DEVELOPMENT AND EVALUATION OF MULTI SENSOR DATA FUSION ALGORITHMS FOR TARGET TRACKING

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*This is to certify that the thesis entitled **"Development and Evaluation of Multi Sensor Data Fusion Algorithms for Target Tracking"** is a bonafide record of the research carried out by **Deepa Elizabeth George** under my supervision and guidance at the Department of Computer Science, in partial fulfillment of the requirements for the Degree of Doctor of Philosophy under the Faculty of Technology, Cochin University of Science and Technology.*

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Declaration

*I, **Deepa Elizabeth George**, hereby declare that the thesis titled **"Development and Evaluation of Multi Sensor Data Fusion Algorithms for Target Tracking"**, submitted to Cochin University of Science and Technology under Faculty of Technology is the outcome of the original research done by me under the supervision and guidance of **Dr.A. Unnikrishnan**, Scientist H (Retired), NPOL DRDO, Kochi, Kerala. I also declare that this work did not form part of any dissertation submitted for the award of any degree, diploma, associateship, or any other title or recognition from any University or Institution.*

Deepa Elizabeth George

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ABSTRACT

Sensor data fusion helps to derive more specific inferences than what could be achieved using a single independent sensor. The present day technology permits the deployment of large number of sensors of different capabilities. Developments in optimization, machine learning and soft computing have supported the synthesis of innovative ideas in data fusion, with promising results, thereby making the Multi Sensor Data Fusion (MSDF) very topical and seriously perused by research community world over. Some of the challenges in data fusion are data imperfections, outliers and spurious data, conflicting data and data association to name a few. In addition to the statistical advantages gained by improved estimate of a physical phenomenon through additional independent observations, the use of multiple types of sensors increases the accuracy of the observation. Naturally, multi sensor data fusion stands out as a technique to reckon in many practical applications and hence stimulates the requirement to explore further.

Over the last many years, the problem of target tracking has gained wide attention in surveillance and measurement systems, where an estimate of the target state driven by measurements is established. Naturally, the MSDF qualifies as a right choice in improving the estimate. Addressing the area of target tracking, the bearings-only tracking (BOT) problem has gained wide attention of both researchers and implementers working in the areas of radar, sonar systems and satellite surveillance. It also is interesting to note that the BOT is the only choice in the case of some typical surveillance systems as in submarines. The limited observability of the states from the bearing only measurements poses major hurdles in estimating the states of the target.

The present thesis concentrates on improving the estimates in BOT, judiciously incorporating MSDF. Addressing the limitations of the performance of EKF and its derivatives in handling MSDF in the context of BOT, the thesis

identifies that the tendency to diverge in the case of the EKF based techniques is a major issue and develops approaches to overcome the issues of divergence. In full appreciation of the fact that the MSDF can help to improve the observability, thereby reducing the tendency of the tracking algorithm to diverge and also realize a better estimate of the states, two major approaches to fusion viz. Data level and feature level (or state level) are proposed to be examined in detail. In order to alleviate the influence of the initial assumption in the convergence process of the MSDF algorithms for tracking, the Information filter, which is a recast of the Kalman Filter and its extensions to MSDF are taken up through elaborate simulation of different scenarios. The thesis puts forward alternate approaches in overcoming the possible divergence of the tracking solutions using MSDF with Information filter. In this context, adaptable results from Fuzzy set theory are utilized in controlling the divergence of the MSDF techniques using the Information filter. With the success achieved in controlling the tendency to diverge in MSDF, improved estimation of states is realised, even when the target manoeuvres heavily, switching between constant velocity and co-ordinated turn models. Fully acknowledging the fact that the JPDA reported in literature is a bench mark in tracking of multiple targets using MSDF, the thesis also intends to compare the performance of the proposed extension of the Information Filter using the Fuzzy set theory with the JPDA. All the existing ideas reported and the new ideas put forward in the thesis are demonstrated with detailed simulation of different type of scenarios that have close semblance to practical systems.

With all the literature surveys conducted thus far, the development of MSDF algorithms for BOT, with better estimate and without divergence showed up as an interesting area to explore. Accordingly, the objectives of the research were identified as

1. Development of algorithms for MSDF to yield excellent estimate, at the same time sustaining the track without divergence.

2. Integration of existing and improved techniques judiciously for better target tracking and state estimation.
3. Evaluation of the performance of data fusion algorithms under 1 and 2 above on a variety of scenarios, having large maneuver of the target, with a large variation in sensor and plant statistics.
4. Extension of the ideas developed to multi target tracking Further investigations based on the objectives above, led to the following contributions from the thesis:

The research work reported in the thesis has concentrated on a detailed study on the well established MSDF algorithms for target tracking. Starting with the preliminary assessment of the variance based fusion in the context of Kalman filter and also the PDA algorithm, the divergence problem in Kalman filter was identified and taken up for further investigations. The evaluation of the performance of Information filter in target tracking was then taken up, since the filter is known to proceed with the estimation even with relatively poor assumptions of initial values of the parameters and the extension to fusion of multiple measurements is straight forward. Although it could not produce a solace to the divergence problem, the Information fusion filter was observed to be computationally simpler compared to multi sensor tracking using Kalman filter. In order to control the divergence in information fusion filter (IFF), the Fuzzy Information Fusion Filter (FIFF) was subsequently proposed in the thesis. The number of independent simulations of the FIFF showed promising performance in alleviating the divergence problem of MSDF. The performance was demonstrated for a variety of complex trajectories switching between CV and CT models.

The estimation of the turn rate of maneuvering targets from estimated states added credence to the sustained performance of FIFF in MSDF. The performance of FIFF was confirmed using Monte Carlo simulation. The FIFF was demonstrated to track both single and multiple targets following CV and maneuvering tracks. The FIFF was developed from the Fuzzy Information Filter

(FIF). The present research work also investigates the multi target tracking with the Fuzzy Information filter, fusing measurements directly from co-located sensors and compares the performance with the well known JPDA algorithm, through Monte Carlo simulations.

The present thesis has demonstrated the efficacy of using fuzziness in the fusion of information state in the context of MSDF, effectively reducing the tendency of the filter to diverge. The works are largely supported by a number of simulation studies and independent Monte Carlo runs, establishing credibility of the proposed algorithms.

ABBREVIATIONS

CT	:	Coordinated Turn
CV	:	Constant Velocity
EKF	:	Extended Kalman Filter
FIF	:	Fuzzy Information Filter
FIFF	:	Fuzzy Information Fusion Filter
IF	:	Information Filter
IFF	:	Information Fusion Filter
JDL	:	Joint Directors of Laboratory
JPDA	:	Joint Probabilistic Data Association
JPDAF	:	Joint Probabilistic Data Association Filter
KF	:	Kalman Filter
MSDF	:	Multi Sensor Data Fusion
MSE	:	Mean Squared Error
MTT	:	Multi Target Tracking
PDA	:	Probabilistic Data Association
PDAF	:	Probabilistic Data Association Filter

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Chapter - 1
INTRODUCTION

Capability to sense and perceive has been major human trait right from the origin of mankind. The five sensors we have bring in a large amount of information into the human brain, which combines the sensory data to perceive and react. Vision, hearing, touch, taste and smell generate information in different bands scaling to various levels of perception. The diverse sensory data is processed in steps one at a time or together in groups, modulated by the human reasoning process. The natural question that arises is: how does the brain combine all the sensory data, which are of different bandwidth and formats. Multi Sensor Data Fusion (MSDF) is the answer to the question and the term MSDF encompasses all the facets of combining information from several sources to provide a unified picture of an environment or process of interest. Sensor data fusion helps to derive more specific inferences than what could be achieved using a single independent sensor.

The present-day technology permits the deployment of substantial number of sensors of different capabilities. From mica mots to large radar systems sensors with assorted capabilities are now available for deployment. A web of tiny geo sensor ramifies a large area in a terrain, to generate data, which could be handy in seismic assessments. Typical battle fields have diverse types of radar, guns and armored personal carriers, which helps to locate contacts in air and land (Fig. 1.1). The real-time fusion has thus become increasingly viable with the emergence of new sensors, improved hardware and advanced processing techniques. Developments in optimization, machine learning and soft computing have supported the synthesis of innovative ideas in data fusion, with promising results, thereby making the MSDF very topical and seriously perused by research community world over. As a result, the data fusion finds wide applications in many military systems, civilian surveillance and robotics.

Fig. 1.1 Surveillance in a battle field

Over the last many years, MSDF has also helped to strengthen the tracking of contacts. Tracking a contact continuously stems out of the requirement to keep a record of a moving system to

(i) continuously record the data from the sensors kept on board the system,

(ii) capture the status of the system, which may include position, velocity, acceleration and other state variables like spectral components, expected values of parameters like temperature, pressure, salinity and their correlations and

(iii) device control strategies to counter the movement of the system, as in a missile or aircraft or take preventive/corrective actions as in maintenance.

The target tracking, addressed in the present work, refers to the process of estimating the state of one or several objects over a period, using measurements received from one or more sources. The target tracking algorithms generally consists of two sets of equations, one for predicting the state of the target and the other for correcting the predicted state using observations from various sensors. In case of tracking multiple targets, the tracking algorithm also takes care of the data association techniques.

1.1 Estimation and Tracking

Estimation is the process of inferring the value of a quantity of interest from indirect, inaccurate and uncertain observations. This process can be dated long back to the period of Laplace when he addressed the “Sunrise problem”[1]. Probably the first estimation problem was the determination of planet orbit parameters studied by Laplace, Legendre and Gauss [2].Estimation techniques are widely being used for statistical inference, in tracking for determining the position and velocity of a target and in control systems to estimate the state variables, to control a plant in the presence of uncertainty. Other typical instances of using estimation include system identification for determining the model parameters for predicting the states as in the case of weather forecasting, economic analysis for market prediction, communication theory for determining the message received through noisy corrupted channel and in signal and image processing for determining some parameters or characteristics of a signal or image.

Tracking is the special case of estimation. Target tracking refers to the process of estimating the states of one or several objects, observed over a period of time [3, 4]. States could mean any derived information from observation viz. geometric status like position, velocity, acceleration, spectral components, average values, correlation etc. Specific target tracking problems include measurement to track association and sensor registration [3]. The solutions to these problems also require the consideration of computational demand of distributed processing of target tracks. The objects can be ground based targets, ships, underwater targets or aircrafts. Basically, all target tracking algorithms are state estimation algorithms, where the estimate of the state is corrected using measurements from one or more sensors. The commonly used sensors for target tracking applications are radar, sonar, and CCD camera to name a few. Tracking can be performed using measurements from single sensor as well as multiple sensors.

Filtering refers to estimating the current state of a dynamic system from noisy measurements. The computational algorithms that process measurements to yield an estimate of a variable of interest often arrive at an optimal solution with respect to a certain criterion. The general tracking problem from the Bayesian perspective is to recursively calculate the probability that the state X_k at any time k, given the measurements $Z_k = \{z_k\}$ up to time k. i.e. $P(X_k/Z_k)$. A well-known optimal estimator is the Kalman filter [5], which minimizes the prediction error in the observation. The main advantage of an optimal estimator is that it makes the best utilization of the data; the knowledge of the system and the disturbances [4]. The disadvantage is that it is sensitive to modeling errors and might be computationally expensive.

1.2 Data Fusion for target state estimation

Over the last few years, researchers have been working on problems concerning how to combine information from various sources to enhance the efficacy of decision making. The term decision making is used in the wide connotation to include both automated decision making and decisions by humans based on the outputs of the fusion system. In order to prompt exploratory study in the area of Data Fusion and to review the relevant literature in this area, it is essential to have a precise definition for data fusion. Data fusion is the process of combining data or information from multiple sources to estimate or predict entity states, where the physical state of entities is the identity, attribute, motion, location and activity over some past, current or future time period [6, 7]. The data fusion model developed in 1985 by the US Joint Directors of Laboratory (JDL) Data fusion group is the most widely accepted system for categorizing data fusion functions [8, 9].They define data fusion as ‘A multi-level process dealing with the association, correlation, combination of data and information from, single and multiple sources to achieve refined position, identity estimates, and complete and timely assessment of situations, threat and their significance'. According to this model, data fusion is divided into a hierarchy of four processes [10], viz. Level 1, 2, 3 and 4. Of these various levels, Level 1 and 2 are generally concerned with numerical fusion methods based on probability theory. These levels deal with the formation of track, identity or estimation of information and fusion of this information received from multiple sources. They deal with both direct fusion of sensor information (Fig. 1.2) as well as indirect fusion of estimates obtained from local fusion centers (Fig. 1.3). Some of the data fusion problems in these levels include multi- target tracking, track- to track fusion and distributed data fusion methods.

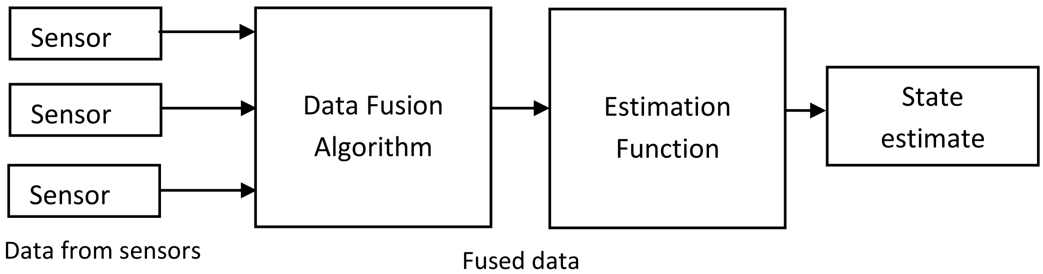

Fig. 1.2 Direct fusion of sensor information

A block diagram representing the direct fusion of sensor information for a target tracking problem is shown in Fig 1.2. Here, the observations received from multiple sensors are fused using an appropriate data fusion algorithm, and the information obtained through fusion is used for estimating track or identity of the object of interest. The sensors used here may be homogeneous, like cameras or microphones or hydrophones of same type or heterogeneous like cameras in different spectral band or mixture of radars and cameras tracking an object.

An indirect fusion technique for the same problem is considered in Fig. 1.3. Here, there are *N* sensors that receive observations or measurements from the target of interest. The continuous measurements received from individual sensors are fed to their respective Estimation Functions, which in this case acts as a state estimator. The block diagram depicted in Fig. 1.3, shows *N* parallel estimators, continuously providing the current estimate of the target. The estimates of the target states given by the individual filters are fused using appropriate fusion techniques at every time instant *k*, so as to obtain the final state estimate. This method of estimation provides better estimates compared to single sensor target tracking. While states form the features triggered by the measurements in Fig. 1.3, other type of features like correlation coefficient, HOS parameters in the case of data streams and cluster size, orientation, intensity distribution in the case of images are also generated through fusion.

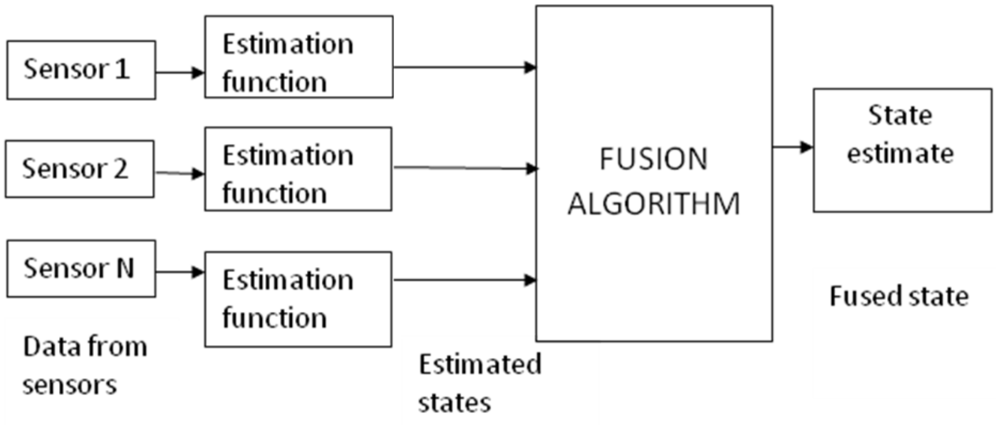

Fig. 1.3 Indirect fusion of estimates

Level 3 and 4 fusions deal with extraction of high level knowledge from low level fusion process, usually referred to as situation awareness [11]. The fusion process at this level includes the incorporation of secondary sources of information, human judgment and formulation of decisions and actions. Thus, it turns out that Level 3-4 data fusion is built on Level 1-2 methods. The problems in Level 3-4 involve the modeling of qualitative information sources and the use of non-probabilistic methods in describing uncertainty and general decision-making process. Even though, the JDL hierarchy had gained wide acceptance, it was found to be more appropriate for military data fusion scenarios and inappropriate for other information fusion problems**,** as the hierarchal structure mislead the study of distributed, decentralized and network centric data fusion structures.

Data fusion techniques have been extensively employed in multi sensor environments with the aim of fusing and aggregating data from multiple sensors to obtain a lower detection error probability and higher reliability [12]. This thesis contributes to distributed data fusion methods and multi target tracking, which belong to Level 1-2 fusion problems.

1.3 Classification of Data fusion methods

Data fusion methods can be classified on the basis of relationships among sources [13] as cooperative data, redundant data and complementary data as illustrated in Fig. 1.4. In cooperative data fusion, the sources provide different data that are fused to obtain a new data, which better describes the reality compared to the original sources. An example of cooperative fusion is estimating the target state based on bearing and range measurements. Redundant data fusion involves the fusion of two or more independent sources that provide the same data in order to provide a more reliable data, thereby increasing the associated confidence. In complementary data fusion, the sources provide data that represent different portions of a broader scene. An example of this fusion is the fusion of data from several cameras to observe different parts of an environment.

On the basis of level of abstraction, data fusion is classified as signal level fusion (usually dealing with single sensors), pixel level fusion (used in image processing tasks), feature level fusion (extraction of attributes from signal) and symbol level fusion (also called decision level fusion). Data fusion methods are also classified as low level, medium level and high level fusion. Signal level and pixel level come under low level fusion, feature level comes under medium level fusion and symbol level fusion comes under high level fusion.

Data fusion is performed with different objectives, such as inference, estimation, and classification. The inference method is applied in decision fusion.

Some of the classical inference methods are based on Bayesian inference [14] and Dempster Shafer belief [15]. The other common methods are fuzzy logic and neural networks.

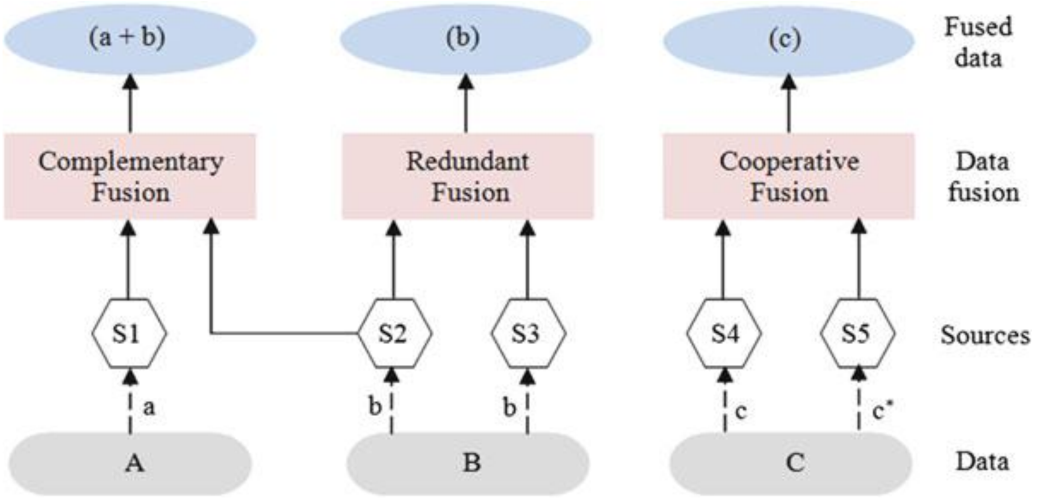

Fig. 1.4 Classification of data fusion methods based on relationships among sources [103]

1.4 Data Fusion for Estimation

Any data fusion problem that one considers, involves an environment, process or quantity, whose true value, situation or state is unknown. The problems usually involve obtaining information indirectly from sources that provide imperfect and incomplete knowledge. In order to utilize the received information to its best effect, it is essential to describe precisely the way how information relates to the state of interest; for example the relationship between the observation and the target state in a target tracking problem.

