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Dynamical aspects of coupled Rossler systems: effects of noise

R. Pravitha *, P. Indic, V.P.N. Nampoori

International School of Photonics, Cochin University of Science and Technology, 682 022 India

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Abstract

Nonlinear time series analysis is employed to study the complex behaviour exhibited by a coupled pair of Rossler systems. Dimensional analysis with emphasis on the topological correlation dimension and the Kolmogorov entropy of the system is carried out in the coupling parameter space. The regime of phase synchronization is identified and the extent of synchronization between the systems constituting the coupled system is quantified by the phase synchronization index. The effect of noise on the coupling between the systems is also investigated. An exhaustive study of the topological, dynamical and synchronization properties of the nonlinear system under consideration in its characteristic parameter space is attempted.

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Keywords: Coupled oscillator; Correlation dimension; Kolmogorov entropy; Synchronization

1. Introduction

Collective dynamics displayed by arrays of oscillators is of great interest in numerous fields of science and technology. In the present study the complex behaviour exhibited by a pair of coupled Rossler systems is investigated. The study reveals several interesting results one of which is that an empirical relation apparently exists between parameters specifying the topology and those associated with the synchronization property of the coupled system. Also significant is the observation that the regime of synchronization is crucially dependent on the coupling factors of the system.

Interacting arrays of oscillators have innumerable applications in diverse fields of science and technology including chemistry [1], electronics [2,3], in engineering in the form of multimode laser arrays [4], in secure communications [5] and so on. Arrays of oscillators can also simulate the biological rhythms of the heart [6], the intestines, pancreas and other biological systems [7]. In most of the studies of coupled arrays of oscillators, however, the invariant parameters of the system including the generalized dimensions D_q or entropies K_q nor the synchronization properties have been sufficiently focused on to derive the wealth of information regarding the system contained in these. The present work investigates the phenomenon of phase synchronization or phase locking

* Corresponding author.
E-mail address: pravitha@cusat.ac.in (R. Pravitha).

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as a function of the coupling constants in the coupled system. If we now assume the behaviour of a group of neurons in the brain to be represented by an array of nonlinear oscillators then the phase locking could be identified as a pattern formation and that a persistent phase locking could be understood as a long lived pattern which ultimately results in the memory formations [8]. It has been shown that the formation of a persistent pattern can be identified as the learning process and the ultimate formation of a concept.

The basis of nonlinear analysis of chaotic systems is the computation of invariant parameters of the attractor of the system in its characteristic phase space. The most widely studied topological parameters are the correlation dimension D_2 and the Kolmogorov entropy, K_2. D_2 refers to the degrees of freedom associated with the system while the entropy is a measure of the information that is lost during evolution of the system at a given instant [9]. Of late, besides the topological parameters, another set of characterizing parameters which deal with the synchronization phenomenon have been of importance [10,12,14] in the case of coupled systems. The current interest revolves over the synchronization resulting from the cooperative behaviour of oscillatory systems [10,11]. Synchronization may in general be divided into a number of classes. The one in which even while the dynamics in time as represented by the amplitude of the measured signal remains chaotic, the states of the different systems of the complex system coincide due to interaction. This is termed as 'complete synchronization' of the chaotic oscillators [12]. Another approach is based on the overlap of power spectra of the signals from the interacting systems [13]; an alternate recent method is the 'phase synchronization' of chaotic systems which studies the correlation between the phases of individual systems and an eventual phase locking that leads to synchronization between them [14].

In the present work, the technique of nonlinear time series analysis is applied to a pair of coupled Rossler oscillators [15] in the chaotic regime to study the dynamical behaviour inherent in such a system. The system behaviour when the oscillators are coupled unidirectionally is inspected first and then the case of mutual or bi-directional coupling is studied. The coupled system in the case of uni-directional coupling contains a coupling parameter, g, while there is a pair of coupling factors g_1 and g_2 in the mutually coupled scenario. The parameter g_1, which forms the feed forward coupling, may be assumed to be an internal parameter of the system while g_2 the feedback coupling factor is considered as the control parameter. In the parameter space of coupling factors the system dynamics is studied by evaluating the correlation dimension and Kolmogorov entropy. The synchronization existing between the subsystems is a measure of the coordination present in the system. The phase synchronization phenomenon exhibited by the system is studied in detail and the empirical relation that may hold between the parameters that characterize the dynamical and interactive behaviour of the coupled system with respect to the coupling parameters is inferred. As a final objective, effects of the presence of noise in the coupled system are investigated with regard to the synchronization phenomenon.

