

Creation and annihilation of fluxons in ac-driven semi-annular Josephson junction

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Abstract. The dynamical behavior of a fluxon in a semi-annular long Josephson junction in the presence of an ac-drive is studied. The non-uniformity due to the non uniform distribution of bias current is investigated. The oscillating potential is found to increase the depinning current. Finite difference method is used for numerical analysis and the response of the system to the ac-bias is studied. The creation and annihilation of fluxon is also demonstrated numerically for the first, second and third Zero-Field step cases.

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1. Introduction

A fluxon in long Josephson junctions (LJJ) is a well-known physical example of a sine-Gordon fluxon. Fluxons, endemic to LJJs, have been employed in the fabrication of devices like constant voltage standards [1, 2], flux flow oscillators [3, 4], logic gates [5, 6] and also in qubits [7, 8]. LJJs of various geometries have been thoroughly studied both experimentally and theoretically in the past. Fluxon dynamical properties like fluxon pinning [9], fluxon trapping [10], and phase locked states have been studied for rectangular [11, 12] and annular [13] LJJs.

The non rectangular Josephson junction has been in the focus of fluxon dynamics study in recent years because of the non uniformity caused by the shape. Semicircular geometry for Josephson junction has been proposed and fluxon dynamics has been studied both analytically and numerically and its various applications has been discussed [14]. It has been shown that in the presence of an external magnetic field applied parallel to the dielectric barrier of such a geometry, the ends of the junction has opposite polarities and because of that opposite polarity fluxons can enter the junction from the ends under a properly biased dc current. If the direction of the current is reversed, flux penetration and progression is not possible and flux free state exists in the junction. This unique phenomenon cannot be achieved in any other geometry and thus this junction behaves as a perfect diode. The effect of in-plane static and rf-magnetic field on fluxon dynamics in a semiannular Josephson junction has also been studied [15]. The response of a fluxon to an ac-drive has investigated by several authors. It was shown that in a system with periodic boundary condition average progressive motion of fluxon commences after the amplitude of the ac drive exceeds a certain threshold value [16]. Complex switching distributions has been obtained for ac-driven annular JJs and theoretical explanation has been provided for the multi-peaked experimental observations [17]. The behavior of fluxon under two ac forces has been studied and it was shown that the direction of motion of fluxon is dependent on ratio of frequencies, amplitudes and phases of the harmonic forces [18]. In this work, we study the effect of an ac-bias applied in the plane of a semi-annular Josephson junction. In section II we discuss the equation representing the junction and arrive at an expression for the potential of the junction. The numerical results are also presented. In section III we demonstrate creation and annihilation of fluxons in semiannular JJs in the presence of an ac bias and an external magnetic field. Section IV deals with the results and discussion.

2. Perturbation analysis of a fluxon in a semiannular junction

The dynamical equation for a semiannular LJJ in a harmonically oscillating field applied in its plane is

$$\varphi_{tt} - \varphi_{xx} + \sin \varphi = -\alpha\varphi_t + \beta\varphi_{xxt} - b \cos(kx) - \gamma + i_0 \sin(\omega t) \quad (1)$$

where $\varphi(x, t)$ is the superconducting phase difference between the electrodes of the junction with the spatial coordinate x normalized to λ_J , the Josephson penetration

depth and time t normalized to the inverse plasma frequency ω_0^{-1} and $\omega_0 = \frac{\tilde{c}}{\lambda_J}$, \tilde{c} being the maximum velocity of the electromagnetic waves in the junction. R is resistance per unit length, L_p is the inductance per unit length, C is the capacitance per unit length, and $\gamma = \frac{j}{j_0}$ is the normalized amplitude of a dc bias normalized to maximum Josephson current j_0 and $i_0 \sin(\omega t)$ is the applied ac biasing. α is the quasiparticle tunneling loss and β is the surface loss term in the electrodes and their values vary from 0.001 to 0.3 in experiments. The surface loss term is important and it will dominate fluxon propagation in some cases. The term $b \sin(kx)$ is due to the semicircular geometry of the junction and $k = \frac{\pi}{l}$ and $b = 2\pi\lambda_J \Delta B k / \Phi_0 = 2k(B/B_{c1})$, where $B_{c1} = \frac{\Phi_0}{\pi\Delta\lambda_J}$ is the first critical field of the Josephson junction. $\Phi_0 = \frac{h}{2e}$ is the flux quantum and its value is 2.064×10^{-15} . The extra term $b \sin(kx)$ corresponds to a force that drives fluxons towards the left and anti fluxon towards the right. Thus in the absence of an external field a flux free state will exist in the junction as any static trapped fluxon present in the junction will be removed [15]. In the absence of any perturbation (1) reduces to simple sine-Gordon equation with fluxon solution given by