The terms, world', 'state', 'information' and 'observation', which frequently appear in the Data Fusion paradigm is elucidated as follows[10].

- The quantity of interest described by *x*, describes an environment, process, statement or single number. The quantity *x* is called the state of nature or simply state. The state can take a variety of values contained in the set of all possible states, $x \in X$. The model of the environment consists of this set X, together with the knowledge of how the elements of this set are related.

- In order to obtain information of the state, the quantity that we observe, called observations or measurements, are described by z. These measurements can take different values contained in the sample space Z, such that $z \in Z$. An observation model is one that completely defines the sensing process; i.e. $z=z(x) \in Z$.
- The goal of a data fusion process is to infer the underlying state, x, using the observation z. Here we need to define a decision function, δ, that maps the observations to state; $\delta(z) \rightarrow x \in X$. This information model comprises of the information about the nature of observation, the accuracy and error in the state of the world and prior beliefs about the world. The function δ considers all this information to produce a final decision.

Estimation is one of the important problems in sensor data fusion, where we wish to find some estimate of the true state of the environment we are observing [10]. Among many estimation approaches, the Kalman filter attains greater presence in literature, since the filter can be designed to estimate the states also from measurements. Though the data fusion results in better quality of estimates, the complexity of data fusion system increases as the number of sensors incorporated to the system increases. This led to the concept of distributed data fusion architecture.

1.5 Challenges and issues

Multi sensor data fusion is a challenging task and some of the issues that make it challenging are summarized below [16].

i). Data imperfections: This arises due to impreciseness in the deployment as well as uncertainty in the measurements provided by the sensors. Wide variations in the data arising out of the above limitation have to be contained, while fusing the data [17].

ii). Outliers and spurious data: These are caused by ambiguities and inconsistencies present in the environment [18]. Bayesian approach in modelling can be used to identify the inconsistency in sensor data so as to minimize the spurious data from fusion process, thereby leading to a better estimate of the desired state variable.

iii). Conflicting data: Fusion of conflicting data can be highly misguiding, especially, when the fusion system deploys evidential belief reasoning as in Dempster's rule of combination [19].

iv). Data corruption due to correlated noise: Typically, in wireless sensor networks, some nodes are likely to be exposed to external correlated noise and hence their measurements are likely to be biased. The data fusion algorithms in such systems should consider the data dependencies as well.

v). Data alignment: It is also called data registration, which is the process of transforming a sensor data from sensor's local frame into a common frame before fusion occurs. Radiographic and geometric corrections of frames received in satellite image form a representative example of the corrections of this category.

vi). Data association: This is a problem that arises mostly in multi-target tracking systems or when tracking single targets in clutter environment. The association problem is mainly classified as two types: measurement-to-track and track-to-track association. The former identifies from which target, if any, a measurement has originated, while the latter deals with distinguishing and combining tracks [18].

vii). Processing Framework: The two standard frameworks used for data fusion process are the Centralized and the Decentralized systems. While Centralized systems are preferred generally for surveillance, the decentralized systems fit better in wireless sensor networks. High computational and data handling capabilities are required for centralized systems whereas decentralized system can manage with limited processing capability.

viii). Operational timing: The different operating frequencies of the sensors and asynchronous nature can lead to out-of-sequence arrival of data. This necessity the use of multiple time scales and proper resampling.

ix). Data dimensionality: The preprocessing of measurement data, either locally at each sensor node, or globally at the fusion centre helps to compress the data into lower dimensional data, assuming a certain compression loss. This pre-processing helps to save on the communication bandwidth and power required for transmitting data [20].

Though there are several challenges involved in data fusion process and the process is computationally demanding, the fusion of data from multiple sensors provides advantages over single sensor data in many applications. The cost effectiveness and ease of deploying considerable number of sensors motivates MSDF. In addition to the statistical advantages gained by improved estimate of a physical phenomenon through additional independent observations, the use of multiple types of sensors increases the accuracy of the observation. Naturally, MSDF stands out as a technique to reckon in many practical applications and hence stimulates the requirement to explore further.

1.6 Problem statement

The major challenges and issues in applying multi sensor data fusion for target tracking using the bearing only measurement leads to the following definition of the problem to be addressed in the thesis. The available literature suggests two major approaches based on (i) Kalman filter and its variances and (ii) Information filter. The reported literature brings out the divergence of the Probabilistic data association filter derived out of the Extended Kalman filter which merits attention in the thesis. The Information filter, though has many advantages over the Kalman Filter in respect of sensitivity to initial assumptions of the states and its properties, also is also not totally free from the hurdle of divergence.

Utilization of soft computing techniques like Fuzzy systems, which can handle the uncertainty associated with the state estimation suggests itself a major area to be explored. The evaluation of the performance of the algorithms on different scenarios involving complex maneuvers, through Monte Carlo runs is required to demonstrate veracity of techniques developed in the thesis. Finally a comparison of the methods developed in the thesis with state of art algorithms is required to substantiate the new results reported in the thesis.

1.7 Major contributions of the thesis

The present thesis work concentrates on MSDF algorithms for target tracking applications, based on bearing only measurements. Popularly known in literature as the BOT, (Bearing Only Tracking) problem, the topic has posed several challenges both in researches and implementation in ensuring sustained tracking on straight course and also during maneuver. In this work, the well-established Kalman filter (KF) is re-visited and a recast of it, the Information filter (IF) is experimented for target tracking application. Like the KF, the IF is also seen to experience a tendency to diverge. Applying the Fuzzy logic to IF, to create the Fuzzy Information Filter (FIF), it is shown to be effective in alleviating the problem of divergence. To enhance the observability for sustained tracking, the thesis thereafter examines the fusion of measurements to improve the tracking. Utilizing the advantages of MSDF, the FIF is improved to Fuzzy Information Fusion filter (FIFF).This filter has been shown to be performing better over other versions of fusion filter, as sensor fusion using FIFF is computationally less demanding and involves simpler mathematics. The effectiveness of FIFF in tracking target, following Constant Velocity(CV) model and maneuvering using Coordinated Turn(CT) models are also experimented and demonstrated to fair with better convergence and low tracking error. FIFF was also tested on targets that switch between CV and CT model, where it employs Chi-square test for maneuver detection. The plant model in tracking switches after detecting the model, assuming the turn rate. Subsequently, the turn rate was also adaptively estimated to give an improved version of the FIFF. The performance of the filter is seen to be well in line with the expectation of no divergence and very low tracking error.

In the context of multi target tracking (MTT) problem, this thesis proposes a technique of associating multiple measurements using FIFF, using a novel method of fusing measurements. A comparison with the well known Joint Probabilistic Data Association Filter (JPDAF) shows that the proposed method is an effective alternative to JPDAF in multi target tracking applications. The performance of the FIFF is seen to be comparable to the JPDAF.

All the evaluations of the performance simulations of the techniques proposed in the thesis have been validated through independent Monte Carlo simulation over long durations. The low tracking errors and sustained convergence adds credence to the propositions of the thesis.

1.8 Thesis Organization

The rest of the thesis is organized as follows. Chapter 2 provides a background and a literature review of the existing techniques in MSDF for target tracking. In Chapter 3, the Extended Kalman filter (EKF) is revisited and the salient features of MSDF is verified by experimenting with (i)the Probabilistic Data Association Filter (PDAF),which is an EKF based filter for estimation in the presence of measurement origin uncertainty and (ii) the commonly used variance based fusion technique in conjunction with EKF. Chapter 4 examines the Information Filter (IF) [4], as an alternative to the EKF in target tracking. The performance of IF using single sensor and Information Fusion Filter (IFF) employing multiple sensors are studied in detail, simulating various scenarios. Chapter 5 introduces the proposed method, the divergence correction using fuzzy technique, leading to the Fuzzy Information Filter (FIFF). The proposed method is further extended to track targets that switch between CV and CT models in

Chapter 6. Two techniques are presented in this chapter for tracking maneuvering targets, FIFF using (i) Chi-square test and (ii) the adaptive turn rate model for maneuver detection. Chapter 7 further extends the target tracking problem to multiple targets. Here a computationally less demanding fusion strategy using FIF is proposed and compared with the JPDA filter. Chapters 3, 4, 5, 6 and 7 also contain a brief review of the literature relevant to the proposed approach, and end up with a series of simulation results that assess these approaches. The conclusions of the thesis and direction for further research are summarized in the Chapter 8.

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Chapter - 2

BACK GROUND AND LITERATURE REVIEW

In order to facilitate communication among researchers, the US Joint Directors of Laboratories, Data fusion group, developed the JDL data fusion model in 1985, which was later revised and generalized in 1998. The most popular frame work for fusion systems is the JDL model [8,21] even though a number of other conceptualizations for fusion systems exist. Multi sensor fusion, also known in literature as Level 1 fusion, according to the JDL data fusion process model, implies a process which generally employs both correlation and fusion processes to transform sensor measurements into updated states and co variances for entity tracking. D. L. Hall and J. Llinas [22] have succinctly differentiated the Sensor Data Fusion and Information fusion. To quote, "Properly said, fusion is neither a theory nor a technology in its own [22]. It is a concept which uses various techniques pertaining to information theory, artificial intelligence and statistics". Information fusion deals with the process of acquiring, processing and intelligently combining information gathered by various sources and sensors to provide a better understanding of the phenomenon under consideration. On a wider canvas, the fusion is also capable of handling diverse data and can be described as a process by which the tracked entities are associated with environmental, doctrinal and performance constraints, or a structured multi-perspective assessment of the distributions. Hence they come under the Level2, heralding the concept of situation assessment and Level3 pointing to threat assessment, of the fusion paradigm. The performance of fusion in terms of probability of detection of target, false alarm rate and classification accuracy is dependent on the validity of the target models, delivered by data mining process [23].

2.1 Multi sensor Data fusion

Generally, sensor data fusion deals with gathering observations of the world and drawing inferences from them [24]. Many definitions of Data fusion exist in literature. The Joint Directors of Laboratories (JDL)[8] defines data fusion as a "Multi level, multifaceted process handling the automatic detection, association, correlation estimation and combination of data and information from several sources". A general definition is given by Klein [25], stating that data can be either provided by a single source or multiple sources. The authors present a review and discussion of many data fusion definitions in [26]. B.Khaleghi et al.[16] proposed a definition of information fusion in 2013 as: "Information fusion is the study of efficient methods for automatically or semi-automatically transforming information from different sources and different points in time into a representation that provides effective support for human or automated decision making".

JDL classification originated from the military domain and is based on the input data and produced outputs. The fusion process in the original JDL model consists of four increasing levels of abstraction, namely object, situation, impact and process refinement. Though JDL model acquired great popularity, it has many shortcomings, such as being too restrictive and especially tuned to military applications. This has led to several extension proposals [9, 27] attempting to alleviate them. Dasarathy's framework [28] was an alternative to the JDL model which views the fusion system, from a software engineering perspective, as a data flow characterized by input/output as well as functionalities or processes. Goodman et al. [29] has given another generalization of fusion based on the notion of random sets. This frame work has the distinctive feature of combining decision uncertainties with decisions themselves and also presents a fully generic scheme of uncertainty representation. Abstract fusion is the most recent fusion frame work presented by Kokar et al. [30] and is also considered as the first step towards development of a formal theory of fusion. This framework is based on category theory and is claimed to be sufficiently general to capture all kinds of fusion, including data fusion, feature fusion, decision fusion and fusion of relational information. The major novelty of this frame work is the ability to express all aspects of multi-source information processing.

Multi sensor data fusion has several military and non-military applications [31]. Sensor fusion was traditionally used in military applications like target identification and acquisition. Data fusion plays a critical and fundamental role in defense and national security, mainly in areas of surveillance and intelligence analysis for timely situational awareness. Currently military data fusion is a highly sophisticated field [32, 33]. The network centric warfare is an emerging operational concept that deals with significant role of information. The paper [34] compares the concept of conventional and network centric grid system and also discusses the importance of sensor fusion in network centric warfare. The non-military applications include fault detection in systems, central monitoring systems, Robotics and Unmanned Ariel Vehicles, and medical field etc. Another established application of sensor fusion is weather forecasting [35] and habitat monitoring [36]. Marzullo in his paper [37] proposes a model for fusing overlapping sensors to obtain a single fault tolerant sensor. He has also shown a relationship between agreement in sensor network and distributed consensus. One of the popular applications of sensor network is location tracking that includes tracking of objects, people, robots etc. The authors in [38, 39] have proposed a number of techniques for this problem. The data fusion methods employed in robotics are often based on probabilistic methods, which are now considered as the standard approach in robotic applications [40, 41].

Another critical problem in wireless environment is power management and synchronization for sensor fusion. Romer [42] in his paper has proposed a power efficient synchronization protocol for use in wireless sensor networks. Researchers have proposed effective content based form of data routing that affect subsequent routing decisions [43].Models for MEMS based sensor networks using NS2 simulator has been proposed by authors in [44].In order to manage the sensor attributes as well as the data they produce, data management facilities are required[45, 46].The special security requirements of sensor networks have been explored by researchers at UC Berkely [47].Some other applications of sensor fusion include smart spaces for children [48] and biomedical sensor implants [49].

Application	**Dynamic system**	**Sensors used**	**Supplementary data**
Process control	Chemical plant	Pressure, temperature, flow or gas analyzer	Production data
Flood prediction	River and back waters	Water level, rain gauge, weather radar, flow details from tributaries.	Previous history of flooding
Medical diagnosis	Human body	Blood pressure, body temperature, ECG and EEG, CAT and MRI scans	Patient history and diagnostic history
Tracking	Space craft	Radar, imaging systems, telemetry on speed with time stamp	Launch data
Navigation	Ship/Air craft	Radar, Sonar, gyroscope, accelerometer	GPS data

Table 2.1 Typical system that utilize MSDF in decision making

The main advantage of multi sensor data fusion over single sensor data is that it improves the accuracy and precision of the received data, reduces the uncertainty and hence also supports effective decision making [22, 50]. Also the availability of sensors and even senor suits, with sufficient processing power has motivated the research community think seriously about fusing data or any other derived information from data. Some of the typical data fusion applications, that exploit the largesse in the multi sensor data, arise in estimation problems in process control, flood prediction, seismic assessments, distributed tracking and navigation. To exemplify the above observation, some of the emblematic dynamic systems and commonly used sensors for each application are tabulated in the Table 2.1 [16].

Since errors are inherent part of any measurement, each sensor has a sensor model to take care of the uncertainty and error in the data received from each sensor. The main challenge in multi sensor data fusion then boils down to devising strategies to reduce the uncertainty [51, 17, 52].

Data fusion process can be categorized into mainly 3 classes based on the level of abstraction used for fusion as measurement fusion, feature-level fusion and decision-level fusion. The measurement fusion or sensor data fusion involves direct fusion of data received from the sensors. This type of fusion is used in applications where the sensors measure the same physical phenomenon and is primarily limited to fusion of homogeneous modalities. Feature level fusion involves the extraction of representative features from the sensor data. The extracted features are then combined into a single concatenated feature vector that is given as input to a fusion node. N. Wichit and A. Choksuriwong [53] have proposed a novel multi-sensor based activity recognition approach with fuzzy logic fusion sensors to recognize human behavior. Other works in this level involves activity recognition systems for wireless sensor networks [54, 55]. Decision level fusion is comparatively a higher-level fusion compared to the previous two classes. Here each sensor makes a preliminary determination or decision of an entity’s location, attributes and identity. Suitable decision level fusion algorithms like weighted decision, Bayesian inference and Dempster -

Shafer’s method are used for combining the decisions to get a better and more reliable decision [56].

The demand for new methods and algorithms for multisource remote sensing data fusion are increasing due to the fast development in remote sensor technologies like very high resolution optical sensors, LiDAR, SAR etc. [57]. Even though, there have been a number of developments in sensor technologies, multi source image fusion remains challenging due to varying spatial and temporal resolution. One of the applications of data fusion of remotely sensed data is for urban area characterization. The paper [58] discusses feature fusion as a way to combine information from multiple sensors, with multiple spatial resolutions and multi temporal acquisitions. The paper [59] indicates that multi source information can significantly improve the interpretation and classification of land cover types and refined Bayesian classification is a powerful tool to increase the classification accuracy. Multi sensor data fusion helps in detecting falls which are a serious concern for aging society. The paper [60] reviews in detail the multi sensor fusion based methods to determine falls and compares it with single sensor based approaches. Fusion of images captured through multiple cameras, operating in the same or different bands have gained prominence in survelliance. The book on image fusion algorithms [61] provides a collection of recent advances in the field of image fusion and also discusses and evaluates various spatial and transforms domain fusion methods.

The main advantage in data fusion involve enhancement in data authenticity or availability. Examples of the former are improved detection, confidence and reliability as well as reduction in data ambiguity, while latter extends spatial and temporal coverage. Data fusion also provides specific benefit to some application contexts. Wireless sensor networks are often composed of a large number of sensor nodes, hence posing a new scalability challenge posed by potential collisions and transmissions of redundant data. Distributed sensor networks are also designed to operate efficiently in adverse environment using limited battery power and resources. Hence it is important that these sensors process information efficiently and share information so that decision accuracy is improved [62].

In multi sensor tracking systems, the sensor data fusion is generally carried out at either measurement level or track level method [63].Track fusion is attractive in multi sensor multi target tracking compared to measurement fusion because of its robustness and flexibility. In measurement level method, the measurements from the sensors are fused in a central site to obtain a combined or weighted measurement and then fed to an estimating filter (eg. Kalman Filter) to get a final optimal estimate based on fused measurement. This method is highly sensitive to sensor failure and requires great computational resource [64, 65].In the latter method; the local sensors require sufficient computational capability to estimate the target state. The estimated states of the local sensors are communicated to the fusion centre wherein track association and track fusion are performed at a global level. Most of the track fusion methods make the assumption that the sensors in the system are synchronous. But that is not the practical case as in most cases the targets or the sensors are moving. In [63], an asynchronous approach for track fusion is proposed, that provides solution to combine tracks estimated by multi scale sensors. In [64], various track to track fusion techniques have been evaluated for various operating conditions that can be used in designing a fusion system. In general, measurement level fusion is optimal but computationally less efficient, and the track level method is more efficient but suboptimal.

Bar-Shalom [66] found that estimated state vectors from each sensor are not independent due to common process noises, and hence proposed an algorithm to compute cross covariance of the track estimate. He has derived an exact likelihood function for the track to track association problem. There are many fusion algorithms in literature which use track level fusion[67, 68, 69, 70] of which the commonly used approaches to fuse state vectors are Weighted Covariance(WC)[64, 66, 67],Information Matrix(IM) [71] and Covariance

Intersection(CI) [72, 73]. Bar-Shalom in 1986, proposed a WC algorithm in which the fusion weights of the local estimates are calculated by taking into account, the error covariance between the local estimates [67]. An IM algorithm was proposed by Chang [71] that estimates cross covariance but it was found to be computationally expensive.