2. Model system and method of analysis

The system under consideration is the coupled Rossler system [16] described by the flow equation

$$
\begin{aligned}
&\alpha \dot{x}_i = a + x_i(y_i - \mu),\\
&\alpha \dot{y}_i = -x_i - \omega_i z_i,\\
&\alpha \dot{z}_i = \omega_i y_i + b z_i + g_j(z_j - z_i),
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
&i, j = 1, 2; \quad i \neq j,\\
&\omega_i = 1, \quad i = 1,\\
&\omega_i = 1.1, \quad i = 2,\\
&\alpha = 0.013, \qquad a = 0.2, \qquad b = 0.412,\\
&\mu = 5.7.
\end{aligned}
$$

This nonlinear set of equations exhibits chaotic behaviour for the choice of parameters made in this study. The mutually coupled Rossler systems exist as independent systems in the chaotic regime in the uncoupled state.

In the initial case of uni-directional coupling, only the feed forward coupling is present and the systems form a drive–response set and hence we have $g_2 = 0$ and $g_1 = g$ as the sole coupling factor. While considering bi-directional coupling, both the coupling parameters come into play. The equations are integrated by the fourth-order Runge–Kutta method with a step of 0.0005. The variables y_1 and y_2 are generated as time series from the first and second systems for varying values of coupling parameters g_i. It is well known that any single parameter in a coupled set of equations ($i = 1$ or $i = 2$) would represent the entire characteristics of the whole system. That is the reason why we choose y_1 and y_2 for the two distinctly different oscillations. For convenience, we use the parlance of uni-directionally coupled systems and name system 1 as the drive and system 2 as the response bearing in mind that in the study of the bi-directional case, the two systems are mutually coupled and each acts as a response to the drive from the other. The generated time series after removal of transients are subjected to a time delay embedding [17] and the technique of nonlinear time series analysis is employed to evaluate the relevant invariant parameters of the subsystems. The invariant parameters included in this study are the correlation dimension D_2 that characterizes the dimensionality of the attractor in the system and the Kolmogorov entropy K_2, the dynamical parameter [17]. D_2 and K_2 of the systems are determined by adopting the Grassberger–Procaccia algorithm [18].

In order to understand the coordination existing between the different units of the composite system, their synchronization is studied. Phase synchronization in the system may be defined as the appearance of phase locking of interacting units even in the absence of apparent amplitude entrainment [14]. The definition of the phase of a chaotic system remains an ambiguous concept to date. Nevertheless, following Rosenblum et al., an analytic signal concept is made use of in this case and the composite signal is taken as

$$\Psi(t) = s(t) + i\tilde{s}(t), \tag{2}$$

where $s(t)$ is the actual signal and $\tilde{s}(t)$ is the Hilbert transform of $s(t)$ given as

$$\tilde{s}(t) = \text{P.V.}\,\frac{1}{\pi}\int_0^\infty \frac{s(\tau)}{t-\tau}\,d\tau, \tag{3}$$

where P.V. denotes the Cauchy principal value of the integral.

Based on this, the phase is defined as

$$\phi(t) = \arctan\left(\frac{\tilde{s}(t)}{s(t)}\right). \tag{4}$$

A pair of chaotic systems (i, j) is said to be phase locked in the ratio $n : m$ if at any instant,

$$\left|n\phi_i(t) - m\phi_j(t)\right| < \varepsilon, \tag{5}$$

ε being a small arbitrary constant. The $1 : 1$ case of phase synchronization is investigated by considering the relative phase distribution of a pair of corresponding variables of the system. The phase synchronization index is defined as

$$\text{Sp} = \frac{S_{\max} - S}{S_{\max}}, \tag{6}$$

where

$$S = -\sum_{i=1}^{N_b} p_i \ln p_i, \tag{7}$$

in which $p_i = N_i/N$ is the probability of occupancy of the ith bin with N_i as the number of points in it and N the total number of points. Hence we may write $S_{\max} = \ln N_b$ as the normalizing factor [19].

3. Analysis based on topological and synchronization invariants

3.1. The uni-directionally coupled system

Two independent Rossler oscillators are coupled so that there is a drive provided from the first to the second system through a coupling as described in (1). The strength of coupling is represented by the coupling constant g, which is scanned through a range of values and the corresponding system behaviour contained in the invariant parameters D_2, K_2 and Sp is studied.