$$\varphi(x, t) = 4 \tan^{-1} \left[\exp \frac{\sigma(x - X)}{\sqrt{1 - u^2}} \right] \quad (2)$$

where u is the velocity of the fluxon and $X = ut + x_0$ is the instantaneous location of the fluxon. $\sigma = \pm 1$ is the polarity of the flux quantum (which means there are two orientations for the fluxon). A quantum of flux in one direction is called the kink (fluxon) fluxon and that in other direction is called antikink fluxon (antifluxon).

2.1. Expression for potential function

The Lagrangian density of Eq. 1 with $\gamma = \alpha = i_0 = \beta = 0$ is

$$\mathbf{L} = \frac{1}{2} \varphi_t^2 - \frac{1}{2} \left(\varphi_x - \frac{b}{k} \sin(kx) \right)^2 - 1 + \cos \varphi \quad (3)$$

where the first term is the kinetic energy associated with the energy density of the electric field, the second term accounts for the potential energy density associated with the magnetic field and the third term represents the Josephson coupling energy density. From the potential energy density term, the change in potential energy due to the combined effect of fluxon motion and the applied field can be determined by integrating the term $-\frac{b}{k} \sin(kx) \varphi_x$ over the length of the junction [15]. The fluxon induced potential as a function of the fluxon coordinate X may be calculated as

$$U(X) = -\frac{b}{k} \int_{-\infty}^{\infty} \sin(kx) \varphi_x dx \quad (4)$$

The integration over $-\infty$ to ∞ may be justified as the length of the junction is very large as compared to the size of the fluxon. Substituting Eq. 2 in Eq. 4 and integrating we get the expression for potential as

$$U(X) = -2bl \sec h \left(\frac{\pi^2}{2l} \sqrt{1 - u^2} \right) \sin(kX) \quad (5)$$

For $u \simeq 0$ we can write

$$U(X) = -2bl \operatorname{sech} \left(\frac{\pi^2}{2l} \right) \sin(kX) \quad (6)$$

which has a potential well form with the depth of the well depending on b and l . The vortex will be pinned to the potential minima as long as the bias current is smaller than the depinning current. The pinned state of a vortex corresponds to a zero voltage state.

Now we arrive at an expression for the potential function of the perturbed system. The Hamiltonian of the system can be written as a combination of the Hamiltonian of the unperturbed sine Gordon part plus the hamiltonian of the perturbation part [11]. Energy of the unperturbed sine-Gordon system is

$$H^{SG} = \int_{-\infty}^{\infty} \left[\frac{1}{2} (\varphi_t^2 + \varphi_x^2 + 1 - \cos\varphi) \right] dx \quad (7)$$

Substituting(2) in (7) we get

$$\frac{d}{dt} H^{SG} = 8u(1-u^2)^{-3/2} \frac{du}{dt} \quad (8)$$

Due to the perturbational part, energy is dissipated and rate of dissipation is given as

$$\begin{aligned} \frac{d}{dt} (H^p) = & [\varphi_x \varphi_t]_{-\infty}^{\infty} - \\ & \int_{-\infty}^{\infty} (\alpha \varphi_t^2 + \beta \varphi_{xt}^2 + [b \cos(kx) + \gamma + i_0 \sin(\omega t)] \varphi_t) dx \end{aligned} \quad (9)$$

Here the first term on the right hand side accounts for the boundary conditions and vanishes. Substituting (2) in above equation we obtain the equation for rate of dissipation as

$$\begin{aligned} \frac{d}{dt} (H^p) = & 2\pi u (\gamma + i_0 \sin(\omega t)) - \frac{8\alpha u^2}{\sqrt{1-u^2}} \\ & - \frac{8\beta u^2}{3(1-u^2)^{3/2}} - 2\pi b u \operatorname{sech} \left(\frac{\pi^2 \sqrt{1-u^2}}{2l} \right) \cos(kX) \end{aligned} \quad (10)$$

Following the perturbation analysis we get

$$\begin{aligned} \frac{du}{dt} = & \frac{\pi}{4} (\gamma + i_0 \sin(\omega t)) (1-u^2)^{3/2} - \alpha u (1-u^2) \\ & - \frac{1}{3} \beta u - \frac{\pi}{4} b (1-u^2)^{3/2} \operatorname{sech} \left(\frac{\pi^2 \sqrt{1-u^2}}{2l} \right) \cos(kX) \end{aligned} \quad (11)$$

Eq. 11 describes the effect of perturbations on the vortex velocity. The first term represents the effect of applied biasing, the second and the third term represents dissipation and fourth term is the effect of the external magnetic field on the semicircular geometry.