Many researchers have tried to evaluate the performances of multi sensor fusion algorithms. The study of Roecker [74] shows that WC algorithm is consistently worse than measurement fusion method. A similar study by Chang [75] points out that the results of WC algorithm, turns out to be a maximum likelihood estimate. Zhi Liu et al. has compared the performance of Information matrix, Weighted Covariance and Covariance Intersection algorithms [76].Their study shows that the performance of Information Matrix algorithm is better that Covariance Intersection and Weighted Covariance approach.

2.2 Multi sensor Data fusion algorithms

The currently available data fusion techniques are basically classified into 3 categories-Data association, State estimation and Decision fusion. The data association technique deals with the process of assigning and computing weights that relates the observation or tracks from one set to the observation or tracks of another set. Some of the commonly used algorithms in literature for data association are the Nearest Neighbours (NN), Probabilistic Data Association (PDA)[77, 78, 79, 80], Joint PDA [77] and Multiple Hypothesis Test(MHT) [81] etc. The state estimation techniques are also called tracking algorithms, aim to determine the state of a moving target from the given measurement or observation. Popular algorithms used for tracking include Maximum Likelihood (ML), Kalman filter (KF), Particle Filter (PF) and covariance methods to name a few. The decision fusion technique aims to make a high-level inference about the events and activities produced from the detected targets. The commonly used algorithms are the Bayesian methods, Dempster-Shafer inference and the Semantic methods etc .Regardless of the fusion framework, the underlying fusion algorithm must ultimately fuse the input data. Thus, data fusion algorithms have to tackle the data related challenges. The input data to the fusion system may be imperfect, correlated or inconsistent. Various categories of imperfect data have been proposed in literature [82, 51, 83].Some of the techniques used to handle these data are discussed here.

2.2.1 Fusion of imperfect data

The most challenging problem of data fusion systems is the inherent imperfection of data, and thus a number of research works have focused on tackling this issue. There are a number of mathematical theories available to represent data imperfections[84], namely Probability theory[85], Fuzzy set theory[86], Possibility theory[87], Rough set theory[88] and Dempster-Shafer evidence theory[89].These theories or approaches represent specific aspects of imperfect data. For example, uncertainty is represented as a probabilistic distribution, the vagueness of data is represented by fuzzy set theory and evidential belief theory can represent uncertain as well as ambiguous data.

Probability theory has been used for a long time to deal with almost all kinds of imperfect information. Alternative techniques are the fuzzy set theory and evidential reasoning that has been proposed in literature to deal with perceived limitations in probabilistic methods such as complexity, precision of models, to name a few [85].The data fusion algorithm along with their hybridizations aim for a more comprehensive treatment of data imperfections. Some of the hybrid frame works are Fuzzy Rough Set theory [90] and fuzzy Dempster Shafer theory [91]. This thesis proposes a hybrid approach of probabilistic fusion and fuzzy set theory. Hence the theoretical back grounds of the two techniques are reviewed in detail.

Framework	Characteristics	Capabilities
Probabilistic[85, 5, 92]	Represents sensory data using probability distributions fused together within Bayesian framework.	Well established approach to treat data uncertainty.
Evidential[93, 94, 95, 96, 97]	Relies on probability mass to characterize data using belief and plausibility and fuses uses Dempster's combination rule.	Enables fusion of uncertain and ambiguous data.
Fuzzy reasoning [98, 99, 100]	Allows vague representation using fuzzy membership, fusion based on fuzzy rules	Intuitive approach to deal with vague data.
Possibilistic [101, 102]	Similar in data representation to probabilistic and evidential framework and fusion to fuzzy frame work.	Handles incomplete data, common in poorly informed environment.
Rough Set Theoretic [88, 103, 104]	Deals with ambiguous data using classical set theory operators.	Does not require any preliminary or additional information.
Hybridization [89,105,106]	Aims at providing more comprehensive method of dealing with imperfect data.	Deploys fusion framework in a complementary rather than competitive fashion.
Random set Theoretic [107, 108, 109]	Relies on random subset of measurement/state space to represent imperfect data.	Potentially provide a unifying framework for fusion of imperfect data.

Table 2.2 Comparison of imperfect data fusion frame work [16]

2.2.2 Probabilistic fusion

Probabilistic method of fusion relies on the probability distribution or density functions to express data uncertainty. Bayes theorem is the most important result in the study of probabilistic models. It is possible to apply Bayes theorem directly to the integration of observations from several different sources [10].

A Bayes estimator provides a method for computing the posteriori probability distribution or density of the state X_k at time k given the set of measurements $Z^k = \{z_1 ... z_k\}$ (measurements up to time k) and prior distribution.

$$P(X_k / Z^k) = \frac{P(z_k / X_k)P(X_k / Z^{k-1})}{P(Z^k / Z^{k-1})} \quad (2.1)$$

Where $P(z_k / X_k)$ is the likelihood function that is based on given sensor model, $P(X_k / Z^{k-1})$ is called the prior distribution which incorporates the given transition model of the system. The denominator is a normalizing term to ensure that the probability density function integrates to one. Bayes estimator allows fusion of pieces of data. It can be recursively applied each time to update the probability density function by fusing new piece of data. An analytical solution of Bayes estimator is occasionally available, as both the prior and the normalizing term contain integrals that cannot be evaluated analytically in general. The authors in [85, 5, 92] have used probabilistic fusion techniques for various applications.

The well-known Kalman Filter (KF) [5,110] is an exceptional case of the Bayes filter with an exact analytical solution. It is a recursive Bayesian estimator that addresses the general problem of trying to estimate the state of a discrete time process. This has been possible due to enforcing simplifying constraints on the system dynamics to be linear Gaussian, i.e. the measurement and the motion model are assumed to have a linear form and be contaminated with zero mean Gaussian noise[107].The KF estimates a process using a two step recursive algorithm, namely the prediction and the correction step. The prediction step deals with estimating the process state at a certain time, based on the previous measurements and the correction involves obtaining feedback from the noisy measurement. KF fusion methods have gained popularity due to its simplicity, ease of implementation and optimality in a mean squared error sense. Data fusion using KF are mainly measurement fusion or track to track fusion. This filter is not capable of dealing with non-linear system dynamics. In dealing with such systems, one usually has to resort to approximation techniques. Some of the extensions of KF applied to non-linear systems are Extended Kalman Filter(EKF) [5], and Unscented Kalman Filter[111], which are the first order and second order approximations as a Taylor series expansion about the current estimate respectively. Welch and Bishop define EKF as a Kalman filter that linearizes about the current mean and covariance. A feature to be noted in EKF is that it propagates only the relevant component of the measurement information. If there do not exist a one to one mapping between the measurement and the state, the filter will quickly diverge and the process is said to unobservable. Sequential Monte Carlo (SMC) is another method of approximating probabilities. They are also very flexible as they do not make any assumptions regarding the probability densities to be approximated [112]. Particle filters are a recursive implementation of the SMC algorithm [92].They are an alternative to Kalman Filters in dealing with non-Gaussian noise and non-linearity in the system. Particle filters have been shown to be sensitive to outliers in data and requires a set of auxiliary variables to improve their robustness [113].Particle filters are computationally expensive as compared to Kalman Filters. They require a large number of random samples to estimate the desired posterior probability density. Hence, they are not suitable for fusion problems involving a high dimensional state space, as the number of particles required to estimate a given density function increases exponentially with dimensionality.

2.2.3 Fuzzy set theory

Fuzzy logic has found widespread popularity as method for representing uncertainty particularly in applications such as supervisory control and high level data fusion tasks [10,114-116]. Fuzzy logic provides an ideal tool for inexact reasoning [52]. It introduces the novel notion of partial set membership. A fuzzy set $F \subseteq X$, defined by general gradual membership function $\mu_F(x)$ in the interval [0, 1], as $\mu_F(x) \in [0,1] \ \forall x \in X$ .Higher the membership degree, the more *x* belongs to *F*. Fuzzy rules are used to combine fuzzy data to produce fuzzy fusion output. The fuzzy fusion rules are divided into conjunctive and disjunctive categories. Examples of conjunctive category are the standard intersection and product of two fuzzy sets given by Eq. 2.2 and Eq. 2.3 respectively.

$$\mu_1^{\cap}(x) = min[\mu_{F1}(x)\mu_{F2}(x)] \ \forall x \in X \tag{2.2}$$

$$\mu_2^{\cap}(x) = \mu_{F1}(x)\mu_{F2}(x) \ \forall x \in X \tag{2.3}$$

Examples of disjunctive fusion category are standard union and algebraic sum of two fuzzy sets given by Eq. 2.4 and Eq. 2.5 respectively.

$$\mu_1^{\cup}(x) = max[\mu_{F1}(x)\mu_{F2}(x)] \ \forall x \in X \tag{2.4}$$

$$\mu_2^{\cup}(x) = \mu_{F1}(x) + \mu_{F2}(x) - \mu_{F1}(x).\mu_{F2}(x) \ \forall x \in X \tag{2.5}$$

For fusing data provided by equally reliable and homogeneous sources, conjunctive fuzzy rules are considered appropriate. On the other hand, when one of the sources is deemed reliable, though which one is not known, or when fusing highly conflicting data, disjunctive fusion rules are deployed. There are also some fuzzy fusion rules that have been developed as a compromise between the two categories [117]. Similar to probability theory that requires prior knowledge of probability distributions, fuzzy set theory requires prior membership functions for different fuzzy sets. As fuzzy set theory is a powerful tool to represent vague data, it is particularly useful to represent and fuse vague data produced by human experts in a linguistic fashion. Fuzzy set theory can be integrated with probabilistic [118, 98] and D-S evidential [99, 100] to give better results.

2.3 Information measure

It is often valuable to measure the amount of information in a probability distribution. Information is a measure of compactness of a distribution. If a probability distribution is spread evenly across many states, then its information content is low and conversely if a probability distribution is highly peaked on a few states, then the information content is high [10]. Hence information is a function of the distribution rather than the underlying state. Information measures play an important role in designing and managing data fusion systems. Two probabilistic measures of information that are used in data fusion problems are the Shannon information (entropy) and the Fisher information. Shannon information is also extended to other forms like conditional entropy and mutual information [81]. While Shannon information is defined on continuous and discrete distribution, Fisher information may be defined only on continuous distribution.

Shannon information (entropy) $H_p(x)$ associated with a probability distribution $P(x)$ defined on random variable *x* is

$$H_p(x) \triangleq -E\{\log P(x)\} \tag{2.6}$$

For continuous distribution, $$H_p(x) = -\int_{-\infty}^{\infty} P(x)\log P(x)dx \tag{2.7}$$

For discrete distribution, $$H_p(x) = -\sum_{x\in X} P(x)\log P(x) \tag{2.8}$$

Fisher information is defined as second derivative of log likelihood.

$$J(x) = \frac{d^2}{dx^2}\log P(x) \tag{2.9}$$

In general, if *x* is a vector, *J(x)* will be a matrix called Fisher information matrix. Fisher information describes the information content about the values of *x* contained in the distribution *P*(*x*).Fisher information measures the bounding region containing probability mass. Entropy is a single number and it measures a volume, whereas Fisher information is a series of numbers, measuring the axes of the bounding surface.

If $P(x)$ describes all the information we have about quantity x, then the smallest variance that we can have on an estimate of the true value of x is known as Cramer-Rao lower bound, and is equal to $J(x)$(Fisher information).

2.4 Decentralized estimation –Information filter

A Decentralized system does not require a central controller for fusing information. A sensor or node in a decentralized system does not have any information regarding the location of other sensors. In Decentralized information fusion, information measures are used as a means of quantifying, communicating and assimilating data obtained from sensors. Decentralized estimation of continuous valued states is implemented in the form of information filter. Conventional KF estimate state $X_{i/j}$ together with a corresponding estimate variance $P_{i/j}$ .The information filter deals with information state vector $y_{i/j}$ and information matrix $Y_{i/j}$, which are related to the estimate variances as

$$y_{i/j} = P^{-1}{}_{i/j} X_{i/j} \tag{2.10}$$

$$Y_{i/j} = P^{-1}{}_{i/j} \tag{2.11}$$

These information quantities are shown to be similar [10] to the probability distributions associated with the estimation problem. The complete estimation and updating equations of the information filter (IF) are discussed in Chapter 4 of the thesis.

The information filter (IF) is mathematically identical to the conventional KF. It has a set of recursive equations for information state and information matrix, which can be derived directly from the KF. The highlight in the IF update stage is that the update equations are computationally simpler compared to KF.

The update stage is a straight addition of information from a prediction and from an observation [119]. This simplicity gives IF its advantages in multi sensor estimation problems. The advantage in using IF over KF for multi sensor target tracking is discussed in Chapter 4.

It is possible to add information states and information matrices from different sensors, while it is not possible to add innovations without accounting for cross correlation (the case of KF).Thus IF provides a far more natural means of assimilating information, than does the conventional KF. It is also a far simpler method of dealing with complex multi sensor data fusion problems. Also, IF provides a better method of mapping estimation equation to different architectures [10].

2.5 Information filter in multi sensor estimation

The application of information filter in estimation is well known in literature [120, 121]. However its use in data fusion has largely been neglected, in favor of conventional state based Kalman filtering methods. The reason appears to be somewhat specious based largely on incorrect hypothesis [10] that it is 'cheaper' to communicate innovation information, which is of the dimension of observation vector, than to communicate information state vectors, which is of the dimension of state vectors.

An IF is an improved form of KF. In MSDF, the IF fuses the information obtained from the sensors rather than the measurements as such [81,122]. Hence it is a more sophisticated technique than the KF. The term 'information' used in information filter is as defined in section 2.3 and section 2.4. In this filter, integrating information to update the predicted state involves only some simple arithmetic [81]. The information fusion algorithm has the advantage that it can be easily modified to incorporate any number of sensors. Thus, an Information Fusion Filter (IFF) provides a simple fusion technique, by retaining all the benefits of the KF. This thesis concentrates on IF for MSDF. Details of IFF and its modified versions are explained in Chapters 4 and 5.

2.6 Multi target tracking (MTT) algorithms

In multi target tracking in clutter, there are more than one measurement available for updating the state of a single target [123].The problem of data association is tackled using Bayesian and non-Bayesian approaches as reported in literature [65]. The most commonly used framework in the Bayesian approach is the Joint Probabilistic Data Association filter (JPDAF) [124, 125]. The non-Bayesian approaches used are Strongest Neighbor filter (SNF), Nearest Neighbor Filter (NNF) and Multiple Hypothesis Tracking MHT.

Data association is crucial in tracking targets with less than unity probability of detection in the presence of false alarms [126,127,128]. A number of algorithms have been developed to solve this problem. The SNF and NNF are two simple solutions to this problem. In the SNF, the signal with the highest intensity among the validated measurements is used for track update and others are discarded. In NNF, the measurement closet to the predicted measurement is used. Though these techniques work reasonably well for tracking targets in sparse scenarios, they begin to fail as false alarm rate increases or with low observable maneuvering targets [129]. An alternative and a very effective technique in tracking a single target in clutter is the Probabilistic Data Association Filter (PDAF). In PDAF, instead of using only one measurement among the received ones and discarding the others, all the validated measurements are used with different weights [77]. In Fuzzy Recursive Least Squares-Probabilistic Data Association (FRLS-PDA), the fused measurement is generated using PDA and FRLS is used to estimate the current state of the target. This technique is found to be better than PDAF and IMM-PDA filter [78].Data association becomes more difficult with multiple targets as a measurement itself can be validated by multiple tracks. The Joint Probabilistic Data Association (JPDA) algorithm is used to track multiple targets by evaluating the measurement to track association probabilities, and combining them to find the state estimate [127]. The JPDA algorithm uses a gating procedure at each step to prune away those infeasible hypotheses. The states are then updated on the basis of a joint probability that is calculated over the remaining association hypothesis. Due to the computational complexity of JPDA algorithm, several versions of JPDA were developed, namely the ad-hoc JPDA [79], Sub-optimal JPDA [130] and Joint Integrated Probability Data Association Filter (JIPDAF) [131]. Another algorithm [132] derived by integrating the Interactive Multimodal (IMM) estimation algorithm with the JIPDA approximation and linear multi target is the IMM-IPDA method, developed for multi target tracking. A Bayesian algorithm that handles the multi target tracking problem is the Multiple Hypothesis tracking (MHT) [128, 129]. Though MHT is a very powerful algorithm, it is much more complex than the JPDA algorithm.

Many researchers have investigated data fusion algorithms based on KF, but very few literature is available on the scope of IF [3, 81, 122] in target tracking. IF [10] is seen to be advantages over KF in MSDF applications. Hence literature study reveals a scope for investigating the applicability of IF for target tracking applications.

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Chapter - 3

KALMAN FILTER FOR SENSOR FUSION

R.E Kalman, in 1960 published a paper describing a recursive solution to discrete data linear filtering problem [110]. Since then, Kalman filter (KF) has been the subject of extensive research and application. Extensions of the theory of KF to fuse data from multiple sensors also have appeared in literature subsequently. A scholastic introduction to the general idea of KF can be found in Chapter 1of [120] and also in [5, 111]. While more exhaustive coverage on the theory of KF is available in literature [133, 134, 120, 135], the paper by Greg Welch and Gary Bishop [5], provides a detailed practical introduction to KF and Extended Kalman filter (EKF). A KF, which is a Bayesian approach of estimation, is used in cases where the system state cannot be measured directly. Thus, a KF estimates the process state optimally from indirect measurements. As the thesis concentrates on fusion of data, largely using the Information Filter (IF), which is a variant of the KF, a brief introduction to the theory of KF and EKF would be relevant to progress with the discussions in the thesis. Extensions of the theory of KF to the fusion of inputs from multiple sensors are also discussed in the present chapter. The performance of KF and EKF in estimating processes following linear and non-linear stochastic difference equation and also the fusion of inputs from multiple sensors using these filters is also evaluated by simulating examples using MATLAB.

3.1 Kalman Filter as a Stochastic Estimator

The Kalman filter (KF) addresses the general problem of estimating the state $X \in \Re^n$, of a discrete time controlled process governed by the linear stochastic difference equation [5].

$$X_k = AX_{k\text{-}1} + Bu_{k\text{-}1} + w_{k\text{-}1} \qquad (3.1)$$

where, A is the matrix that relates the current state (X_k) and previous state (X_{k-1}). The matrix B relates the optional control input u to the state. The measurements $Z \in \Re^m$ from the system are given by

$$Z_k = HX_k + v_k \qquad (3.2)$$

H is the matrix that relates the state and the measurements. w_k and v_k are random variables that represent process noise and measurement noise respectively, drawn from, white Gaussian distributions, viz. $p(w) \sim N(0,Q)$ and $p(u) \sim N(0,R)$

Given a series of measurements Z_k = {z_1, z_2, … z_k} up to time k, the KF finds out the a-posteriori estimates P(X_k/Z_k) .In essence, the KF finds a solution to the problem of finding the estimate of the state variable, maintaining the first two moments of the state distribution viz. the mean and the variance [5] denoted by $E[X_k] = X_{k/k}$ and $E[(X_k - X_{k/k})(X_k - X_{k/k})'] = P_{k/k}$

In the KF terminology, $X_{k/k}$ refers to the estimate of X at time k, based on the last k measurements. All other variables similarly designated have the same implication. The a posteriori state estimate mentioned above reflects the mean of the state distribution which is normally distributed, if the process noise w_{k-1} and the measurement noise v_k are also normally distributed as noted above.

The filter executes a sequence of predictor - corrector operations given by

Predictor

$$X_{k/k\text{-}1} = AX_{k\text{-}1/k\text{-}1} + Bu_{k\text{-}1} \quad (3.3)$$

$$P_{k/k-1} = AP_{k-1/k-1}A^{'} + Q \quad (3.4)$$

where, " ( ) ′" corresponds to transpose.