In determining the parameters, we have, as mentioned before adopted a time delay embedding technique and used the Grassberger–Procaccia algorithm. In the time delay embedding undertaken here, the embedding dimension was chosen on the basis of the method of false nearest neighbours [17]. The average mutual information criterion [20] was used in the choice of the appropriate time delay, which was found to hold in each case of coupling. A modified version of the Grassberger–Procaccia algorithm [21] was employed for the determination of D_2 and K_2 for the systems. The coordination between the coupled systems

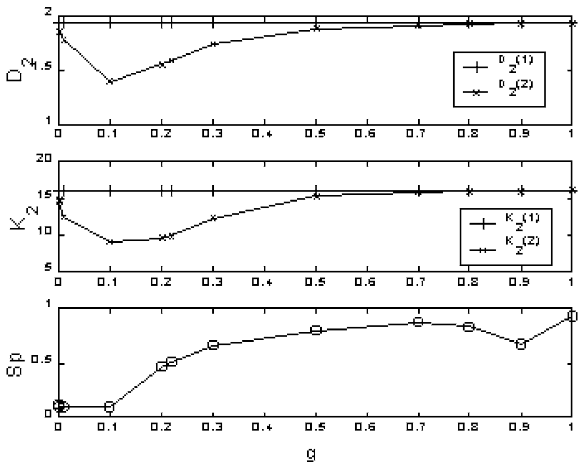


Fig. 1. The variations in D_2, K_2 and Sp with the coupling factor g for a uni-directionally coupled Rossler system. 1 refers to the drive system and 2 to the response.

is reflected in the phase synchronization existing between them. This aspect of the coupled system is investigated and the synchronization property of the system is quantified by a phase synchronization index, Sp, as defined in (6).

The correlation dimension for the independent Rossler system is determined to be 1.93. This is in general agreement with the value determined in [22] as 1.89 and the small discrepancy in the estimated and reported values of D_2 arises from the finite length of the data set used in the evaluation of D_2. Fig. 1 gives the variations in correlation dimension, Kolmogorov entropy and phase synchronization factor for the different coupling values. The D_2 for the drive system remains at a fixed value while that for the response varies as a consequence of the coupling between the two systems. For values of low feedback, $g < 0.5$, D_2 for the response system varies almost randomly but beyond a critical feedback value D_2 for the drive and response systems appear to coincide. A close look at the plot of Sp vs. g reveals that this phenomenon occurs at the value of g where the subsystems begin to synchronize. This establishes the underlying fact that synchronization between the systems translated into attractor space, is a convergence between the individual attractors of the subsystems.

The Kolmogorov entropy, K_2, exhibits a random variation in the response system initially but beyond a certain coupling it becomes identical with the K_2 value of the drive system. This phenomenon seems to suggest that flow of information from the coupled systems occurs in an identical manner.

3.2. *The bi-directionally coupled system*

In the study concerning system dynamics of a pair of bi-directionally coupled oscillators, the coupling parameter g_1 is kept fixed while the feedback factor g_2 is scanned through a range of values corresponding to very weak feedback from the response to the drive to that giving complete feedback, i.e., $g_2 \sim 0.1$–100% of magnitude. The whole procedure is repeated after raising g_1 to the next higher order of magnitude. The system behaviour in the parameter space of (g_1, g_2) with g_1 varying from 0.0001 to 1 is studied by analyzing the variations in D_2, K_2 and Sp of the subsystems. The two systems are embedded in their respective phase spaces and the topological parameters corresponding to each of the systems is determined in these spaces. In general, once the systems are synchronized, any one of these spaces effectively acts as the phase space for the coupled system [23]. However, in this case, we are focusing on studying the evolution of the subsystems in an independent manner, under the effect of coupling. Hence, with an embedding dimension of 5, chosen on the basis of the false nearest neighbourhood criterion [17], each of the subsystems is embedded in the space of lagged coordinates and D_2 and K_2 are evaluated.

Fig. 2 depicts the variations in D_2, K_2 of the constituent systems as well as changes in Sp of the composite system as g_2 is varied from 0 to 1 for fixed values of g_1. In Fig. 2, the g_1 value increases steadily form 0.0001 to 1. However, there is great deal of parallelism in the behaviour beyond a threshold g_2 value of 0.5. The phase synchronization index, Sp becomes saturated to a value around 0.9 and the saturation starts for $g_2 \sim 0.5$. This implies that the systems get synchronized almost perfectly beyond $g_2 \sim 0.5$. Further, it may be noticed that as the value of forward coupling g_1 is steadily increased to higher orders of magnitude, the parallelism between the constituent systems sets in at earlier values of threshold feedback coupling. This effect is noticeable when compared with the uni-directional case as well in which synchronization is decided by a single coupling factor. This is the cause for the delay in the onset of synchronization in the