In perturbational analysis, a vortex is considered as a non-relativistic particle of rest mass $m_0 = 8$ moving in one dimension. Therefore the effective potential can be obtained using the force relation

$$\frac{\partial U_{eff}}{\partial X} = -m_0 \frac{du}{dt} \quad (12)$$

Substituting Eq. 11 in Eq. 12 and integration yields to the expression for effective potential of the form

$$U_{eff}(X_0) = -2bl \sec h\left(\frac{\pi^2}{2l}\right) \sin(kX_0) - 2\pi(\gamma + i_0 \sin(\omega t))X_0 \quad (13)$$

The potential energy function $U_{eff}(X)$ has a well form in the absence of external biasing and the fluxon will remain pinned to the centre of the junction under such a potential. As the biasing is increased the potential gets tilted finally favoring the motion of the vortex. The dc bias at which the zero voltage switches to a finite voltage is called the depinning current. Fig. 1 shows the form of the potential for $b = 0.1$ for a junction of length $l = 15$ for different external biasing. It can be seen that in the presence of an external bias the potential gets tilted favoring motion of the fluxon. While moving through such a potential, the fluxon(antifluxon) after bouncing from the edge turns into an antifluxon (fluxon) and hence will move in the opposite direction. In the presence

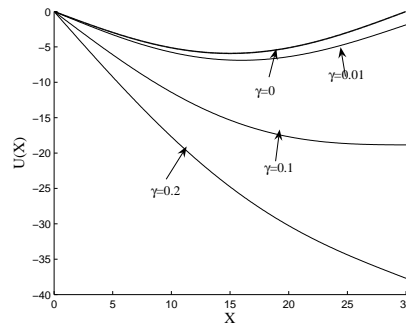


Figure 1. Potential well form for a JJ of length $l=15$. Other parameter values are $b = 0.1$, $i_0 = 0$. The γ value is increasing from top to bottom line.

of an ac, the potential gets oscillating with a frequency equal to the frequency of the applied field and the shape of the potential depends on the amplitude of the applied ac and dc biasing. In the presence of external ac biasing along with the dc, the potential gets time varying as shown in fig2. In Fig. 2(a), the applied ac has an amplitude of

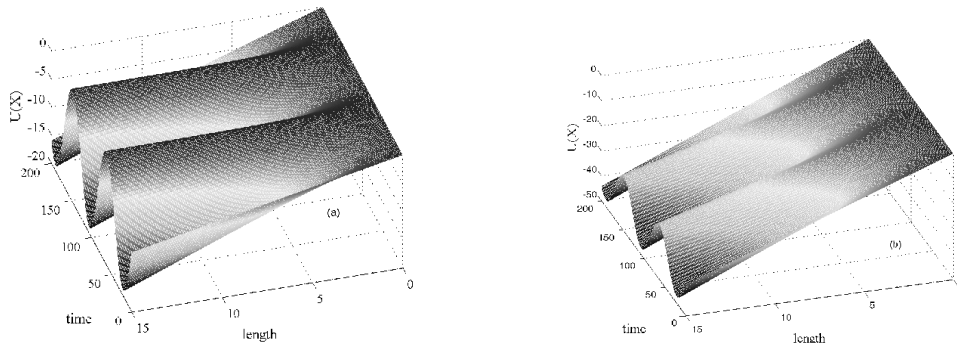


Figure 2. Form of the oscillating potential for a JJ of length $l=15$. Parameter values are $b = 0.1$, $i_0 = 0.2$, $\omega = 0.3$, $\gamma = 0.1$. a) applied dc bias is $\gamma = 0.1$ b) $\gamma = 0.4$

0.1 and it can be seen that as time goes on, the potential gets a well form in a time period equal to the period of oscillation of the applied field. The applied dc bias is 0.1 in this case. Hence progressive motion of fluxon does not occur for this value of biasing as the average velocity of a fluxon moving in such a potential would be zero. However as the dc-biasing is increased though the form of the potential is still oscillating, there is a definite tilt which makes progressive motion of fluxon possible. The response of the system to such a potential can be investigated by measuring the velocity of the fluxon in the potential. Thus when an ac biasing is applied in such a form to the semi-annular JJs, progressive motion occurs only when dc bias value exceeds certain threshold value.