Corrector

$$X_{k/k} = X_{k/k\text{-}1} + K_k(Z_k \text{-} H X_{k/k\text{-}1}) \quad (3.5)$$

where, $$K_k = P_{k/k-1}H^{'}(HP_{k/k-1}H^{'} + R)^{-1} \quad (3.6)$$

and $P_{k/k-1}$ is given by Eqn. 3.4. The time update equations project the state and covariance estimates forward from time step *k-1* to step *k*. The term $(Z_k \text{-} H X_{k/k\text{-}1})$, which accounts for the error between the measurement predicated based on the available estimate $X_{k/k\text{-}1}$ and the actual measurement Z_k, is called residual.

The variance of the state estimate is also corrected as

$$P_{k/k} = (I - K_k H_k)P_{k/k-1} \quad (3.7)$$

Starting with $X_{0/0}$ and $P_{0/0}$, the filter progressively computes the next updates, cycling through predication and correction. Therefore in conclusion, the KF yields

$$P(X_k / Z_k) = N(X_{k/k}, P_{k/k}) \quad (3.8)$$

Thanks to the recursive nature of the computation, the KF became more acceptable to the implementers than the Weiner filter [136].

3.2 Extended Kalman filter (EKF)[5]

The general problem of trying to estimate the state $X \in \Re^n$ of a discrete time controlled process that is governed by linear stochastic difference equation is addressed by the KF. If the discrete time controlled process is governed by a non-linear stochastic difference equation or the measurement relationship to the process is non-linear, one has to go for an Extended Kalman Filter (EKF). The process governed by a non-linear stochastic difference equation, whose state vector $X \in \Re^n$ is represented as

$$X_k = f(X_{k-1}, u_{k-1}, w_{k-1}) \tag{3.9}$$

with a measurement, $Z \in \Re^m$.

$$Z_k = h(X_k, v_k) \tag{3.10}$$

where , w_k and v_k are sampled from a Gaussian Process, as mentioned in Sec. 3.1. The EKF turns out to be a KF that linearizes the non linear functions f or h about the current mean and covariance, and simply approximates the optimality of Bayes rule because of linearization.

The equation that linearise Eq. 3.8 and Eq. 3.9 are given by [5].

$$X_k \approx X_k + A_k(X_{k-1} - X_{k-1}) + W_k w_{k-1} \tag{3.11}$$

$$Z_k \approx Z_k + H_k(X_k - X_k) + V_k v_k \tag{3.12}$$

$$X_k = f(X_{k-1}, u_{k-1}, w_{k-1}) \tag{3.13}$$

$$Z_k = h(X_k, v_k) \tag{3.14}$$

X_k and Z_k are the actual state and measurement vectors, X_k and Z_k are the approximate state and measurement vectors obtained by Eq. 3.12 and Eq. 3.13 , X_k is the a posteriori estimate of the state at step k and w_k and v_k represent the process and measurement noise.

A_k and W_k are the Jacobean matrix of partial derivatives of f with respect to X and w respectively which are defined as follows.

$$A_k = \frac{\partial f}{\partial X}(X_{k-1}, u_{k-1}, 0) \tag{3.15}$$

$$W_k = \frac{\partial f}{\partial w}(X_{k-1}, u_{k-1}, 0) \tag{3.16}$$

H_k and V_k are the Jacobean matrix of partial derivatives of h with respect to X and v respectively.

$$H_k = \frac{\partial h}{\partial X}(X_k, 0) \tag{3.17}$$

$$V_k = \frac{\partial h}{\partial v}(X_k, 0) \tag{3.18}$$

Here the Jacobeans A_k, W_k, H_k and V_k are time varying.

Prediction error $\tilde{e}_{x_k}$ and measurement residual $\tilde{e}_{z_k}$ are defined as

$$\tilde{e}_{x_k} = X_k - X_k \tag{3.19}$$

$$\tilde{e}_{z_k} = Z_k - Z_k \tag{3.20}$$

In comparison to KF Eq. (3.3) to Eq. (3.7), the equations of EKF are summarized with minor variations, as follows.

Prediction

$$X_{k/k-1} = A_k X_{k-1/k-1} + Bu_{k-1} \tag{3.21}$$

$$P_{k/k-1} = A_k P_{k-1/k-1} A_k' + W_k Q_{k-1} W_k^{-1} \tag{3.22}$$

$$K_k = P_{k/k-1} H_k' (H_k P_{k/k-1} H_k' + V_k R_k V_k')^{-1} \tag{3.23}$$

Correction

$$X_{k/k} = X_{k/k-1} + K_k (Z_k - H X_{k/k-1}) \tag{3.24}$$

$$P_{k/k} = (I - K_k H_k) P_{k/k-1} \tag{3.25}$$

3.3 Simulation of KF and EKF

The performance of KF and EKF was studied by simulating two typical cases in MATLAB. The first simulation corresponds to the KF estimating a linear stochastic process, while the second one demonstrates an EKF in estimating a process following non-linear measurement model.

3.3.1 Case 1-KF

Following the example given in [5], consider the problem of an incorrect Analog to Digital Converter, whose measurements are corrupted by a 0.1V RMS white Gaussian noise. A simple KF is used to remove this noise and obtain a good estimate of the digital measurement. The following configuration is assumed for the simulation:

i. Since the quantity to be estimated is a constant in this case and A is taken to be 1.
ii. As there is no control input , u is taken as 0
iii. The state in this case is same as the measurement, thus making H=1.
iv. Process noise variance Q, and initial error covariance $P_{0/0}$ are assumed to be *1e-5* and 1 respectively and the measurement noise is taken as, R=0.01 V^2.
v. Actual value of voltage X is taken as 14V.

Kalman filter starts with an initial estimate of 14.06Vand runs for 25 iterations, estimating the ADC output. From Fig. 3.1, it can be seen that the estimate converges to the actual value to be digitized, even though the measurements are corrupted by noise. The true value of the measurements, the noisy observations and KF estimated measurements are presented in Fig. 3.1 to confirm the performance of a simple KF.

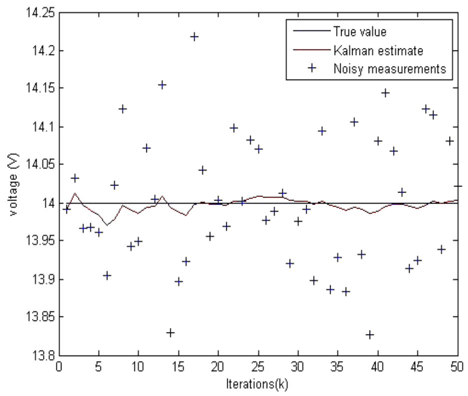

Fig. 3.1 True observations, noisy measurements and Kalman filter estimate

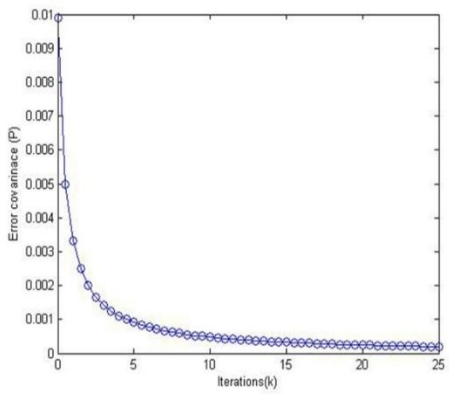

Fig.3.2 Plot of estimate error covariance, which progressively comes down.

It is understood that the values of initial state estimate *X*, *Q* and *R* have a large influence in the performance of the KF. When the process error covariance *Q* is very less compared to the measurement error covariance *R*, the filter believes *Q* more than *R* and the estimate is seen to follow *Q*. In such cases, the variances of the measurement error are very less and poor initial estimate may lead to bad estimate of the process. When *R* is very less compared to *Q*, the filter believes the measurements more than the process model. In such cases, the estimate follows the measurements and it is observed that the estimate variance is larger. Thus proper selection of *Q* and *R* are very important in the performance of a KF. In the case considered here, both *Q* and *R* are chosen to be moderately low values. Hence the KF is seen to give good performance. It is obvious from the estimate error covariance in Fig. 3.2, that the variance decreases and settles to a very low value after 21 iterations, which shows the convergence of the filter.

3.3.2 Case 2- EKF

The problem of tracking a moving target following linear path using bearing only measurements obtained from a single sensor is considered here for estimation using the EKF. Since the bearing angle and the target state have a non-linear relation, an EKF is used in this case to estimate the process.A discrete time linear dynamic system, described by a vector difference equation with additive white Gaussian noise is used for modeling the target [136, 137]. Gaussian noise here models the unpredictable disturbances in the target tracking scenario. The target is assumed to follow a constant velocity (CV) model. The target state evolve in time according to the model

$$X_k = A_{k-1} X_{k-1} + w_k \tag{3.26}$$

where the state vector at time *k* is given by $X_k = \left[x_k, y_k, \mathrm{v}_{xk}, \mathrm{v}_{yk}\right]'$, where x_k, y_k, v_{xk} and v_{yk} are the *x* position, y position, *x* velocity and *y* velocity respectively at time *k* and w_k denotes zero mean white Gaussian process noise with covariance *Q*, as defined in section 3.1. The x and y positions represent the Cartesian coordinates in metre (m) and velocities in m/s.A_k is the process transition matrix defined as

$$A=\begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.27}$$

where, T is the sampling period. The target is observed by a sensor according to the non linear observation model,

$$Z_k = h(k, X_k) + v_k \tag{3.28}$$

The measurement equation is given by

$$Z_k = \tan^{-1}(y_k / x_k) + v_k \tag{3.29}$$

where , v_k is a zero- mean mutually independent white Gaussian noise sequences with known covariance matrices $R_k = E[v_k v_k^{'}]$.

The measurement Jacobean matrix H_k is given by,

$$H_k = \left[\frac{y_k}{x_k^2 + y_k^2}, \frac{-x_k}{x_k^2 + y_k^2}, 0, 0 \right] \tag{3.30}$$

The matrices A, Q and R and the function h are assumed to be known and time varying. The initial state of the targets, $X_{0/0}$ is assumed to be modeled as random vectors, Gaussian distributed with known mean and covariance. The noise sequences v_k and w_k are initially assumed to be mutually independent.

A target moves from an initial state, $X = [50, 50, 5, 6]^{'}$, with following specifications:

i. Initial position: (50 m, 50 m) in Cartesian coordinate system.
ii. The target moves with a constant velocity, where v_x=5 m/s and v_y=6 m/s respectively.
iii. The initial estimate is assumed as $X_{0/0} = [40, 45, 5.1, 6.1]^{'}$.

iv. The error covariance matrix is a 4 x 4 matrix and the value of initial error covariance is, taken as

$$P_{0/0} = \begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 0 \\ 0 & 0 & 0.001\dfrac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\dfrac{\text{m}^2}{\text{s}^2} \end{bmatrix}$$

The measurement error covariance, R=0.5 rad^2 and process error covariance, Q=1e-5 (diagonal). The actual and estimated track of the target, the Mean Squared Error (MSE) in position estimate of the target and velocity estimate are shown in Fig. 3.3, Fig.3.4 and Fig.3.5 respectively.

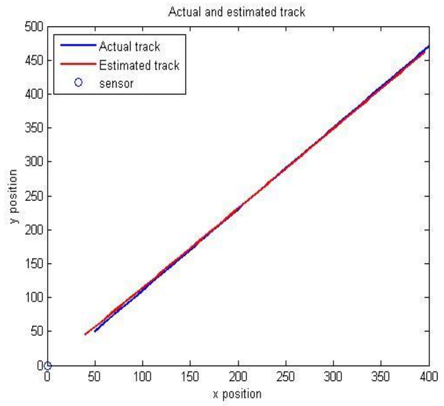

Fig. 3.3 Actual and estimated track of the target

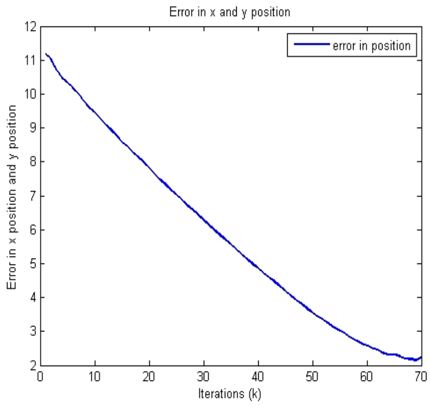

Fig. 3.4 Error in x and y position estimates

The filter starts with an initial error of 10 m and 5 m respectively in x and y directions. The tracking of the filter is observed from the plot of position estimate and error in position estimate by running the filter for 70 iterations. The Mean Squared Error (MSE) in position estimate is calculated as $\sqrt{(x(k)-x(k))^2+(y(k)-y(k))^2}$, in terms of the actual values of x and y positions known to the simulator. It can be seen from the MSE plot in Fig. 3.4, that the filter settles to an error of 2.2 m in 65 iterations from an initial MSE of 11.1 m, which shows that the EKF is able to track the target well. The stability in velocity estimate of the filter adds to the laudable performance of the filter in tracking a contact, moving with constant velocity, from the bearing only measurements.

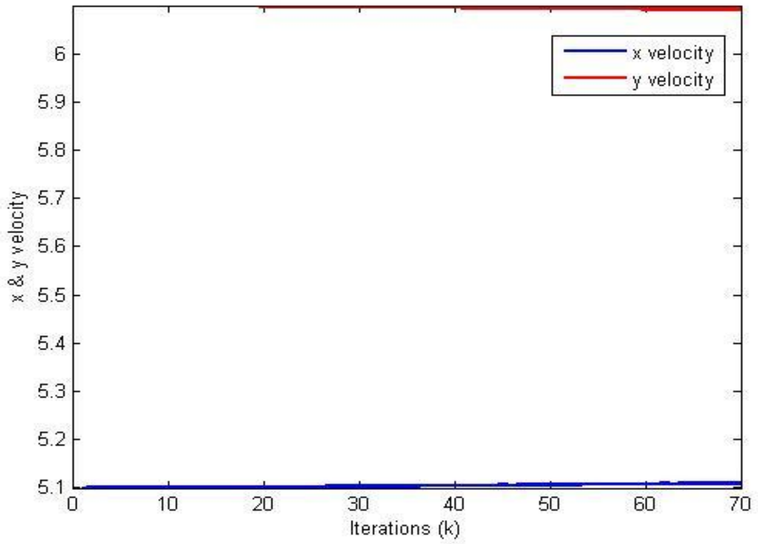

Fig. 3.5 Plot of the velocity estimate;

***x* velocity ≈ 5 m/s and *y* velocity ≈ 6.1 m/s.**

3.4 Multi sensor data fusion (MSDF) by directly fusing the senor data

The advantages of MSDF in target tracking applications were explained in Chapter 1. One of the reported approaches in MSDF is to fuse the sensed data at its origin and then use the fused data for further processing like tracking or feature extraction as illustrated in Fig. 3.6.Consider data from N independent sensors given by the vector $X_k = [x_{1k}, x_{2k} ... x_{nk}]'$. Fused data obtained by minimizing the variance given by [138].

$$x_{fk} = W_k{}' X_k \tag{3.31}$$

where

$$W_k = \frac{R_k^{-1} U}{U' R_k^{-1} U} \tag{3.32}$$

$$\text{and } R_k = E[X_k \; X_k{}'] \tag{3.33}$$

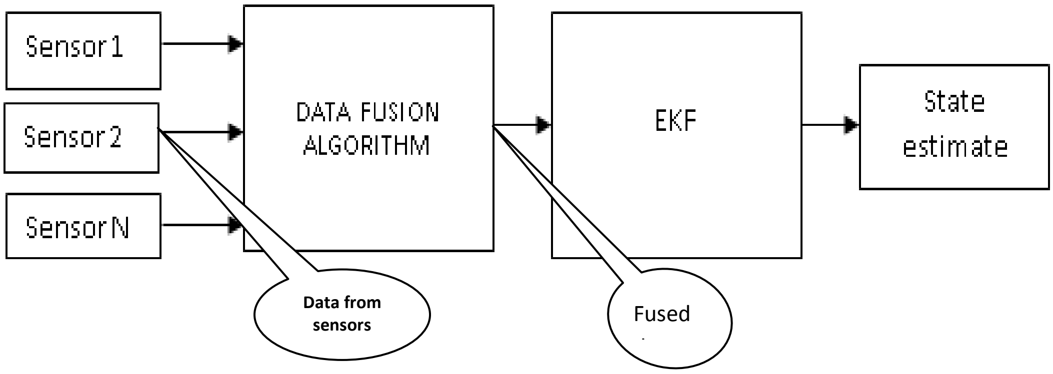

Fig.3.6 Multi sensor target tracking using EKF from fused measurements

The basic concept of variance based fusion is demonstrated through simulation in the example below. Three sensors, having different measurement variances are considered. The sensors measure a 1V sinusoidal signal of frequency 10 Hz. The following configuration is assumed for the simulation:

1. The measurement error variances of the 3 sensors were assumed as 0.2 V^2, 1 V^2 and 0.5 V^2 respectively.
2. *R* is calculated using Eq.3.32. The weights for each of the sensors are computed as $W_k = \frac{R_k^{-1} U}{U' R_k^{-1} U}$.
3. The instantaneous fused signal is computed as $x_{fk} = w_k' X_k$

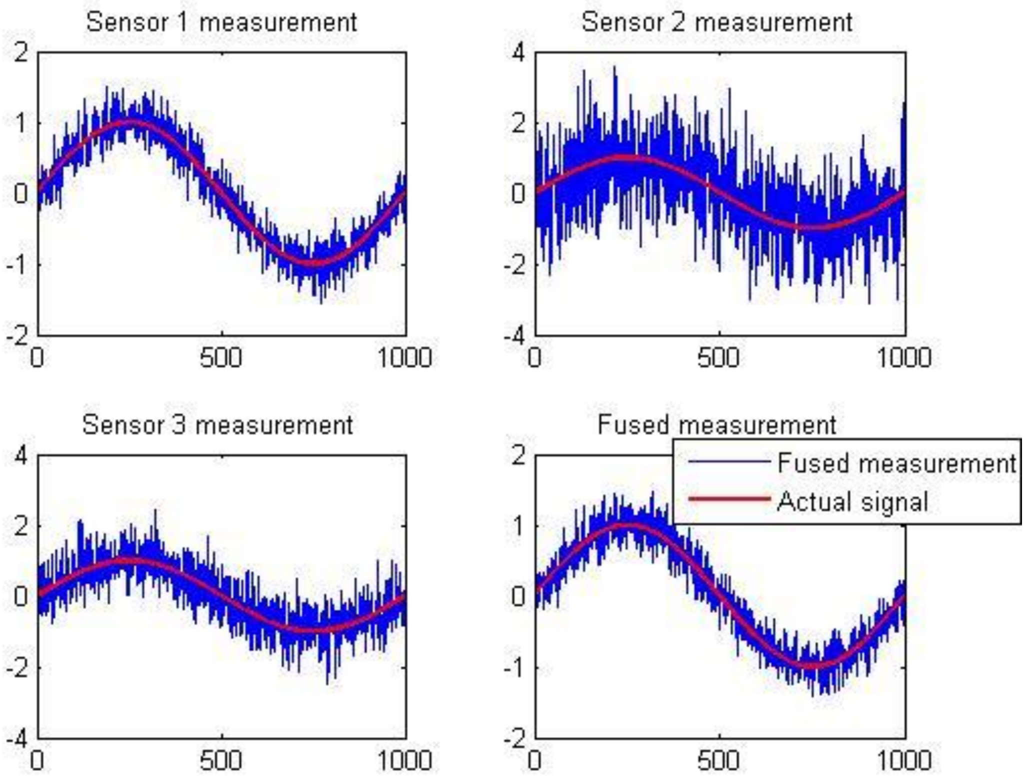

Fig. 3.7 An example illustrating variance based fusion of sensor measurements

Figure 3.7 presents the signals, corrupted by measurement noise, measured by the three sensors in blue along with the actual signal in red. The blue trace in the fourth subplot of the figure is the fused measurement obtained by instantaneously fusing the measurements of the three sensors and the red trace shows the actual signal.