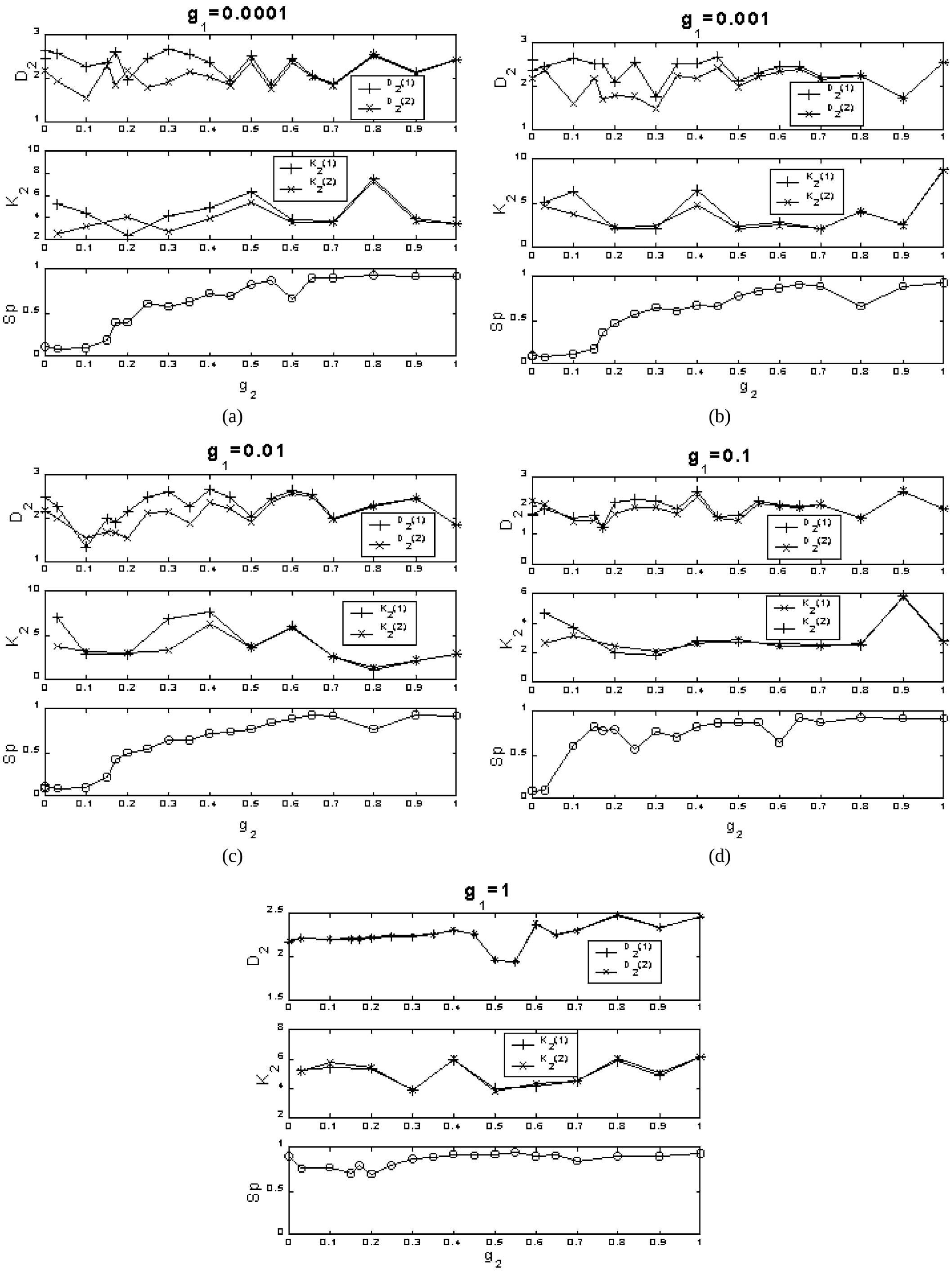


Fig. 2. The plots exhibiting the variations in correlation dimension D_2, Kolmogorov entropy K_2 and phase synchronization index Sp with the feedback fraction represented by g_2 for a mutually coupled system with gradually increasing feed forward coupling, g_1. (a) $g_1 = 0.0001$, (b) $g_1 = 0.001$, (c) $g_1 = 0.01$, (d) $g_1 = 0.1$ and (e) $g_1 = 1$.

uni-directional case. Moreover, once synchronization is achieved the information loss seems to be identical for both systems. Hence, the crucial factor in a mutually coupled set of Rossler oscillators, the synchronization, is mainly determined by the feed forward coupling percentage and the onset value of the synchronization phenomenon gets reduced as the feed forward parameter is increased. The individuality of the attractors is increasingly lost as this parameter is steadily increased. The lower values of D_2 imply that the coupling in effect brings down the number of independent variables required to modulate the dynamics of the system. In the attractor space, the variation in D_2 may be pictured as an expansion or shrinkage of the original attractor as the control parameters vary. In general, weak coupling tends to lower the dimensionality of the system.