2.2. Numerical Results

To solve Eq. 1 numerically, we use an explicit method treating ϕ_{xx} with a five point, ϕ_{tt} with a three point and ϕ_t with a two point finite difference method. The boundary conditions are treated by the introduction of imaginary points and the corresponding finite difference equation is solved using standard tridiagonal algorithm[19]. Numerical simulations are carried out on the JJ of normalized length ($l=15$). The time step was taken as 0.0125 and the space step was 0.025. The numerical results were checked by systematically halving and doubling the time steps and space steps. Details of the simulation can be obtained from [12, 14]. After the simulation of the phase dynamics for a transient time, we calculate the average voltage V for a time interval T to be

$$V = \frac{1}{T} \int_0^T \varphi_t dt = \frac{\varphi(T) - \varphi(0)}{T}$$

Also for the faster convergence of the averaging procedure, the phases $\varphi(x)$ in the equation were averaged over the length of the junction. The spatial averaging increases the accuracy in the calculation of the voltages in cases where the the time period over which integration is made is not an exact multiple of the time period of oscillation. Once the voltage averaging for a current γ is complete, it is increased in small steps of 0.01 to calculate the next point of the characteristic graph. The average velocity of the fluxons can be calculated from the average voltage using the relation $u = V(l/2\pi)$

Taking β to be zero, the velocity change with increase in dc biasing is observed. Fig.3 shows the velocity change with dc biasing for different values of amplitude of the ac biasing. In the presence of ac biasing the averaging interval T was taken as a multiple of the ac drive's period $2\pi/\omega$ [16]. If an ac-biasing is present, the depinning current is found to increase which can be seen from Fig.3. Constant voltage steps are observed for an ac-bias of amplitude 0.2 and 0.3.

In the presence of external magnetic fields, the velocity versus dc bias is shown in Fig. 4. The value of dc bias to cause a finite velocity for the fluxon in a JJ with a magnetic field of $b = 0.1$ and no ac-biasing is 0.1 while for $b = 0$ the depinning current is 0.04. Thus the external magnetic field also increases the depinning current value. The depinning current increases for higher values of damping parameter $\beta = 0.035$. The depinning current to be applied to the semi-annular JJ in the absence of ac, and

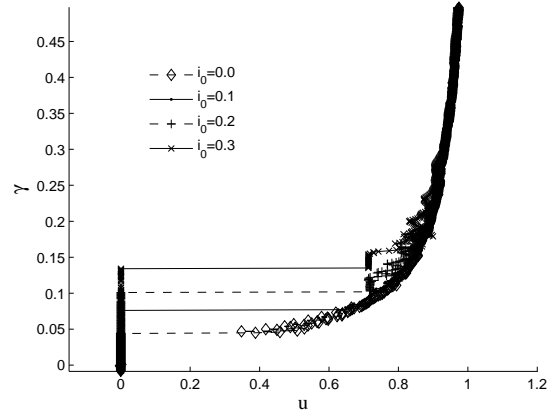


Figure 3. The velocity- bias characteristics of a LJJ of length $l=15$ with no external magnetic field applied. Other parameter values are $\omega = 0.3, \beta = 0.02, \alpha = 0.05$

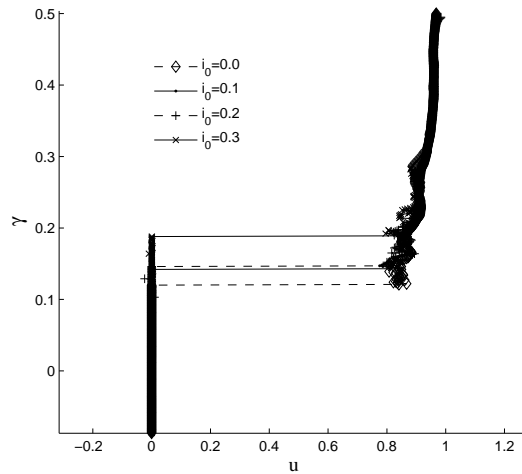


Figure 4. The velocity- bias characteristics of a LJJ of length $l=15$ in the presence of an external magnetic field $b = 0.1$.

a magnetic field of $b = 0.1$ is 0.125. Also for an ac bias of amplitude 0.2 and 0.3 the velocity shoots to a higher value even for a dc-biasing of 0.46 and 0.36. For dc bias values of more than these values, quasiperiodic or chaotic motion may exist in the system.