It is observed that the fused measurement has a lower variance, in amplitude, compared to the signals from individual sensors. The reduction in variance is further confirmed from the plot of the instantaneous error plotted in Fig. 3.8. Here instantaneous squared error is computed as $\sqrt{(y_k - y_{mk})^2}$; where y_k represent the actual signal being measured and $y_{m\mathrm{k}}$ represent the sensor measurement at time instant *k*.

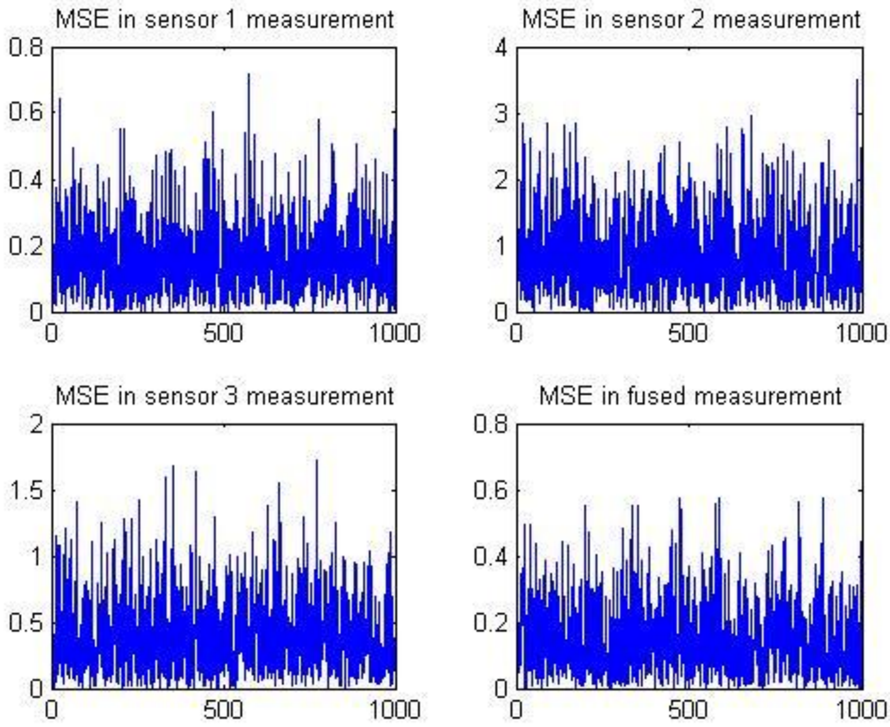

Fig. 3.8 Squared Error plot of individual sensors and fused sensors

Simulation results of Fig.3.7 and the instantaneous error in Fig. 3.8 show that the noise is the lowest among all the three, in sensor 1. The variance based MSDF results in the instantaneous error, which is lower than all the three sensors, thereby underscoring the effect of the fusion technique used. The MSE of each of the sensors and the fused data in Table 3.1 below also confirms the merit of the variance based fusion.

Sensor 1	Sensor 2	Sensor 3	Fused data
0.1616 V	0.7604 V	0.4012 V	0.1576 V

Table 3.1 MSE of the sensors and the fused data

3.5 Multi sensor fusion for target tracking using EKF

Multi sensor data fusion using KF and its variants have been reported [139] , with applications in areas such as radars, sonar systems, guidance and control of autonomous vehicles, medical diagnosis and a variety of smart systems [53]. In target tracking applications, observations from one or more sensors are used to refine the estimate of target's position and velocity [139]. Similarly, observations of the target's attributes may be used to assess the relative motion with respect to the observer, thereby allowing the determination of the intent of the target, (eg. Intruding or evading). Considering the advantage in using multiple sensors, the performance of EKF in the context of multi sensor target tracking is explored below.

Two types of approaches are possible in dealing with multiple sensors, while tracking with EKF.

i. In the first approach, each sensor suite is assumed to be autonomous and the measurement at the suite is used to independently estimate the state of the moving target using separate EKF, running in each sensor suite. Extending the fusion technique reported in [138] and demonstrated in Sec 3.4, the estimates of the individual filters are combined, since each estimate of state is reported along with its variance (earlier shown in Fig.1.3, repeated below for clarity) portrays the approach for fusing three sensors (Fig. 3.9).

ii. The second approach is the Probabilistic Data Association [77] (PDA), which refines the estimate by combining the innovations from multiple sensors as shown in Fig.3.18. The two approaches are discussed in Sec 3.5.1 and 3.5.2 below.

3.5.1 Variance based fusion for target state estimation [89]

In this section, multi sensor target tracking using approach 1 (Fig. 3.9) is considered and the fusion is based on the variances. Here, the target states are estimated by the separate EKF and are fused using the variances of the estimates, to obtain the fused estimate of the target state. Each sensor suite produces the state estimates from independent EKFs, $X_k{}^i = [x_k{}^i, y_k{}^i, v_{xk}{}^i, v_{yk}{}^i]$ at different time instants *k*, along with state co-variance $P^i_{k/k}$, where i=1, 2….*N*, *N* being the number of sensors used. The sensors are located at geographically separate locations and receive only the bearing angle measurements from the moving target. Following [138], the steps involved in determining the fused estimates are as follows. Given a set of states, $X_k{}^i$ with co-variance Σ^i_k, from *i*=1 to *N* sensors at any time *k*, the fused data is given by (in terms of each state variables)

$$x_j^f = \sum_{i=1}^{N} w_{jk}^{i\,'} x_{jk}^i \qquad (3.34)$$

where, j = 1 … number of state variables. Each state variable x^i_{jk} is estimated with a variance of σ^i_j, by the EKF. Considering Σ_{jk} = diagonal (σ^i_j) over *i*, the weight vector for each state variable *j* in the sensor *i* is computed as

$$w^i_{jk} = \frac{\Sigma^{-1}_{jk} U}{U' \Sigma^{-1}_{jk} U} \qquad (3.35)$$

The fused error covariance for the state variable *j* is given by

$$\frac{1}{\sigma^f_j} = \sum_{i=1}^{n} \frac{1}{\sigma^i_j} \;, j = 1 \;\ldots \text{ number of state variables} \qquad (3.36)$$

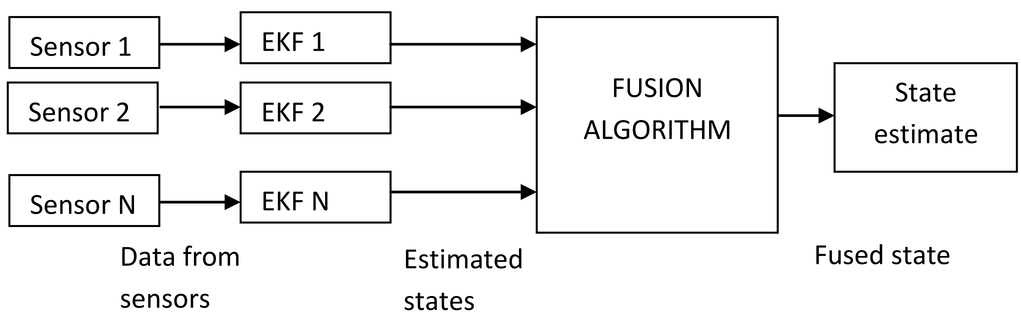

Fig.3.9 Multi sensor target tracking using EKF- approach 1

As each sensor estimates the target position by using the prediction and update equations of the EKF, the state covariance matrix $P_{k/k}$ is directly available from the KF estimating the states for each instance *k*.

A target tracking problem using bearing only measurements received from 4 sensors, using variance based fusion, described above is demonstrated in Sec. 3.5.1.1.The estimated state of the target and MSE in position estimate for different cases of this problem has been simulated and compared.

3.5.1.1 Simulation and Results- Case 1:

The performance of EKF with variance based fusion for target tracking using bearing only measurement received from 4 sensors has been demonstrated by experimenting four cases. This multi sensor target tracking problem tries to estimate the state of a moving target following a CV model as defined in section 3.3.2.

The target state is a 4 dimensional vector representing the *x* position, *y* position, *x* velocity and *y* velocity respectively. The filters are run for 1000 iterations. Here each sensor has its associated EKF for tracking, which is not explicitly mentioned in the different cases discussed below. The MSE in the fused output confirms the improvement in the variance of the fused estimates.

The cases examine how the performance of an EKF is affected by changes in initial state assumption and selection of *Q* and *R* values. Each case also presents the comparison of tracking performance using single sensor and multiple sensors in terms of MSE in position estimate, as defined in section 3.3.2.The simulation scenario for Case 1 is as shown in Table 3.2.

The actual initial state vector of the target. Initial position assumed by the 4 EKFs.	Target: X=[6000m,10000 m,4.7 m/s,4.8 m/s]´ $X1_{0/0}$ = [5800 m,9800 m,4.7 m/s,4.8 m/s]´ $X2_{0/0}$ = [5800 m,9800 m,4.7 m/s,4.8 m/s]´ $X3_{0/0}$ = [5800 m,9800 m,4.7 m/s,4.8 m/s]´ $X4_{0/0}$ = [5800 m,9800 m,4.7 m/s,4.8 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the 4 sensors	$R1$=1 rad^2, $R2$=1 rad^2, $R3$=1 rad^2, $R4$=1 rad^2	Measurement error covariance R
Sensor positions	(1000 m, -500 m),(800 m,1000 m),(100 m,1500 m) and (500 m, 3000 m)	(x, y) position of the 4 sensors.
Q matrix	$Q=\begin{bmatrix} 10e-6\text{m}^2 & 0 & 0 & 0 \\ 0 & 10e-6\text{m}^2 & 0 & 0 \\ 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix, Q

Table 3.2 Simulation scenario for variance based fusion- Case 1

In this case, the initial position estimates of the target differ by 200 m in *x* and *y* coordinates with respect to the actual initial position of the target, which has been considered to be a moderately good initial estimate. Fig. 3.10 shows the actual and estimated paths of the target using variance based fusion. Though the actual track of the target, variance based fused estimate and the estimates of the individual sensors are shown in different colours, the estimated tracks of all sensors and the fused estimate overlap and hence are not clearly distinguishable. The initial assumptions and measurement variances of all the 4 sensors which are assumed to be the same, account for this situation. On the other hand, the MSE plot (Fig.3.11), where the MSE of the fused estimation is plotted along with the MSE in estimation performed by individual sensors, gives a better picture of the performance of variance based fusion. The initial assumptions and measurement variances of all the sensors which are assumed to be the same, account for this situation. On the other hand, the MSE plot (Fig.3.11), where the MSE of the fused estimation is plotted along with the MSE in estimation performed by individual sensors, gives a better picture of the performance of variance based fusion.

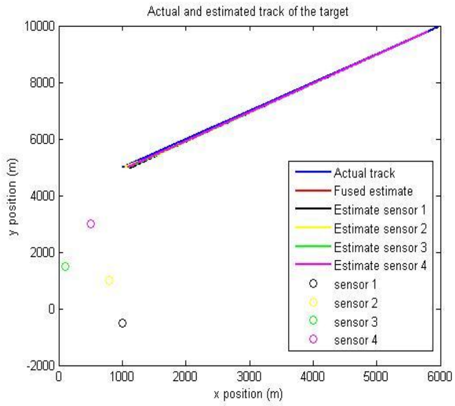

Fig.3.10 Actual and estimated track of the target in Case1

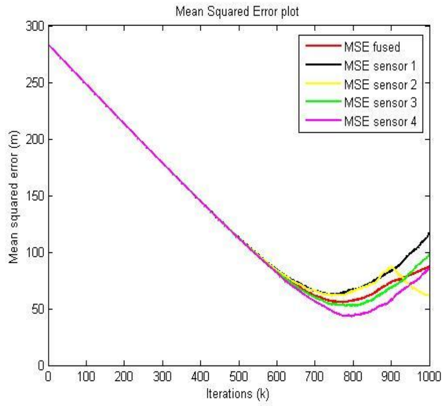

Fig. 3.11 Mean Squared Error in position in Case 1

As the initial assumptions of all sensors are the same, it is observed that the error in the fused estimate is almost the same as those of single participating sensors most of the time and generally takes a mean position between the extremes. It is interesting to note that the error in the fused position estimates (in red) is better, when compared to the estimates of sensor 1 and 2 (in yellow and black), after 500 iterations. Even when the error is showing tendency to diverge, it is interesting to note that the variance based fusion provides a better estimate of the order of 50. As can be seen, the individual filters and the fused output shows signs of divergence, possibly due to observability issues, considering the different geographical location of the individual sensors. The tendency of the error to increase with increase in iterations, after stabilizing , is due to the divergence problems in Kalman filters [135].

3.5.1.2 Simulation and results - Case 2

In this case, the initial state estimates of the sensors 1,2 and 3 are considered to be poor , while the initial estimate of the 4th sensor is relatively good. The simulation scenario for Case 2 is as shown in Table 3.3. The actual track, estimate of the 4 sensors and the fused track are presented in Fig. 3.12. As can be seen, the fused track takes a mean position compared to the extreme performance of others.

The corresponding MSE of the estimates given in Fig. 3.13 further brings out that

i. the MSE of the estimates of sensors 1,2 and 3 are very large initially, however reduces to lower values as time progresses,

ii. the sensor 4 with good initial estimate start with an initial MSE of 300 m and reduces to reduces to 50 m, consistently in 800 iterations and

iii. the fused estimate has an initial MSE of 750 m and reduces to 500 as time progresses.

The actual initial state vector of the target. Initial position assumed by the 4 EKFs.	Target : X = [6000m, 10000 m, 5 m/s, 5 m/s]´ $X1_{0/0}$ = [5400 m, 9400 m, 4.7 m/s, 4.8 m/s]´ $X2_{0/0}$ = [5200 m, 9200 m, 4.7 m/s, 4.8 m/s]´ $X3_{0/0}$ = [5000 m, 10000 m, 4.7 m/s, 4.8 m/s]´ $X4_{0/0}$ = [5800 m, 9800 m, 4.7 m/s, 4.8 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix}1 & 0 & -T & 0\\ 0 & 1 & 0 & -T\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix}10\text{m}^2 & 0 & 0 & 0\\ 0 & 10\text{m}^2 & 0 & 1\\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0\\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2}\end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the 4 sensors	$R1$=1 rad^2 , $R2$=1 rad^2,$R3$=1 rad^2 , $R4$=1 rad^2	Measurement error covariance R
Sensor positions	(1000 m, -500 m), (800 m,1000 m), (100 m,1500 m) and (500 m, 3000 m)	(x, y) position of the 4 sensors.
Q matrix	$Q=\begin{bmatrix}10e-6\text{m}^2 & 0 & 0 & 0\\ 0 & 10e-6\text{m}^2 & 0 & 0\\ 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} & 0\\ 0 & 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2}\end{bmatrix}$	Process noise covariance matrix, Q

Table 3.3 Simulation scenario for variance based fusion- Case 2

As observed in the previous case, the sensors with good initial estimates provide good estimation of the target state, while the poor initial assumptions of the filters have led to poor tracking. It is also seen that the fused estimate is always better than all the poor estimates, but not as good as the best estimate, as is seen in the MSE plot. With increase in iterations, variance based fusion filter catches track of the target, as the better estimates from sensor, has greater effect on the fused estimate. The remark goes well with the fact that the weights for fusion are calculated using the inverse of the state covariance matrix, thereby making the good estimates getting higher weights in fusion compared to the poor estimates.

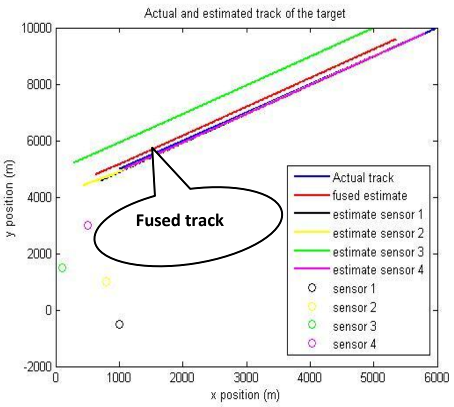

Fig.3.12. Actual and estimated path of the target in Case 2

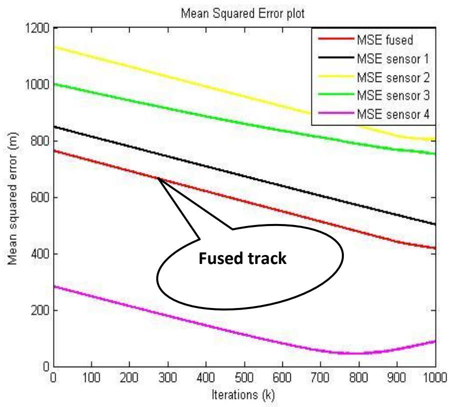

Fig.3.13 Mean squared error in position in Case 2

3.5.1.3 Simulation and Results- Case 3

In this case, the effect of different values of measurement error variances in the tracking problem is demonstrated. Though the initial estimates are assumed to be good as in Case 1, the measurement variances are assumed to be large, for three sensors. The simulation scenario for Case 3 is as shown in Table 3.4.

The actual initial state vector of the target. Initial position assumed by the 4 EKFs.	Target : X = [6000m,10000 m,5 m/s,5 m/s]´ $X1_{0/0}$ = [5800 m,9800 m,4.7 m/s,4.8 m/s]´ $X2_{0/0}$ = [5900 m,9900 m,4.7 m/s,4.8 m/s]´ $X3_{0/0}$ = [5900 m,10000 m,4.7 m/s,4.8 m/s]´ $X4_{0/0}$ = [5900 m,9900 m,4.7 m/s,4.8 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix}1 & 0 & -T & 0\\ 0 & 1 & 0 & -T\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix}10\text{m}^2 & 0 & 0 & 0\\ 0 & 10\text{m}^2 & 0 & 1\\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0\\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2}\end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the 4 sensors	$R1$=1 rad^2 , $R2$=10 rad^2,$R3$=10 rad^2 , $R4$=10 rad^2	Measurement error covariance R
Sensor positions	(1000 m, -500 m),(800 m,1000 m),(100 m,1500 m) and (500 m, 3000 m)	(x, y) position of the 4 sensors.
Q matrix	$Q=\begin{bmatrix}10e-6\text{m}^2 & 0 & 0 & 0\\ 0 & 10e-6\text{m}^2 & 0 & 0\\ 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} & 0\\ 0 & 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2}\end{bmatrix}$	Process noise covariance matrix, Q

Table 3.4 Simulation scenario for variance based fusion- Case 3

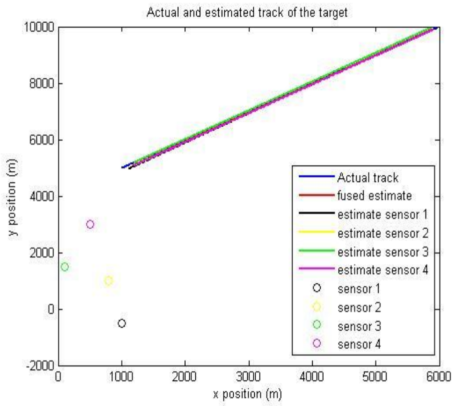

Fig. 3.14 Actual and estimated path of the target in Case 3

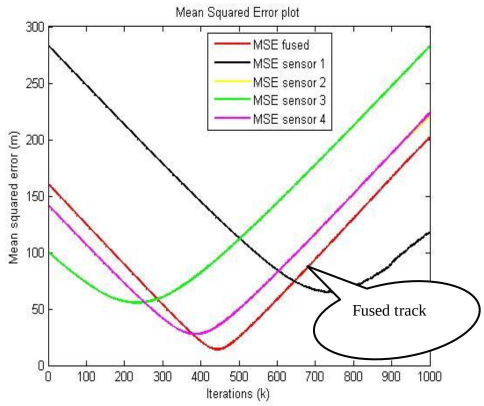

Fig.3.15 Mean squared error in position estimate in Case 3

The actual and estimated tracks and MSE in estimation for Case 3 are presented in Fig. 3.14 and Fig. 3.15. It is observed from the plot that all the sensors track the target well and the fused estimate is also good, since the initial estimates are good. However, the effect of *R* on the performance of the filter is interesting to be noted (Fig.3.15).