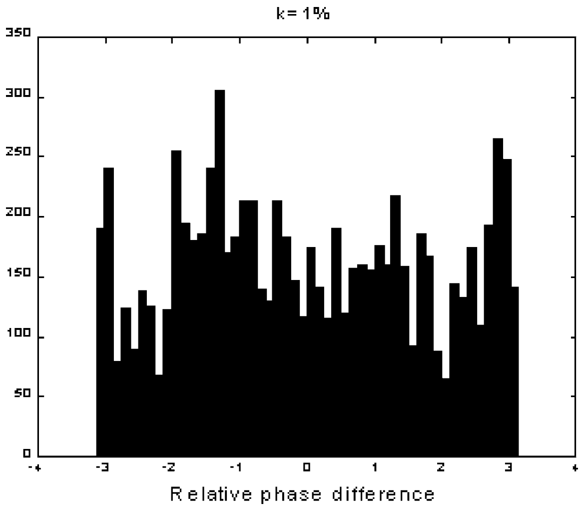
The point of interest is that while in cases with $g_1 \sim 0.001$, while Sp becomes 0.6–0.7 at $0.3 \leqslant g_2 \leqslant 0.4$, D_2 and K_2 values of the drive and the response coincide only from about $g_2 \sim 0.7$. This also corresponds to the region of strong phase synchronization as indicated by high Sp value ∼0.8–0.9. At higher $g_1 = 0.1$, Sp ~ 0.8 at $g_2 = 0.4$ beyond which the variations in the correlation dimension and Kolmogorov entropy for the two systems follow identical variations. Thus the coincidence of the topological parameters for the coupled oscillators as well as strong phase entrainment between them sets in simultaneously at critical values of the coupling parameters. While phase synchronization by itself does not imply coincidence of topological parameters, at sufficient values of coupling, strong phase synchronization is followed by an identical behaviour of the D_2 and K_2 parameters of the coupled subsystems.

3.3. Effect of noise on the behavior of the coupled Rossler system

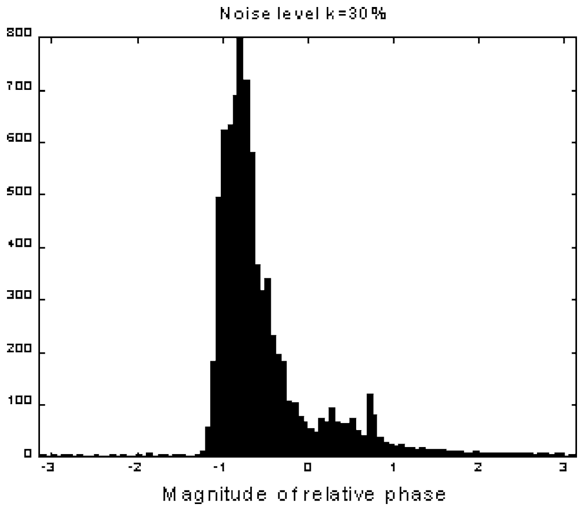
The study, as of now, has been concentrated on deriving the dynamical aspects of a coupled Rossler system in its characteristic coupling parameter space depending on whether it is a uni-directionally or mutually coupled system. Taking into account the fuzzy nature of coupling factors, we incorporate noise at specified levels into the coupling. The effects on the behaviour caused by the randomness due to the presence of noise in the system are analyzed. In this study, noise at a pre-determined level, represented by k is introduced into the system in the form of an additive term to the coupling factor g in the uni-directional case. The effective coupling in such a situation is $g + kr_n$, wherein r_n is the noise added to the system at a percentage level k. An identical noise factor is added to the feedback coupling, g_2, for the mutually coupled system.

We have considered for analysis three types of noise—normally distributed noise (Nml.), uniformly distributed noise of mean 0 (Unif.0.) and, as a special case, uniformly distributed noise of finite mean (Unif.). The system response in each case as contained in the synchronization is studied.

