S.m.2. MERCY K JACOB.–A study of Discrete Pseudo Analytic Functions–1983–Dr. Wazir Hasan Abdi and Dr. T. Thrivikraman

This thesis is a study of discrete pseudoanalytic functions defined on the lattice $\{(q^m x_0, q^n y_0) / m,n \in Z\}$ where (x_0, y_0) is a fixed point in C and q a fixed number in (0,1).

The theory of discrete functions had its start from R.P. Isaac's work (1941). Isaac and people like Ferrand, Duffin and Abdullaev developed the theory mainly on the Gaussian lattice. q-difference theory was developed by Jackson, Hahn and Abdi and using this theory C. Harman (1972) developed a discrete analytic function theory on the lattice $H = \{(q^m x_0, q^n y_0)\}$.

The theory of pseudoanalytic functions is a generalisation of the theory of analytic functions. When the generator becomes the identity ie. (1,i) the theory of pseudoanalytic function reduces to the theory of analytic functions. This thesis develops a discrete analogue of this theory. In the first chapter of the thesis, an outline of the theory of pseudoanalytic functions in the classical continuous case and also a survey of the discrete function theory are given.

The second chapter deals with the definition of Holder-type discrete functions and generating vecotrs. Their properties have been examined. Using q-difference equations modulo g, whre g is a generating vector, definitions of discrete g-pseudoanalytic functions of the first and second kind are given and their properties studied. We denote the class of all discrete g-pseudoanalytic functions of the first kind in a discrete domain D by $P_D(g)$ and that of the second kind by $P_D(g)$. The real and imaginary parts of the elements of $P_D(g)$ satisfy a linear elliptic system of partial q-differenc equations of the second order with Holder-type coefficients.

Concepts of g and p_g-integration in the discrete system are introduced and their properties examined. In chapter 3, it is established that the g-integral of a discrete function is an element of $P_D(g)$ and p_g-integral of a discrete function is an element of $P_D(g)$.

Solutions of partial q-difference equations modulo-g and an analogue of Beltrami's equations are discussed. Properties of solutions thus obtained are established through examples in the fourth chapter.

The discrete g-derivative of an element of ${}_1P_0(g)$ is not in general an element of ${}_1P_0(g)$. However, there does exist a generating vector $g^{(1)}$ such that the discrete g-derivative is an element of ${}_1P_0(g^{(1)})$. We call $g^{(1)}$, a successor of g and g a predecessor of $g^{(1)}$. It is shown that if $g = [g_1\ g_2]$ then $[g_1/i\ g_2/i]$ is a successor of g. Also any generative vector equivalent to $[g_1/i\ g_2/i]$ is also a successor of g. We have discussed the concept of a generating sequence and the periodicity of the generating sequence. It is established that if $w \in {}_1P_0(g)$ is not a g-pseudoconstant then g can be embedded in a generating sequence of minimal period one if and only if the first component of the generating vector is equal to the product of the second component and a function of y alone. It is also established that any generating vector g can be embedded in a generating sequence of minimal period 2.

A product of two elements of ${}_1P_0(g)$ is not in general an element of ${}_1P_0(g)$. In the last chapter of the thesis, some sufficient conditions under which w^2, aw + b are elements of ${}_1P_0(g)$ where $w \in {}_1P_0(g)$ and a and b are complex constants. Also sufficient conditions for a quadratic in $w^2 + aw+b$ to be an element of ${}_1P_0(g)$ are obtained. Also these for a cubic and in general an n^{th} degree polynomial to be an element in ${}_1P_0(g)$ are obtained.