The filter catches the track in less iteration for sensors 2, 3 and 4, which have relatively larger value of *R*. The MSE plot of sensor 2 (yellow trace) is not visible as it overlaps with sensor 4, on account of same initial estimate and *R*. The sensors with larger measurement error variance and good initial position estimate, catches track in 450 iterations compared to sensors with low measurement variance, which track in 800 iterations. However, the fused track gives the best estimate after 450 iterations, even when the filter is showing the inclination to diverge.

3.5.1.4 Simulation and Results- Case 4

The simulation scenario for Case 4 is as shown in Table 3.5.

The actual initial state vector of the target. Initial position assumed by the 4 EKFs.	Target : $X = [6000\text{m}, 10000\text{ m}, 5\text{ m/s}, 5\text{ m/s}]'$ $X1_{0/0} = [5200\text{ m}, 9200\text{ m}, 4.7\text{ m/s}, 4.8\text{ m/s}]'$ $X2_{0/0} = [5200\text{ m}, 9500\text{ m}, 4.7\text{ m/s}, 4.8\text{ m/s}]'$ $X3_{0/0} = [5900\text{ m}, 10000\text{ m}, 4.7\text{ m/s}, 4.8\text{ m/s}]'$ $X4_{0/0} = [5900\text{ m}, 9900\text{ m}, 4.7\text{ m/s}, 4.8\text{ m/s}]'$	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0} = \begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the 4 sensors	$R1$=1 rad^2 , $R2$=1 rad^2,$R3$=10 rad^2 , $R4$=10 rad^2	Measurement error covariance R
Sensor positions	(1000 m, -500 m),(800 m,1000 m),(100 m,1500 m) and (500 m, 3000 m)	(x, y) position of the 4 sensors.
Q matrix	$Q=\begin{bmatrix} 10e-6\text{m}^2 & 0 & 0 & 0 \\ 0 & 10e-6\text{m}^2 & 0 & 0 \\ 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix, Q

Table 3.5 Simulation scenario for variance based fusion- Case 4

The initial state estimates of sensors 1 and 2 are considered poor, while the measurement variance, *R* is assumed to be good for both these sensors. The sensors 3 and 4 are assumed to have good initial estimates but poor measurement variances.

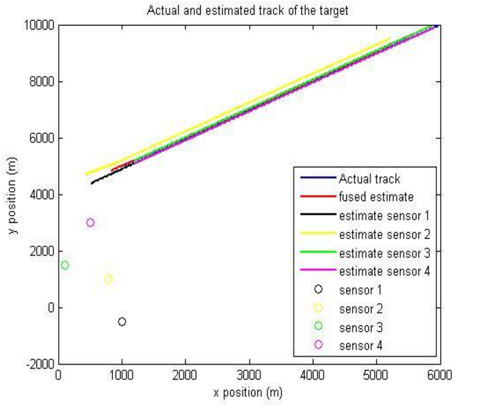

Fig. 3.16 Scenario plot in Case 4

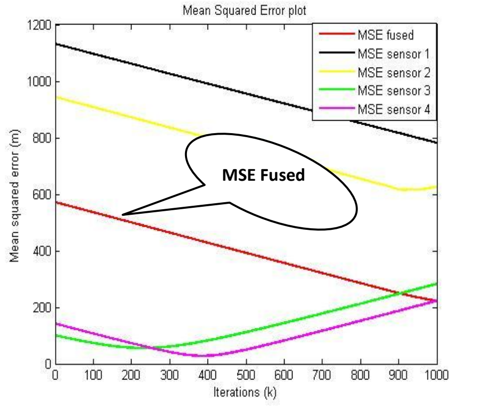

Fig.3.17 MSE in position estimate in Case 4

The scenario plot and the MSE in estimation for this case are shown in Fig. 3.16 and Fig. 3.17.The individual EKFs are seen to catch track in 200 iterations, for the sensors 3 and 4, having very good initial estimate, though with a large value of *R*. For sensors 1 and 2, having large errors in initial estimate, the filters get the track later only, though they have relatively lesser values of *R*. But the fused estimate based on variances, consistently maintains a better track with low error.

Summarizing the performance of variance based fusion, in target tracking; it is observed that the MSE in position of the fused estimate is better than the individual state estimates of sensors, even with poor initial estimates and large measurement variance. Though poor initial estimates and large measurement variances can lead to loss of track in due course and also a poor variance in the fused state estimate, it is seen that fusion helps to produce estimates with less MSE compared to the individual sensors. The application of variance fusion in target tracking application is a contribution of this research.

3.5.2 PDA algorithm

The Probabilistic Data Association Filter (PDAF) [77] is a multi sensor target tracking algorithm based on EKF. This fusion algorithm is used in cases, where there is measurement origin uncertainty. Uncertainty in measurement occurs, when the signal from the target is weak, and the detection threshold has to be reduced to detect it, which leads to the detection of background signals and noise [80]. Hence the biggest challenge in this type of tracking problem is data association. PDAF filter, which attaches weights to the innovations from the measurements of each sensor, corrects state using the weighted sum of innovations. Fig. 3.18 illustrates the technique.

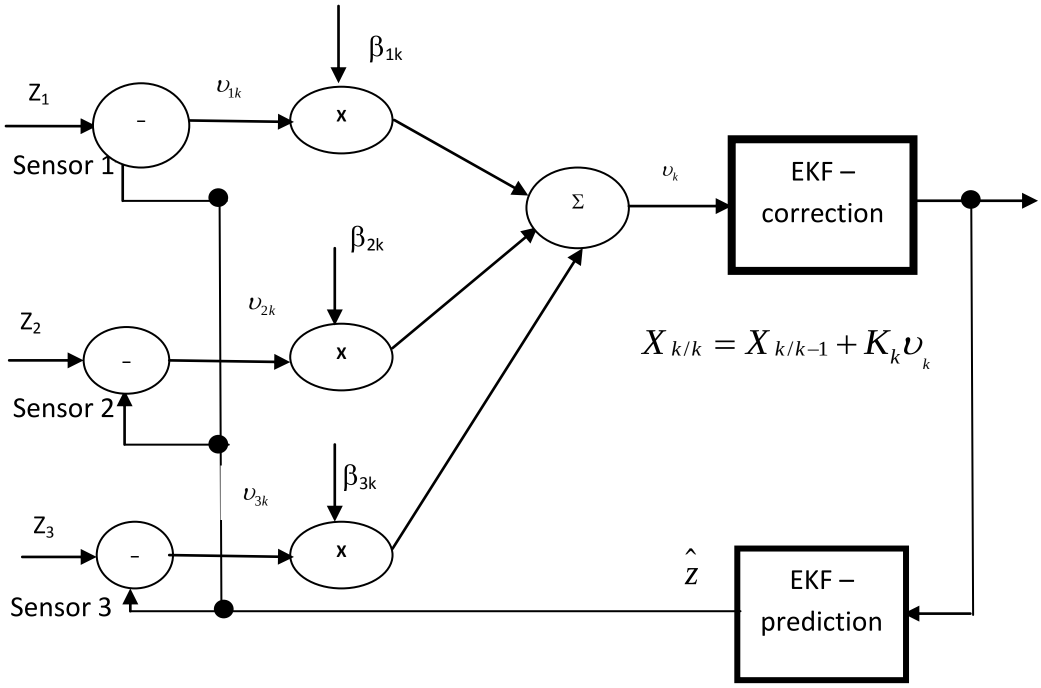

Fig. 3.18 PDA technique

The main steps involved in PDAF are as follows

1. Calculates the association probabilities (β_{ik}) to the target being tracked for each validated measurement at time *k*, as shown in Fig. 3.18.

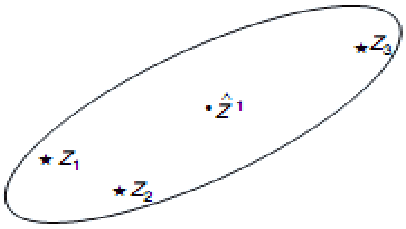

Fig. 3.19 Several measurements Z_i in the validation region of a single target

2. Calculates the association probabilities to the target being tracked for each validated measurement at time *k*. The validation region is an ellipse centered at the predicted measurement Z^1 as in Fig.3.19.
3. This probabilistic or Bayesian information is used in the PDAF tracking algorithm, which accounts for the measurement origin uncertainty.

$$X_k = \left[x_k, y_k, v_{xk}, v_{yk}\right]' \tag{3.37}$$

The association probability for Z_{ik} to a correct measurement is computed as

$$\beta_{ik} = \begin{cases} \dfrac{L_{ik}}{1 - P_D P_G + \sum_{j=1}^{m_k} L_{jk}} ; i = 1, 2 \ldots m_k \\ \dfrac{1 - P_D P_G}{1 - P_D P_G + \sum_{j=1}^{m_k} L_{jk}} ; i = 0 \end{cases} \tag{3.38}$$

where, P_D is the target detection probability , P_G is the gate probability and likelihood function is defined as

$$L_{ik} = \frac{N[Z_{ik}; Z_{k,k-1}, S_k] P_D}{\lambda} \tag{3.39}$$

where λ is the parameter in Poisson clutter model and S_k is as given in Eqn. 3.34. For the PDAF, the state updation equation of the EKF gets modified as given by the equation.

$$X_{k/k} = X_{k/k-1} + K_k \upsilon_k \tag{3.40}$$

where, K_k is the Kalman gain

$$K_k = P_{k/k-1} H_k ' S_k^{-1} \tag{3.41}$$

and υ_k is the combined innovation.

Here the innovation covariance matrix,

$$S_k = H_k P_{k/k-1} H_k ' + R_k \tag{3.42}$$

and combined innovation υ_k is,

$$\upsilon_k = \sum_{i=1}^{m_k} \beta_{ik} \upsilon_{ik} \tag{3.43}$$

The combined innovation covariance is used for updating the estimated state of the target.

As in the case of EKF, the covariance of the updated state is given by

$$P_{k/k} = P_{k/k-1} - K_k S_k K_k ' + P_k \tag{3.44}$$

where P_k is the spread of innovation term which is computed by

$$P_k = K_k [\sum_{i=1}^{m_k} \beta_i \upsilon_{ik} \upsilon_{ik} ' - \upsilon_k \upsilon_k '] K_k ' \tag{3.45}$$

3.5.2.1 Simulation and Results

In order to illustrate the performance of a PDAF, in target tracking application, the scenario described in Table 3.6.

The actual initial state vector of the target. Initial position assumed by the PDAF.	Target : X = [6000m,10000 m,5 m/s,5 m/s]´ $X1$ = [5900 m,9900 m,4.9 m/s,4.9 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the 4 co located sensors	$R1$=2 rad^2 , $R2$=3 rad^2,$R3$=4 rad^2 , $R4$=5 rad^2	Measurement error covariance R
Sensor positions	(0,0)	(x, y) position of the 4 co-located sensors.
Q matrix	$Q=\begin{bmatrix} 10e-6\text{m}^2 & 0 & 0 & 0 \\ 0 & 10e-6\text{m}^2 & 0 & 0 \\ 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 10e-6\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix, Q

Table 3.6 Simulation scenario for PDAF

The PDAF receives 4 bearing angles simultaneously from the co located sensors. The actual and estimated track of the target is given in Fig. 3.20.The PDAF estimates the track well as can be seen from the plot, where the estimated track overlaps the actual track. This is further confirmed from the MSE plot of Fig. 3.21, where the error in position estimate reduces from 150 m to as low as 22 m in 1000 iterations. The MSE in position estimate is calculated as in section 3.3.2.

The variance of the individual bearing measurements of the 4 sensors and the fused bearing measurement are plotted in Fig. 3.22.It is the fused bearing measurement that is used for updating the state of the moving target. It is clear from the variance plot in Fig. 3.22, that the fused bearing in blue approximates the actual bearing measurement of the target in red, and has the least variance compared to the individual sensor measurements. The traces are not clearly distinguishable in the plot as the blue trace overlaps the red trace.

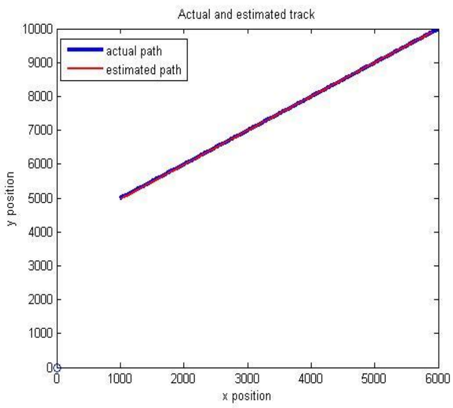

Fig. 3.20 Actual and estimated track of the target

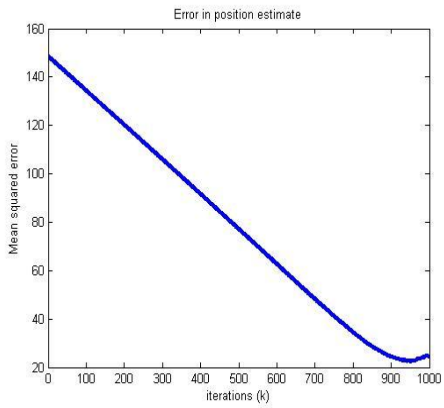

Fig. 3.21 MSE in position estimate

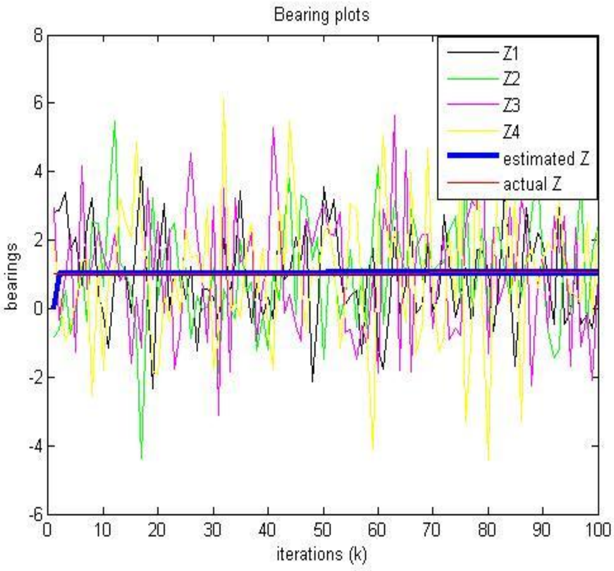

Fig. 3.22 Variance of the fused measurement (in blue is minimum)

3.6 Divergence of Kalman filter

It is convincingly seen that proper selection of Q, R and initial estimates are essential for obtaining good estimates using Kalman and EKF and fusion techniques discussed in the present chapter do not provide any panacea to mitigate the problems arising out of divergence. The divergence in Kalman filter and the EKF is tougher in the case of the bearing only tracking (BOT) and divergence may be attributed to one of the following.

A true divergence may be observed due to system modeling errors. When the process model is not correct, the Kalman filter will not be able to estimate the system parameters.

1. An apparent divergence is observed in Kalman filter when the initial state of the system is badly estimated.
2. There can be numerical divergence due to round off errors in filter computations and also due finite precision arithmetic.
3. Combination of any of the above three reasons may also lead to divergence.

Literature provides a number of techniques to alleviate the problem of divergence in Kalman filter [140, 141]. There are techniques for determining the optimal value of Q and R of the filter [140] and hence apparent divergence can be controlled to a large extent. It is to be noted that, true divergence can be avoided only by proper system modeling, which is an essential requirement for KF and EKF. There are also cases where the filter has to estimate the parameters of the system that switches between different models. Multiple models can also be integrated with EKF for state estimation. Though KF has been the area of extensive study for almost five decades, overcoming divergence, especially for the BOT problem is an area that is still eluding a comprehensive solution.

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Chapter - **4**

INFORMATION FUSION FILTER IN TARGET TRACKING

The advantage of multi sensor fusion was demonstrated in Chapter 3, where in the fusion techniques, based on the EKF for target tracking was evaluated. Considering the influence of initial assumptions on the performance of the filter, the information filter (IF) [81], which is a modified version of KF, the effectiveness of using IF for fusing multiple sensors for target tracking is examined in the present chapter. Instead of fusing the received measurements or the states estimated from measurements, the IF fuses the information content of the received measurements in target tracking. This fusion technique uses simple arithmetic and is shown to be quite efficient [81], compared to the various fusion techniques described in literature. Unlike a KF, the IF carries out the recursive computation of the inverse of the covariance matrix. It is less demanding computationally for systems and is preferred in those cases, where dimension of the measurement vector is larger than that of the state [4]. It also has the advantage that the filter can start its estimation without an initial estimate, unlike KF. In IF, with no initial estimate, we can start the initial information matrix as zero i.e. $P^{-1}_{0/0} = \mathbf{0}$.This results in a non-informative prior, because of the infinite uncertainty associated with it. This chapter presents a comparison of the IF and KF algorithm and examines the performance of information fusion filter (IFF), in terms of mean square error (MSE) in estimating the track of a moving target.

4.1 Decentralized algorithm [81]

Many decentralized data fusion algorithms are surprisingly more efficient in terms of computation and communication, compared to conventional centralized fusion systems [81]. The conventional fusion algorithms employ the concept of state, for example position, velocity, acceleration etc. to learn the measurements and estimate the process. In decentralized fusion algorithms, the concept of information is used for understanding the sensor measurements. The information is defined in the sense of Fisher and Shannon information [10]. The benefit of using information measure is that separating the new information from the prior knowledge is a straight forward technique and also the assimilation of information measure is additive. Since this fusion technique does not give much concern to the order of information received from sensors or assimilation, it is said to be associative and hence can be decentralized. This leads to the concept of information fusion filter. The conventional data fusion algorithm like the PDA based on Kalman filter, performs state fusion in which the time information and how the estimates are constructed are important. Such fusion algorithms are said to be non-associative [10]. The decentralized fusion algorithm finds wide applications in tracking and controlling mobile robots and Unmanned Ariel Vehicles. The IF is the most important tool in decentralized data fusion systems. The information fusion filter (IFF) uses very simple associative algorithms for assimilating the information measures that are communicated from various sensors. The section to follow gives a brief introduction to the Information Filter, bringing out the major differences from the KF.

4.2 Information filter

This section discusses the algorithm of information filter for tracking a moving target using measurements received from multiple sensors. The target state is taken as a four dimensional vector consisting of the *x* and *y* position of the target and the velocities in the *x* and *y* directions. The target state vector at any time, k is defined as $X_k = [x_k, y_k, \mathrm{v}_{xk}, \mathrm{v}_{yk}]'$.The target in this experiment is assumed to follow a CV model [135], where the process governed by a non-linear stochastic difference equation, whose state vector $X \in \Re^n$ is represented as

$$X_k = f(X_{k-1}, u_{k-1}, w_{k-1}) \tag{4.1}$$

with a measurement $Z \in \Re^m$.

$$Z_k = h(k, X_k) + v_k \tag{4.2}$$

where, h is the measurement function that relates the measurement to the target state. The process noise w_k. and the measurement noise v_k are un-correlated noise as mentioned in Sec. 3.1 [4].The term information in information filter is used in the sense of Cramer-Rao lower bound, where the Fischer information matrix is the inverse of the covariance matrix.