The normally distributed noise with zero mean and unit variance when introduced into the coupled system is found not to affect the behaviour to any great extent. To undertake a comparative study between the noise free system and that with noise introduced, we arbitrarily choose the threshold of phase synchronization to be at Sp $= 0.5$. With this criterion, we may say phase synchronization sets in at the coupling factor g or, correspondingly, at that g_2 for specified g_1 in bi-directionally coupled case at which Sp tends to 0.5. Moreover, in the evaluation of phase synchronization, we have taken into account the random phase slips that may be introduced by the strong noise added to the system. We determine the distribution of the relative phase as in [24,25] and find that in cases where there is no phase entrainment between the systems the distribution is flat while phase entrainment gives rise to a unimodal distribution. Fig. 3 exhibits plots of the relative phase distribution in the bi-directionally coupled system at a noise level of 1% that produces no apparent phase entrainment at low coupling parameters as compared to that at a high noise level of 30% that gives rise to significant phase entrainment even when the systems are very weakly coupled. With a noise level of $k = 70\%$, normally distributed noise causes the uni-directionally coupled systems to synchronize around $g = 0.25$ which is the g value at which the noise free system synchronizes as well. At lower values of coupling, the noise does not cause any significant change in the system behaviour. Identical behaviour is observed in the presence of uniformly distributed noise with zero mean. Fig. 4 gives the plot of Sp vs. g for the uni-directional system with various types of noise added at the constant level of $k = 70\%$. The plot reveals the interesting phenomenon of syn-



(a)



(b)

Fig. 3. The distribution of the relative phase difference in the weakly coupled system with $g_1 = 0.0001$ and $g_2 = 0.001$. The additive noise of finite mean is added to a level of (a) 1% and (b) 30%.

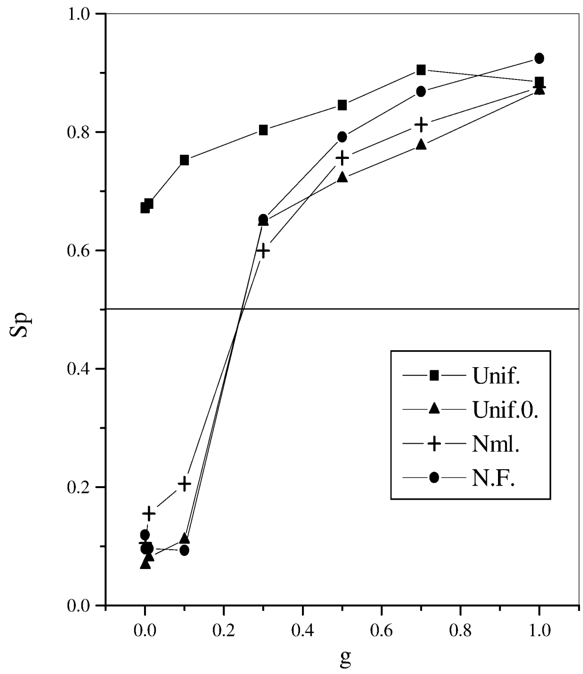


Fig. 4. The synchronization, Sp, versus coupling, g, for the uni-directionally coupled system with various types of additive noise in the coupling. The noise level, k, is 70% in each case. In the figure, N.F. refers to noise free case, Unif. to additive uniform noise of finite mean, Unif.0. to additive uniform noise of zero mean and Nml. to normally distributed noise present in the system.

chronization at low coupling values in the system induced when uniformly distributed noise of finite mean is added to it. It is observed that Sp $\sim$ 0.65 occurs between the drive and response systems even when the coupling between them is as low as 0.001. However, once the systems synchronize, i.e., Sp beyond 0.5, the noise seems to play no significant part in the system dynamics. The behaviour exhibited by the system in the absence and presence of noise beyond the onset of synchronization is almost identical.

Fig. 5 gives the synchronization values against the coupling for the mutually coupled system for a typical case with feed forward coupling $g_1 = 0.1$. In this case, too, the uniform noise of finite mean causes the systems to synchronize even at weak coupling levels. A probable reason for this is that at high enough noise levels k, the coupling g gets added to the mean noise level to form a higher coupling in general, at which the systems are in synchrony in the noise free state. This is substantiated in Fig. 7, which shows the behaviour of the system in the presence of uniform noise of finite mean at different noise levels. It is clearly seen that the system synchronizes sooner at higher noise levels and the noise level required to produce synchronization is lower if the system synchronizes faster in the noise free case itself. Nevertheless, the presence of phase synchronization by itself does not ensure that the dynamical parameters of the subsystems are coin-