3. Creation and Annihilation of Fluxons

An annular LJJ preserves the number of trapped fluxons in it. However in an open ended geometry the number of fluxons is not a conserved quantity. In this section we investigate the creation and annihilation of fluxons in semiannular JJ with open boundary conditions in the presence of an external field and an ac and dc biasing.

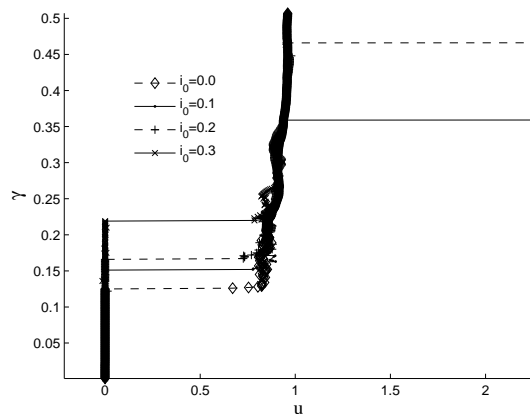


Figure 5. The velocity- bias characteristics of a LJJ of length $l=15$ in the presence of an external magnetic field $b = 0.1$. The damping parameter $\beta = 0.035$

3.1. First Fiske Step

The collision of fluxons with localized obstacles leads to creation and annihilation of fluxons. The fluxon creation and annihilation process for a single kink solution as input is described here. A kink solution is launched from the centre of the junction with an initial velocity of $v = 0.6$. For each value of biasing the fluxon is allowed to propagate for some time in order to stabilize its motion in the junction. The kink fluxon gets reflected from the boundaries and moves on till $\gamma = 0.57$. Above this biasing, no solitonic propagation is observed for an external magnetic field of strength 0.1.

However, the presence of an ac bias creation and annihilation of fluxon was observed for values of dc which gave one fluxon solution earlier. For a γ value of 0.1 the fluxon

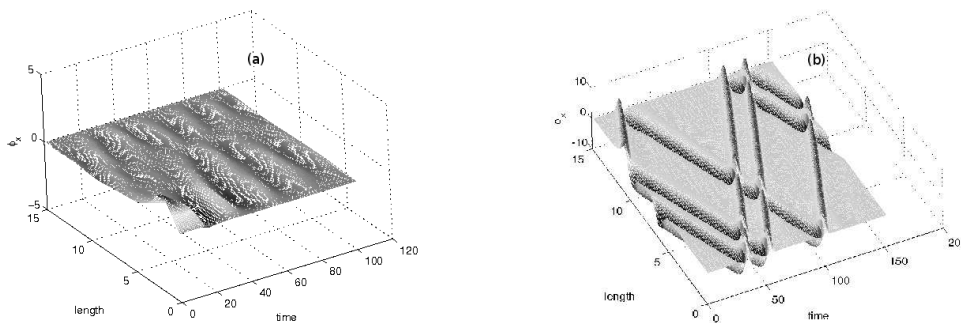


Figure 6. (a) The pattern shows annihilation of fluxon propagating in a JJ with $l=15$ for a dc bias of $\gamma = 0.1$ and $i_{ac} = 0.2$. (b) Creation of fluxon with $\gamma = 0.1$ and $i_{ac} = 0.1$. Other parameter values are $\omega = 0.3, \beta = 0.02, \alpha = 0.05, b = 0.1$

propagates through the semiannular junction, while an ac bias of 0.2 destroys the fluxon as can be seen from Fig. 6(a). Similarly Fig. 6(b) shows that a fluxon is created for $\gamma = 0.5$ and $i_{ac} = 0.1$.