4.2.1 Information filter for tracking with inputs from a single sensor

The standard version of conventional KF estimates the state of the target at time k, $X_{k/k}$, given all the observations up to time k, along with a corresponding estimate covariance, $P_{k/k}$, while the IF calculates recursively the inverse of the covariance matrices $P_{k/k}^{-1}$ for both prediction and update equations [4]. The information filter formulation of KF can be obtained by re-writing the state variable and covariance in terms of two new variables y_k and, Y_k called the information state and corresponding information matrix respectively. These variables relate to the state and covariance as [18, 140].

$$Y_k = P_k^{-1} \tag{4.3}$$

$$y_k = Y_k X_k \tag{4.4}$$

The information associated with an observation Z_k are i_k and I_k, given by

$$i_k = H_k' R_k^{-1} Z_k \tag{4.5}$$

$$I_k = H_k' R_k^{-1} H_k \tag{4.6}$$

where , H_k is the Jacobean of the measurement function defined in Eq.3.17 and it is obvious that $I_k = E[i_k i_k^{'}]$.The predicted information state y_k and information matrix Y_k are given as

$$y_{k/k-1} = Y_{k/k-1} X_{k/k-1} \quad (4.7)$$

$$Y_{k/k-1} = \left(A Y_{k/k-1}^{-1} A^T + Q \right)^{-1} \quad (4.8)$$

The estimated information state and information matrix [76] are updated by

$$y_{k/k} = y_{k/k-1} + i_k \quad (4.9)$$

$$Y_{k/k} = Y_{k/k-1} + I_k \quad (4.10)$$

The advantage of IF over KF is evident from the relatively simpler update equations. In the case of IF, the updating terms have the dimension of the state, while in conventional KF, the updating terms have the dimension of the observation vector (modified by the Kalman Gain).

4.2.2 Simulation of Information filter for target tracking

The problem of tracking a moving target following CV model using bearing only measurements obtained from a single sensor is considered here for comparing the performance of IF and EKF. The target scenario is as shown in Fig. 4.1.

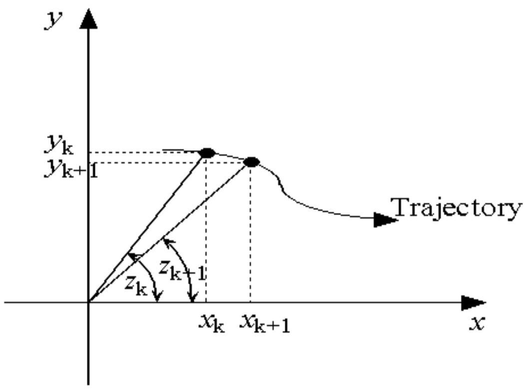


Fig. 4.1 Target scenario

The simulation scenario is shown in Table 4.1.

The actual initial state vector of the target. Initial position assumed by the IF.	Target : X = [8000m,10000 m,5 m/s,3 m/s]′ $X_{0/0}$ = [7900 m,9900 m,4.9 m/s,2.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 100\text{m}^2 & 0 & 0 & 0 \\ 0 & 100\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensor.	R=10 rad^2	Measurement error covariance R
Sensor position	(-2500 m,-500 m)	(x, y) position of the sensor.
Q matrix	$Q=\begin{bmatrix} 0.001\text{m}^2 & 0 & 0 & 0 \\ 0 & 0.001\text{m}^2 & 0 & 0 \\ 0 & 0 & 0.0001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.0001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix, Q

Table 4.1 Simulation scenario for comparing performance of IF and EKF

The IF is run for 700 iterations and the scenario plot, velocity estimate and error in position estimate of the information filter are shown in Fig. 4.2(a), Fig 4.2(b) and Fig 4.2(c) respectively.

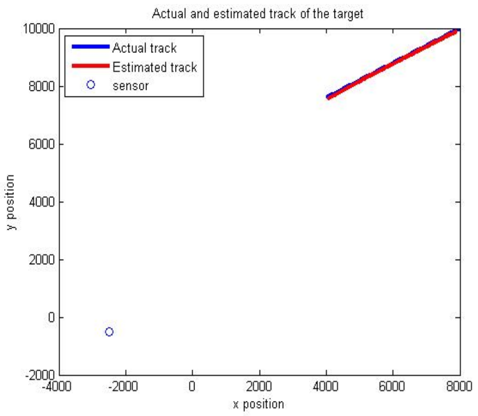


Fig. 4.2(a) Scenario plot –IF

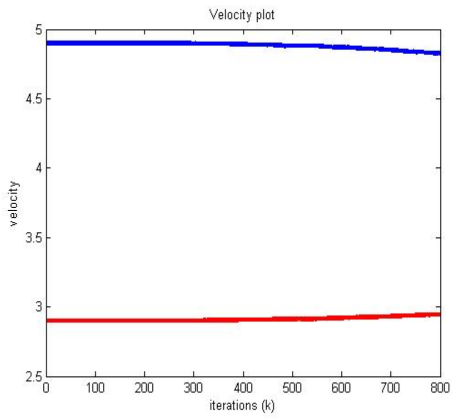


Fig. 4.2(b) Velocity estimate –IF

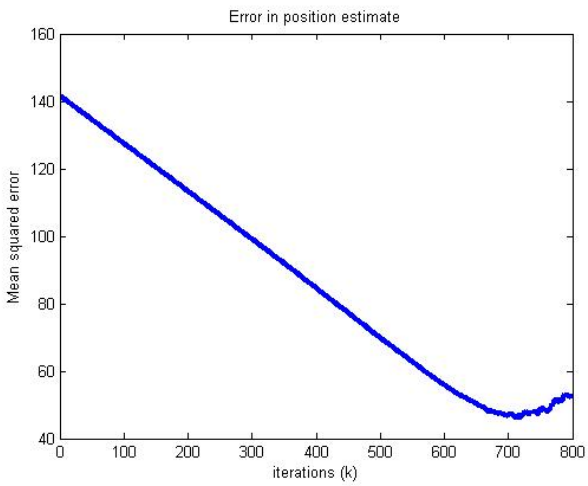


Fig. 4.2 (c) MSE in position estimation-IF

The estimated track of the target, MSE in position estimate and velocity estimate of an EKF for the same scenario are plotted in Fig 4.3(a), Fig 4.3 (b) and Fig 4.3(c) respectively.

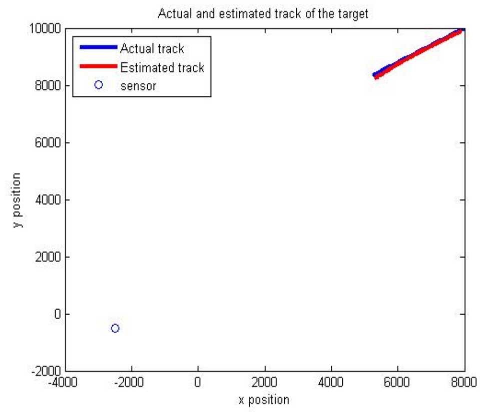


Fig. 4.3(a) Scenario plot –EKF

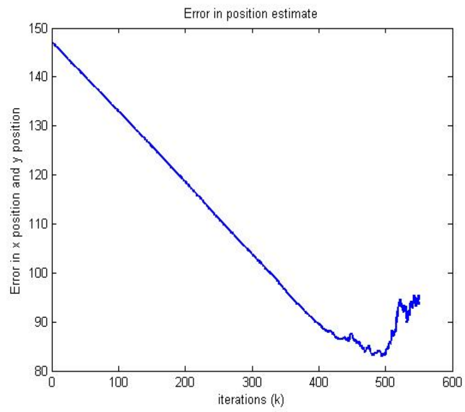


Fig. 4.3(b) Error in position estimation-EKF

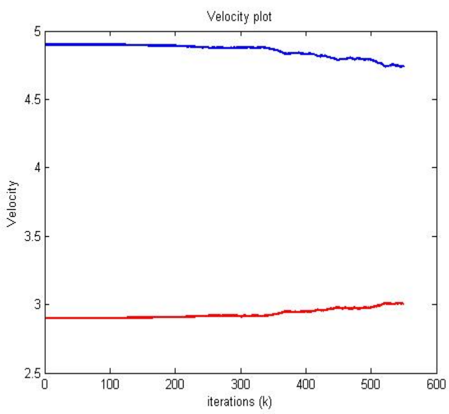


Fig. 4.3(c) Velocity estimate –EKF

It is observed from the simulation results that the IF, like the KF keep track of the target for some period of time and then begins to lose track. This is evident from Fig. 4.2(c), where the error in position estimate reduces to as low as 60 m in 700 iterations and then begin to increase. This indicates the tendency of IF to diverge, which is also confirmed from the instability in velocity estimate after 700 iterations (Fig. 4.2 (b)).

The tracking results of the EKF for the same scenario is plotted in Fig 4.3.It is observed that the performance in KF is similar to section 3.3.2.Here, it is observed that EKF shows tendency of divergence in about 450 iterations as observed in the MSE plot and velocity estimate (Fig.4.3(b) and Fig. 4.3(c)).

4.3 Information filter for multi sensor fusion

The IF scores over the conventional KF in multi sensor estimation problems. It was noted in Sec. 4.2. , that the information generated from any given sensor , in the form of i_k and its co-variance I_k can be added to the information state and information matrix respectively, as given by Eq. 4.9 and Eq. 4.10. Extending the concepts to multiple sensors, the information from different sensors are linearly combined to compute the update estimate of the information state and the information matrix Eq. 4.11 and Eq. 4.12 [122]. Therefore, in multi sensor estimation of states, the information state and information matrix are updated by

$$y_{k/k} = y_{k/k-1} + \sum_{i=1}^{N} i_k^{\,i} \tag{4.11}$$

$$Y_{k/k} = Y_{k/k-1} + \sum_{i=1}^{N} I_k^{\,i} \tag{4.12}$$

where, N refers to the number of sensors. Finally, the estimated state of the target is given by

$$X_{k/k} = Y_{k/k}^{-1} y_{k/k} \tag{4.13}$$

The $X_{k/k}$ estimated may be contrasted with the estimate in the case of PDA (which uses the EKF for fusing multiple sensor inputs. In the case of PDA, the estimate cannot be constructed directly summing the contributions from each sensor; but uses a weighted sum of individual sensor contributions in the form of innovations, where the weights are likelihood functions). While the innovations generated from sensors in KF based fusion are correlated, the information from each sensor is uncorrelated [10]. Since, the IF combines the information from multiple sensors; the filter in this case is referred to as Information Fusion Filters

(IFF). The fusion can be done at one central place or at each of the sensor suites independently, if all information is made available to the sensor suites. The IFF can thus execute a decentralized fusion algorithm.

As was note above, the information measures computed by each sensor are communicated to all other sensors for estimating the state of the target. In a typical BOT problem, sensor suites which are located a different locations exchange the information they gather from the bearing observation. The measurement in Eq. 4.2 is a one dimensional vector. Typical scenario could consist of four stationary sensor suites, which are located at geographically separate positions tracking a moving target (Fig. 4.4). At any time *k*, each sensor gets only the bearing update, Z_k^i, *i*=1,2,3 and 4, relative to its location. The sensors are assumed to have different measurement variances associated with them.

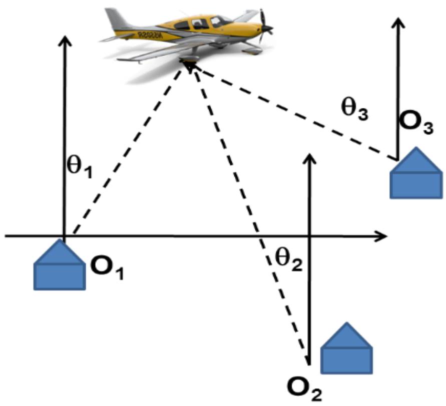


Fig 4.4 Simulation scenario

The simulations below for the scenario illustrated in Fig. 4.4, demonstrates the fusion of measurements from multiple sensors, which also give interesting leads to the dependence on initial assumptions on target positions, errors in plant and measurement, in obtaining sustained tracking, using the IFF.

4.3.1 Simulation and results of IFF for single target tracking -Case 1

The scenario for Case 1 is as shown in Table. 4.2.

The actual initial state vector of the target. Initial position assumed by the IFF.	Target : X = [8000m,10000 m,5 m/s,3 m/s]′ $X1_{0/0}$ = [7900 m,9900 m,4.9 m/s,2.9 m/s]′ $X2_{0/0}$ = [7900 m,9900 m,4.9 m/s,2.9 m/s]′ $X3_{0/0}$ = [7900 m,9900 m,4.9 m/s,2.9 m/s]′ $X4_{0/0}$= [7900 m,9900 m,4.9 m/s,2.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 100m^2 & 0 & 0 & 0 \\ 0 & 100m^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{m^2}{s^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{m^2}{s^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensor.	$R1$=5 rad^2,$R2$=10 rad^2,$R3$=15 rad^2,$R4$=20 rad^2	Measurement error covariance R
Sensor positions	(-2500 m,-500 m), (-5000 m,-1000 m), (-4000 m,-1500 m) and (-3000 m,1500 m)	(x, y) position of the sensor.
Q matrix	$Q=\begin{bmatrix} 0.001m^2 & 0 & 0 & 0 \\ 0 & 0.001m^2 & 0 & 0 \\ 0 & 0 & 0.0001\frac{m^2}{s^2} & 0 \\ 0 & 0 & 0 & 0.0001\frac{m^2}{s^2} \end{bmatrix}$	Process noise covariance matrix, Q

Table 4.2 Simulation scenario for IFF- Case 1

The performance of the IFF is gauged in terms of the mean squared error in the estimation of position, for different scenarios. As in the previous cases, the mean squared error (MSE) in estimation is calculated as $\sqrt{(x_k - x_k)^2 + (y_k - y_k)^2}$, where x_k and y_k refer to the actual position of the target and x_k and y_k refer to the estimated positions of the target. The location of the sensors and the target are represented in Cartesian coordinate system. The initial assumption of the target position by all sensors is considered to be good as it differs from the actual position by only 100 m each in x and y directions. The filters are run for 800 iterations. The plot of actual and estimated tracks of the target and the corresponding MSE in estimation are presented in Fig. 4.5.

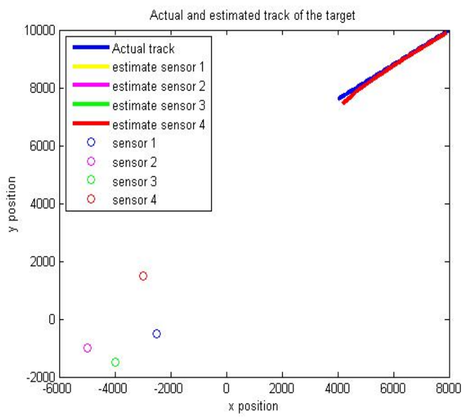


Fig.4.5 (a) Scenario plot – case 1

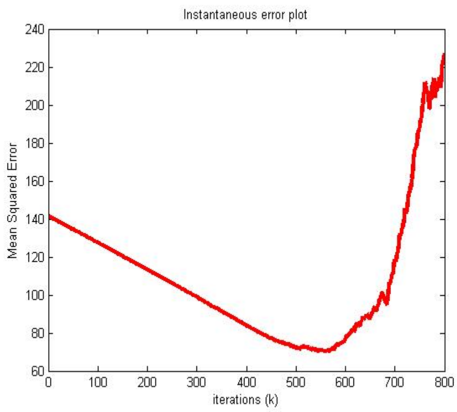


Fig.4.5 (b) MSE in position estimate – case 1

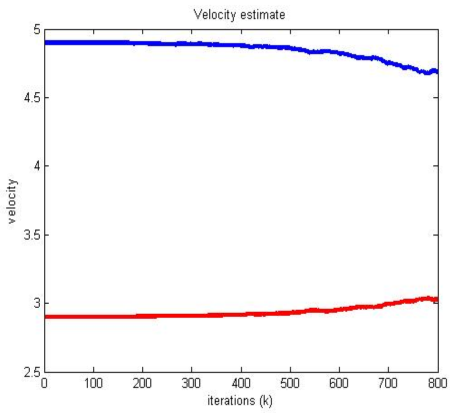


Fig.4.5 (c) Velocity estimate – case 1

Fig.4.5. Results of tracking using IFF – case 1

In Fig. 4.5(a) above, only the track of the sensor 4 is visible, since the initial estimates of all the sensors were assumed to be the same and so the estimated track of the individual sensors overlap. It is seen from this plot that the sensors track the target initially and then shows a tendency to diverge. It is observed from Fig. 4.5(b), that the MSE in position estimate reduces from 140 m to 70 m in 500 iterations and then begin to increase drastically. The velocity estimates of in Fig. 4.5(c) shows that the filter provides a good velocity estimate initially; but the estimate becomes unstable, when the filter begins to diverge. The divergence could be attributed to the fact that all the observations posts are located on one side, when the observability may be getting reduced as the tracking progresses.

4.3.2 Simulation and results of IFF for single target tracking -Case 2

The scenario considered is same as in Case 1, except that the initial state of the target (all sensors) is assumed as [7700 m ,9700 m,4.9 m,2.9 m]´ which is not as good as in Case 1. The filter is run for 850 iterations; the actual and estimated track of the target, the MSE in position estimate and velocity estimate are plotted in Fig.4.6.

In this case, as observed in *Case1*, the filter is seen to track the target for some time and then shows tendency to diverge. The poor initial estimate of the filter is obvious from the scenario plot in Fig. 4.6(a).The MSE in position estimate reduces from 420 m to 320 m in 750 iterations and then begins to diverge (Fig. 4.6(c)). On comparing with *Case 1*, it is observed that the error in position estimate reduces only to 320 m before it begins to diverge. This behavior of the filter is due to the poor initial position estimate. The divergence is also clear from the unstable velocity plot in Fig. 4.6(b).

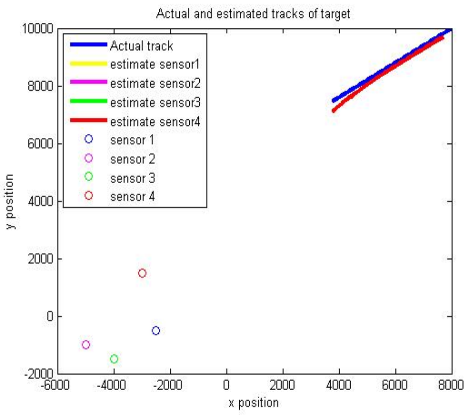


Fig. 4.6(a) Scenario plot, case 2

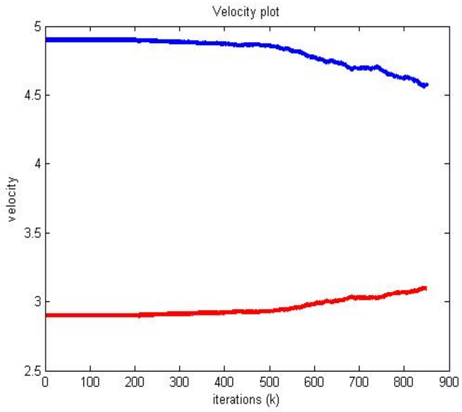


Fig. 4.6 (b) Velocity estimate, case 2

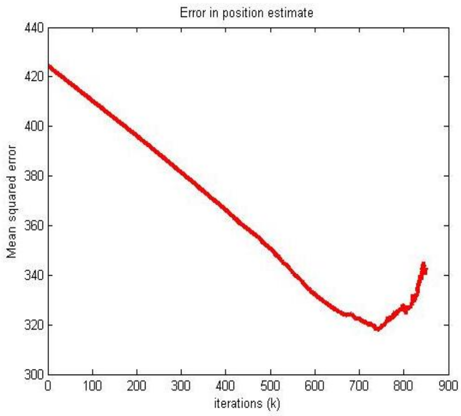


Fig. 4.6 (c) MSE in position, case 2

4.3.3 Simulation and results of IFF for single target tracking - case 3

In this case the scenario considered is same as in Table 4.2, except that $R1=R2=R3=R4=1$ rad^2.