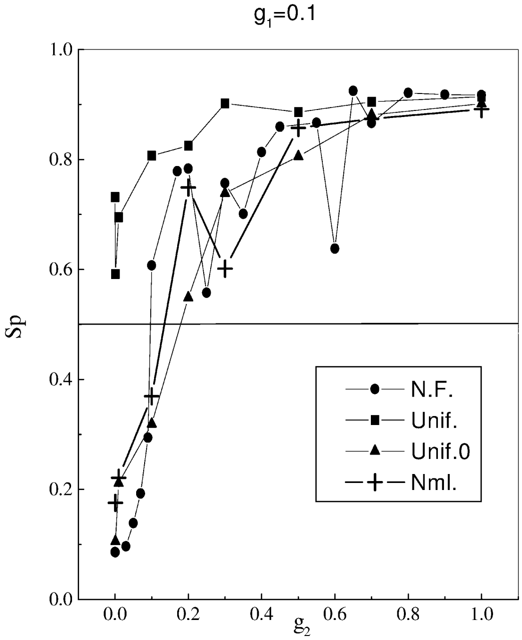


Fig. 5. The effect of different types of noise at level $k = 70\%$ in a mutually coupled Rossler system on the synchronization as a function of the feedback coupling, g_2, for a typical case of forward coupling, $g_1 = 0.1$.

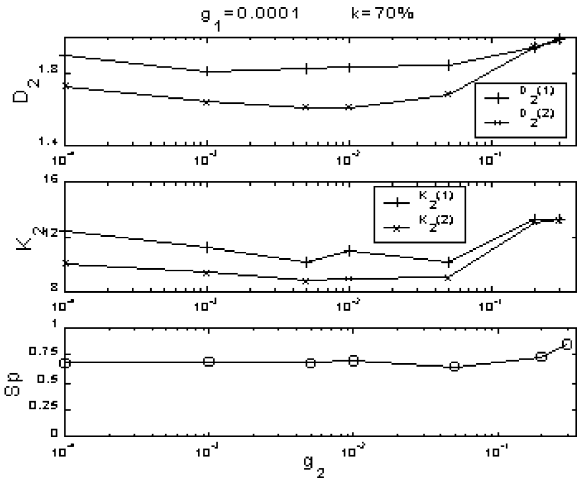


Fig. 6. The D_2, K_2 and Sp variations for the case of coupled system with $g_1 = 0.0001$ and with additive noise at $k = 70\%$. The noise added is uniform with finite mean, Unif.

cident. This is still critically dependent on the coupling as evidenced in Fig. 6. Fig. 7(a) plots the variation of synchronization for very weak forward coupling, $g_1 = 0.0001$, while Fig. 7(b) corresponds to a moderately stronger coupling, $g_1 = 0.01$. It is clear that in the latter case, since synchronization occurs sooner in the noise free state, a noise level of 30% is sufficient to induce synchronization in it as compared to the case with lower feed forward coupling.

On the whole, the coupled Rossler system appears to be unaffected by noisy fluctuations in coupling. The normal and uniform noise with zero mean seem to produce no ill effects in the system with respect to the synchronization. Besides, uniformly distributed noise with finite mean apparently aids the system to synchronize at weak coupling.

4. Discussion

In this Letter, we have considered the effects of coupling on dimension and coordination of a pair of coupled Rossler oscillators. Coupling two independent systems produces a variety of phenomena that throw light on the underlying dynamics of the system. The coupled systems are termed the drive and response and coupling is applied between the two systems through suitable factors. The correlation dimension (D_2) estimation reveals the variation of the corresponding parameter for the two systems in a random manner about the individual system D_2 value. Weak coupling in general lowers the computed correlation dimension D_2 of the response and drive systems in a bi-directionally coupled scenario; thus minimizing the number of degrees of freedom required in specifying it. In the case of the neural system, millions of neurons interact in a complex manner at any instant to generate an output. In spite of there being innumerable variables in the system, a dimensional analysis reveals the dimensionality of the brain to be around 8–12 [25,26]. This agrees with the result obtained in the present case of the coupled nonlinear oscillator that shows a lower dimensionality. This observation throws light on the possibility of modeling complex systems like the neural system as a network of coupled nonlinear oscillators. The main interest is in the synchronization effect produced by the coupling. For a particular range of coupling parameters, we have identified a regime wherein the phases are entrained strongly. The strengths of coupling decide the point at which the systems lose their independent nature and synchronize. The onset