3.2. Second Fiske Step

Two kink solutions were launched with at differ initial points in the junction with intial velocity $v = 0.6$. An dc bias of more than $\gamma = 0.1$ is needed to support motion of two fluxons in the junction. For $\gamma = 0.1$ it is observed that only a single fluxon propagates through the junction as can be seen from Fig. 7(a). A dc bias of $0.12 - 0.45$ supports two fluxon propagation in the junction in the absence of an ac biasing. However if an ac bias of 0.1 is applied along with $\gamma = 0.41$ creation of a fluxon occurs as shown in Fig. 7 (b).

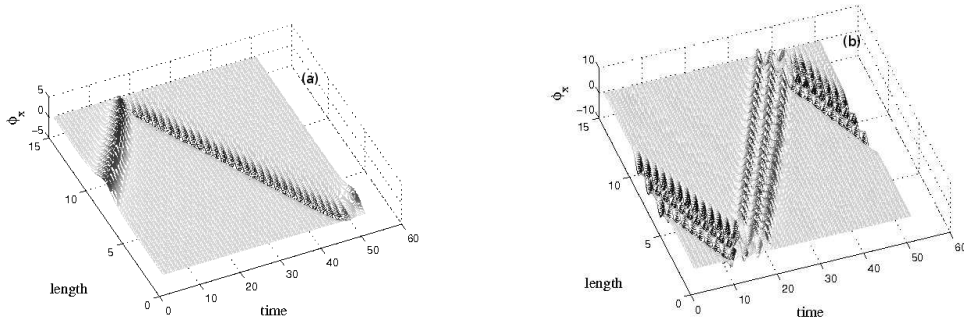


Figure 7. The pattern shows single fluxon propagating in a JJ with $l=15$ for $idc = 0.1$ for 2 fluxon input.(b)Creation of a fourth fluxon. $iac = 0.1, idc = 0.41$

3.3. Third Fiske Step

In this case, three kink fluxons are launched at different initial points. A dc bias of $\gamma = 0.4$ is needed to support the three fluxon propagation in the junction. In Fig.8(a) we numerically show that only two fluxons propagate through the junction for a γ value of 0.3 . Also for $\gamma = 0.4$, if an ac bias is applied annihilation of one fluxon occurs again giving the two solitonic propagation as shown in 8(a). The ac biasing causes annihilation and if the i_0 value is increased to 0.19 or more the solitonic profile is lost.

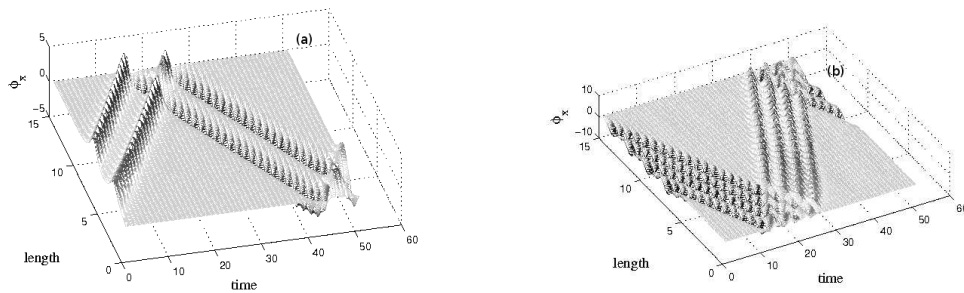


Figure 8. The pattern shows two fluxons propagating in a JJ with $l=15$. Other parameter values are $idc = 0.3$ (b)Creation of fourth fluxon $idc = 0.55$

Creation of fluxon is observed for γ values of 0.55 as shown in Fig. 8(b) with ac

biasing destroying the structure even for $i_0 = 0.1$. it is to be noted that all these effects takes place only in the presence of an external magnetic field in semiannular JJs. In the absence of magnetic fields, we were not able to observe creation and annihilation of fluxons.

4. Conclusions

We have studied the dynamics of a fluxon trapped in a semi annular JJ in the presence of an external magnetic field along with an ac biasing. This method of applying ac biasing offers a much easier and controllable way to induce a harmonic periodic modulation to the junction. In the presence of an external magnetic field the vortex remains pinned in the potential well. The ac biasing modulates the form of the potential and we obtain an oscillating potential with frequency of oscillation equal to the driving field. In the presence of an ac-drive and magnetic field , fluxon creation and annihilation phenomena is observed. This has been demonstrated for one, two and three fluxons and can be extended to higher number solutions. The fluxon creation and annihilation process being crucial for the understanding of the internal dynamics of the junctions, it will have important applications in design and fabrication of superconducting digital devices.

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