
6	600.0	2.0	−3.7
7	500.0	−6.0	−13.0
8	400.0	−12.0	−20.0
Station height, z: 10.0 gpm			

No. of pressure levels in which moist q, T and, z are computed: 10, Pressure interval: 50 mb.
Accuracy of pressure: 0.1 mb, and accuracy of temperature: 0.1 K.

cross sectional area, that is

$$dp = gdM$$

$$dM = dp/g \qquad (14)$$

Therefore, an air column of unit cross sectional area and thickness dp contains $q \times dp/g$ kg of water vapor.

RESULTS AND COMPARISON WITH CONVENTIONAL METHOD

The program was tested with many datasets. The results from this method were compared with the results from the conventional method. Here, a tephigram of the India Meteorological Department (Ananthakrishnan and Yegnanarayanan, 1949; Pisharoty, 1961) was used to test the results. The input data (Table 1), results from the program (Table 2), and results from the tephigram analysis (Table 3) are given for one randomly selected test case.

The values of various parameters evaluated by the program agree with the values obtained from the tephigram analysis. However, the equivalent potential temperature evaluated by the numerical scheme does not agree with the value obtained from the tephigram. The error on the equivalent potential temperature value may be due to errors in the representation of saturated adiabats on the tephigram. The values computed from the numerical scheme are verified with the mathematical formulation cited by Krishnamurty (1986), Betts and Dugan (1973), Simpson (1978), and Wurtelec and Finke (1961) and are in good agreement. The computations of LFC, cloud top and humidity, temperature, and geopotential height at different levels on the saturated adiabat are based on the equivalent potential temperature and hence there are small differences in these values from the tephigram analysis.

CONCLUSIONS

Using this technique all the thermodynamic parameters of the atmosphere can be evaluated to the desired accuracy. This technique is superior to the conventional method in respect to computation time, accuracy and evaluation of additional parameters. The processing time for all these computations on a PC AT 286 machine is 11 sec with a

Table 2. Results from program

LCL = 873.7 mb, z = 1271.8 gpm, T = 290.8 K, iterations = 14
T_w = 296.2 K, i.e. 23.0°C, corresponding saturation q at 1010 mb = 0.01751
Θ_w = 296.0 K, i.e. 22.8°C, corresponding saturation q = 0.01748
T_e = 347.1 K, i.e. 73.9°C, Θ_e = 346.1 K, i.e. 72.9°C
CCL = 765.7 mb, i.e. 2368.3 gpm, T at CCL = 288.8 K, iterations = 16
Condensation temperature = 312.6 K
LFC = 678.4 mb i.e. 3414.6 gpm, temp at LFC = 282.2 K, iterations = 11
Cloud top = 446.8 mb, i.e. 6711.76 gpm, temp at 446.8 mb = 264.0 K, iterations = 11
Water vapour amount = 49.589 kg/sq. m, steps = 62
Thickness from m.s.l. to 400 mb = 7628.78 gpm, steps = 62
Dry bulb temperature, specific humidity, and geopotential height on the saturated adiabat at different pressure levels
1 p = 874 mb, T = 290.8 K, i.e. 17.7°C, q = 0.0145, z = 1271.8 gpm
2 p = 850 mb, T = 289.0 K, i.e. 15.8°C, q = 0.0133, z = 1763.9 gpm
3 p = 800 mb, T = 286.7 K, i.e. 13.6°C, q = 0.0123, z = 2279.2 gpm
4 p = 750 mb, T = 284.1 K, i.e. 10.9°C, q = 0.0110, z = 2822.8 gpm
5 p = 700 mb, T = 281.5 K, i.e. 8.3°C, q = 0.0099, z = 3398.1 gpm
6 p = 650 mb, T = 278.4 K, i.e. 5.3°C, q = 0.0086, z = 4009.4 gpm
7 p = 600 mb, T = 275.0 K, i.e. 1.9°C, q = 0.0073, z = 4661.5 gpm
8 p = 550 mb, T = 271.4 K, i.e. −1.7°C, q = 0.0061, z = 5360.6 gpm
9 p = 500 mb, T = 267.0 K, i.e. −6.1°C, q = 0.0048, z = 6114.4 gpm
10 p = 450 mb, T = 262.6 K, i.e. −10.5°C, q = 0.0034, z = 6933.3 gpm

Table 3. Results from the tephigram

LCL = 872.5 mb
T_w = 22.9°C, Θ_w = 22.75°C,
T_e = 78.8°C, Θ_e = 351 K,
CCL = 765 mb,
Condensation temperature = 39.5°C
LFC = 645 mb
Cloud top = 435 mb
Water vapour amount = 49.57 kg/sq. m,
Thickness from m.s.l. to 400 mb = 7633.8 gpm.

pressure level accuracy of 0.1 mb. The error limit in the iteration procedure can be adjusted by the user while running the program and hence the result can be obtained to the desired accuracy. This technique evaluates parameters such as *LCL*, *CCL*, *LFC*, cloud top in gpm, and temperatures at these levels in addition to parameters evaluated from a thermodynamic diagram. It also evaluates temperature, humidity, and geopotential height of surface air parcel at different levels along a saturated adiabat. The source code of the Fortran program given here can be run on any machine with a Microsoft Fortran compiler of version 3.0 or above. The source code is well documented and developed in separate subprograms for each parameter so as to enable the user to modify them accordingly. This software, with appropriate modifications, can be used for verifying curves on thermodynamic diagrams. The program is available for public access by anonymous FTP from the server IAMG.ORG.

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