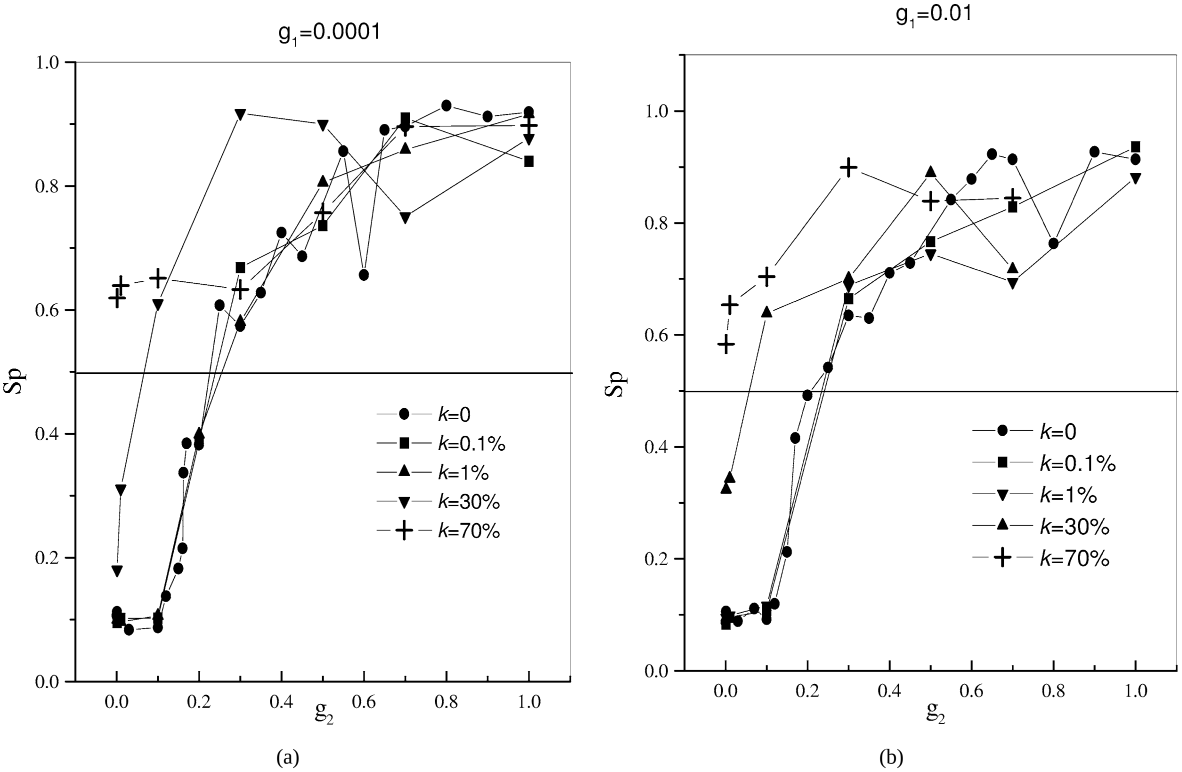


Fig. 7. The dependence of phase synchronization on the coupling factor g_2 in the presence of various noise levels, k, in the case of uniformly distributed noise with finite mean for the mutually coupled systems at two feed forward coupling values. (a) $g_1 = 0.0001$ and (b) $g_1 = 0.01$.

of strong synchronization between the coupled units is translated into a coincidence of the invariant parameter D_2 of the individual systems. Once the systems synchronize, we may infer an overlap of the individual attractors with respect to the D_2 and K_2 parameters. The existence of a mathematical relation between the topological invariant D_2 of the subsystems and the phase synchronization index, Sp, that represents the coordination of the global systems suggests the underlying interdependence of these seemingly different aspects of the coupled systems. The time series data is only a manifestation of a multitude of interdependent factors giving rise to it. It has been proposed that interaction in the human brain generates a large number of attractors specified by various characteristic parameters such as embedding dimension ε_i, attractor basin B_i, generalized metric dimension D_{iq}, generalized entropy K_{iq} and Lyapunov constant λ_{iq}/Lyapunov function [26], where i is the attractor index and q the generalization index. This is a pointer to the fact that the interconnections between the neurons may be likened to a set of coupled nonlinear oscillators and the behaviour may be modeled based on these. The system behaviour in the presence of noise suggests that the system is robust against the randomness that may be produced by most types of noise with a flat spectrum. However, the system responds favourably to uniform noise with a finite mean in the sense that the noise induces the coupled systems to synchronize even with weak coupling. This does not mean a total coincidence of the topological parameters of the individual systems at very weak coupling. The coupled subsystems essentially act as two independent systems albeit with a phase entrainment. We propose to carry out a similar analysis with stress on the amplitude synchronization and this in turn would give the attractor locking due to the degree of coherence. We also propose to continue this with an array of van der Pol oscillators and then make a detailed study of the merits and demerits of the different systems. This will be further applied to pattern formation in brain due to response from an input stimulus.

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