The initial assumption by all sensors differs from the actual position of the target by 200 m in *x* and *y* coordinates. The MSE in position estimate for this case is shown in Fig.4.7. It is observed that the error reduces from 150 m to 100 m in 350 iterations, and then shows tendency to diverge. When the measurement error variance is too low, the estimation of the information fusion filter is observed to diverge earlier, after keeping track for some period.

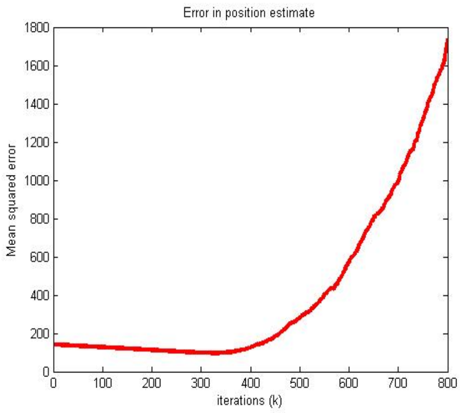


Fig.4.7 Mean Squared error in position estimate, *Case 3*, R=1

4.3.4 Simulation and results of IFF for single target tracking -Case 4

In this case, the effect of very large values of *R*, in the performance of information filter is studied. The simulation scenario is same as described in Table 4.2, except that the measurement error variances of all filters are assumed to be very large, i.e. $R1=R2=R3=R4=25$ rad^2. It is observed from Fig.4.8, that the filter tracks the target well initially, and then begins to diverge after 650 iterations.

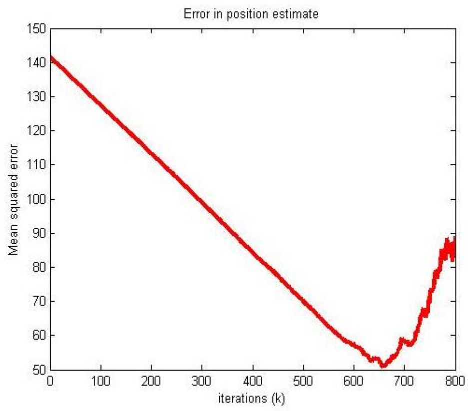


Fig.4.8 Mean Squared error in position estimate, Case 4

The MSE in estimation reduces from 140 m to 50 m in 650 iterations, which gives a better tracking compared to Case 3.On comparing with Case 3, it is inferred that MSE in position estimate reduces to a larger extent, when using sensors with relatively larger measurement variances. In IFF, it is observed that when *R* is fairly large, the MSE in estimation reduces considerably, before the filter begins to diverge, for the same initial estimate as compared to very low value of *R*. It is also observed that the filter takes a larger time to settle to minimum MSE, when *R* is fairly large (Case 4), compared to very low values of *R* (Case 3). Another observation is an information filter is able to track a target even without a good initial estimate, but the MSE in position estimate will not be as low as in cases with a fairly good estimate (Case 2). In all the cases discussed in this chapter, *Q* has been considered to be moderately low, and the effect of *R* and assumption of initial state *X*, in the performance of information fusion filter has been investigated.

By fine tuning the state process error covariance matrix, *Q*, the MSE in tracking using IF can be reduced to certain extent, by not fully. The information filter is not very sensitive to minor variations in *Q*. The filter is sensitive to *Q*, only when the process has been modeled wrongly. The tendency of the filter to diverge after the error settles down to a low value needs further investigation, though many researchers have addressed the divergence of the EKF, literature does not discuss on the divergence of information filter.

4.4 A new fusion technique in Information filter

This chapter proposes a new fusion technique using the relation between i_k and I_k, the information associated with a measurement, defined by equations Eq. 4.5 and Eq. 4.6.

$$
\begin{aligned}
E[i_k i_k^{'}] &= E[(H_k^{'} R_k^{-1} Z_k)(H_k^{'} R_k^{-1} Z_k)'] \\
&= E[(H_k^{'} R_k^{-1} Z_k)(H_k R_k^{-1} Z_k^{'})] \\
&= H_k^{'} R_k^{-1} E(Z_k Z_k^{'}) R_k^{-1} H_k \qquad (4.14)
\end{aligned}
$$

But , $E[Z_k Z_k^{'}] = R_k$, which cancels with R_k^{-1} and hence we obtain

$$E[i_k i_k^{'}] = H_k^{'} R_k^{-1} H_k \qquad (4.15)$$

$$E[i_k i_k^{'}] = I_k \tag{4.16}$$

From Eq. 4.16, we obtain the relationship between i_k and I_k.

This chapter proposes a novel fusion technique for multi sensor data IF based on variance fusion. Instead of fusing the I_k received from all sensors by simple summation (Eq. 4.11); variance fusion of I_k is performed. The Eq. 4.11 is modified as

$$y_{k/k} = y_{k/k-1} + \sum_{i=1}^{N} W_k^i i_k^i \tag{4.17}$$

$W_k^i = U \text{ x } diagonal(I_k^i)$, where U is a unity vector and diagonal (I_k^i) $= I_k^i(l,l)$.Except for the modification in fusion technique of Eq. 4.11; all other equations of IFF remain the same. The performance of modified IFF is analyzed for tracking target following CV model.

4.4.1 Simulation and results of modified IFF for single target tracking

The target scenario of section 4.3.1, Case 1 is simulated for the modified IFF. The performance of the modified IFF is shown in Fig. 4.9. The IFF was run for 1050 iterations and the actual and estimated track, estimated velocity and MSE in position estimate were observed as shown in Fig. 4.9(a), Fig. 4.9(b) and Fig. 4.9(c).

Comparing the performance of modified IFF with IFF described in section 4.3.1 Case1, it is observed that the MSE in IFF reduces to 70 m in 500 iterations and thereafter the filter shows tendency to diverge. The modified IFF gives the same performance up to 500 iterations, but unlike the conventional IFF, this filter reduces the error in estimation to nearly 0 m in 1000 iterations and then shows tendency to diverge. Thus, the modified IFF is able to keep track of the target for a longer period, thus delaying the divergence problem in IFF. But this modification is not capable of controlling the divergence in information filter.

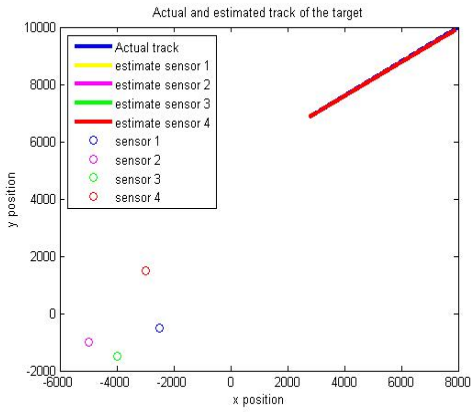


Fig. 4.9(a) Scenario plot- modified IFF

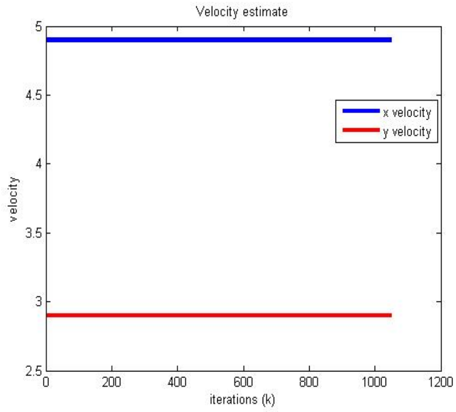


Fig. 4.9(b) Velocity estimate modified IFF

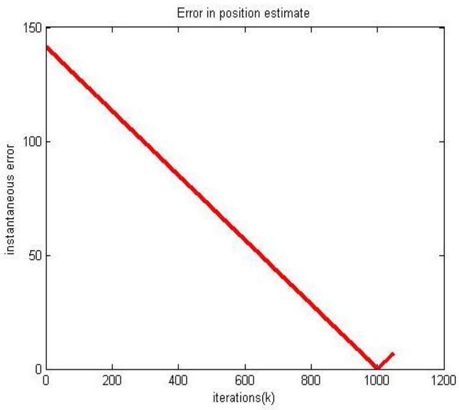


Fig. 4.9(c) MSE in position estimate- modified IFF

4.5 Conclusion

Though the instantaneous error in estimation can be reduced to some extent, by proper tuning of Q and R, the IF is seen to diverge after tracking for some time. This is an area which need to be further studied and hence techniques need to be developed to prevent this divergence in tracking, as IF is advantageous in multi sensor fusion estimation problems compared to KF. This chapter proposes a modified IFF, which was observed to delay the divergence issue, but not capable of controlling it. This thesis proposes a new method in Chapter 5, for controlling the divergence in information filters, using a fuzzy technique. The effectiveness of the proposed technique has been presented using target tracking application.

****✪****

Chapter - 5

FUZZY INFORMATION FUSION FILTER IN TARGET TRACKING

It was observed in the last chapter that the IFF, extended from the IF [81] is an attractive alternative to the EKF based fusion techniques like the PDA, in the context of multi senor data fusion for tracking. However, it was not very cheering to observe that the IFF also diverges in the course of tracking targets, based on the bearing measurement only. In order to alleviate the problem of divergence, the present chapter proposes a solution centered on a correction based on a Fuzzy function. The information update equations of the conventional IF are modified in terms of fuzzy function of error and change of error, and the results have been found to be effective in controlling divergence. It is also encouraging to note that the efficacy of the technique in assuaging the divergence is manifest while tracking a maneuvering target also.

In this chapter, a modified information filter called the Fuzzy information Fusion Filter (FIFF) is introduced to alleviate the problem of divergence in information fusion filter (IFF) [81]. The performance of the FIFF is compared with that of IFF for various scenarios. As introduced in Chapter 4, the application considered here is a BOT problem, using measurements received from 4 stable sensors (like the ground stations).The performance of fuzzy information fusion filter (FIFF) for tracking a CV target and maneuvering target is investigated. Each sensor is assumed to be autonomous and has sufficient computational power to estimate the information state from measurements and the information states are communicated across all sensors [81,140].

5.1 Process and observation model

The target state *X* at time *k*, X_k is assumed to be a 4 dimensional vector representing *x* and *y* position of the target and the velocities in the *x* and *y* directions.

The state vector at any time *k* is defined as

$$X_k = \left[x_k, y_k, \mathrm{v}_{xk}, \ \mathrm{v}_{yk} \right]' \tag{5.1}$$

where, $x_k, y_k, \mathrm{v}_{xk}, \ \mathrm{v}_{yk}$ are the *x* position , *y* position, *x* velocity, *y* velocity respectively at time k . It is assumed that the target follows a CV model, and the state evolves in time according to

$$X_k = A_{k-1} X_{k\text{-}1} + G w_k \tag{5.2}$$

G is defined as shown below:

$$G = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \tag{5.3}$$

where, *T* is the sampling time. The bearing angle measurements at any time *k*, are given by Z_k^i where *i* = 1, 2 3 and 4 corresponds to the 4 sensors. The sensors observe the target according to the non linear observation model given by $Z_k = h(k, X_k) + v_k$, as indicated in previous chapters. The measurement error co-variance, *R* and the co-variance of the plant, *Q* are given below.

$$R = E[v(k)v(k)'] \tag{5.4}$$

$$Q = E[\{\mathrm{G}\, w(k)\}\{\mathrm{G}\, w(k)\}'] \tag{5.5}$$

Q can be simplified as $\sigma^2 GG'$, which is given by

$$Q = \sigma^2 GG' = \sigma^2 \begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix} \tag{5.6}$$

5.2 Fuzzy Logic Based Information Fusion filter (FIFF)

The FIFF works more or less the same way as the IFF. However, a decision function computed on the Fuzzy variables corresponding to the error and the change in error is used to correct the measurement co-variance, is used to correct the information state and information matrix. The theory of fuzzy logic controllers [142] and how fuzzy logic can be used to control divergence in Kalman filters has been referred from literature [143, 144]. The seven fuzzy variables are defined on each of the two variables viz. error.

$$e_k = Z_k - Z_k \tag{5.7}$$

where, Z_k is the predicted bearing measurements from the estimates ,and change of error

$$\Delta e_k = \frac{(e_k - e_{k-1})}{T} \tag{5.8}$$

Where, T is the sampling period. The total support for e and Δe of bearing angle is from -1 to +1. All membership functions of the fuzzy set are represented as a Gaussian function with center c_i and variance σ_i^2 for error e and c_j and σ_j^2 respectively for change of error Δe such that , The membership functions are given by

$$\mu_i(e) = e^{-((e-c_i)^2/2\sigma_i^2)} \tag{5.9}$$

$$\mu_j(\Delta e) = e^{-((e-c_j)^2/2\sigma_j^2)} \tag{5.10}$$

where *i* and *j* varies from 1 to 7 to define 7 fuzzy variables each for *e* and *Δe*.

The definitions of fuzzy variables are given in Table 5.1 below, by uniformly spreading the support of each variable over the range of *e* and *Δe* [122].

Fuzzy term	c_i , c_j	σ_i^2, σ_j^2	Support
LN (Large Negative)	-0.857	2	[-1,-0.714]
MN (Medium Negative)	-0.572	2	[-0.714,-0.429]
SN (Small Negative)	-0.286	2	[-0.429,-0.143]
ZE (Zero)	0	2	[-0.143,0.143]
SP (Small Positive)	0.286	2	[0.143,0.429]
MP (Medium Positive)	0.572	2	[0.429,0.714]
LP (Large Positive)	0.857	2	[0.714,1]

Table 5.1 Definition of Fuzzy variables

The rules are framed as a 7x7 matrix, which is the conjunction of 2 input conditions *e* and *Δe*, to produce the output response at the intersection of row and column. In the case considered, there are 49 possible logical product output responses. Table 5.2 illustrates the rule base used. One typical rule is illustrated.

$Rule_{ij}$: If e is LN and Δe is ZE then output is MN

e / Δe	**LN**	**MN**	**SN**	**ZE**	**SP**	**MP**	**LP**
LN	LN	LN	MN	MN	MN	SN	ZE
MN	LN	MN	MN	MN	SN	ZE	SP
SN	MN	MN	MN	SN	ZE	SP	MP
ZE	MN	MN	SN	ZE	SP	MP	MP
SP	MN	SN	ZE	SP	MP	MP	MP
MP	SN	ZE	SP	MP	MP	MP	LP
LP	ZE	SP	MP	MP	MP	LP	LP

Table 5.2 Rule base for inference [122]

The rules compute the weight and the functional overlap of the inputs and generate output responses. The output responses are combined across all 49 rules and defuzzified to a single value. Accordingly the rule matrix has been taken as

$$C = \begin{bmatrix} -0.857 & -0.857 & -0.571 & -0.571 & -0.571 & -0.285 & 0 \\ -0.857 & -0.571 & -0.571 & -0.571 & -0.285 & 0 & 0.285 \\ -0.571 & -0.571 & -0.571 & -0.285 & 0 & 0.285 & 0.571 \\ -0.571 & -0.571 & -0.285 & 0 & 0.285 & 0.571 & 0.571 \\ -0.571 & -0.285 & 0 & 0.285 & 0.571 & 0.571 & 0.571 \\ -0.285 & 0 & 0.285 & 0.571 & 0.571 & 0.571 & 0.285 \\ 0 & 0.285 & 0.571 & 0.571 & 0.571 & 0.285 & 0.285 \end{bmatrix}$$

Here every $C(i,j)$; i= 1…7, j=1…7 corresponds to the c_i and $c_{j,}$, the fuzzy consequent terms like LN defined in Table 5.2. The defuzzifier function is calculated as

$$q = \frac{\sum_{i=1}^{7}\sum_{j=1}^{7}\mu_i(e)\mu_j(\Delta e)C(i,j)}{\sum_{i=1}^{7}\sum_{j=1}^{7}\mu_i(e)\mu_j(\Delta e)} \tag{5.11}$$

It can be concluded from the experimental results of Chapter 4, that the measurement error covariance affects the performance of the IF, in target tracking application. A fairly large value of R is essential for maintaining a good estimate. It is also observed that the filter diverges after the estimation error settles to a low value. The proposed technique to alleviate divergence, tries to control the value of R, using the generated defuzzifier output q. For every value of e and Δe, produced for each measurement, the membership functions $\mu_i(e)$ and $\mu_j(\Delta e)$ are computed. The generated output, q, obtained using Eq. 5.11, is used to modify the variance of the observation error viz. R, in the equation for calculating the update of information state and information matrix (step 4 below). The FIFF, with the modified measurement covariance, is presented in Fig. 5.1.

5.3 Simulation and results

The simulations in the sections to follow illustrate the performance of the FIFF (Fuzzy Information Fusion Filter) and bring out the improvement in performance compared to the IFF (information fusion filter [122]). The resilience of the FIFF to the divergence is exceptionally encouraging.

As in Chapter 4, the scenario depicts an object flying at different directions and is being monitored by four tracking stations. The sensors at all the observation points watch the relative bearing only. Though real field values were not available at that moment, the scenario depicted illustrates typical tracking of an object. Different scenarios are tried out to evaluate the convergence properties of the algorithm.

1. State is as defined in Eq. 5.1.The state prediction and measurement predictions are done as defined in section 5.1.

2. The two inputs to the fuzzy system are error and rate of change of error, which are defined by Eq. 5.7 and Eq. 5.8.

3. In the information filter, which is a decentralized algorithm, the concept of information measure is being used. The prediction covariance matrix of the KF is related to the information matrix of the IF using Eq. 5.12. The information state y_k is related to the target state by Eq. 5.13.[81, 140].

$$Y_k = P_k^{-1} \quad 5.12$$

$$y_k = Y_k X_k \quad 5.13$$

where $X(k)$ refers to the state estimate.

4. An observation Z_k^i contributes $i_k^{\,i}$ to information state y_k and I_k^i to information matrix Y_k.

$$i_k^{\,i} = H_k^{'} R_k^{-1} q Z_k^{i} \quad 5.14$$

$$I_k^i = H_k^{'} R_k^{-1} q H_k \quad 5.15$$

The q as computed in Eq.5.11 above is used to modify the error covariance R. The difference between IFF and FIFF is the modification of R with q in Eq. 5.14 and Eq. 5.15, the contribution of sensors, $i = 1\ldots N$.

5. Finally $y_{k/k}$ and $Y_{k/k}$ are obtained by adding $i_k^{\,i}$ and I_k^i to $y_{k/k}$ and $Y_{k/k}$ respectively, for $i = 1\ldots N$ (sensor).

$$y_{k/k} = y_{k/k-1} + \sum_{i=1}^{N} i_k^{\,i} \quad 5.16$$

$$Y_{k/k} = Y_{k/k-1} + \sum_{i=1}^{N} I_k^i \qquad 5.17$$

6. The posteriori state estimates are obtained as

$$X_{k/k} = Y_{k/k}^{-1} y_{k/k} \qquad 5.18$$

The estimated state is obtained by the product of estimated state covariance matrix and estimated information state.

Fig. 5.1 The Fuzzy Information Fusion Filter (FIFF) algorithm

5.3.1 Performance of the Fusion filters (IFF vis-à-vis FIFF) - Case 1

The performance of the IFF and FIFF for Case 1 is discussed in this section, to demonstrate the superiority of the FIFF. The actual and estimated tracks of the target, the velocity estimate and MSE in position estimate are shown in Fig. 5.2, Fig. 5.3 and Fig. 5.4 respectively. The simulation scenario for Case 1, is as shown in Table 5.3

Both the IFF and the FIFF are executed. The blue trace in Fig.5.2 corresponds to the actual path of the target, while the yellow trace is the estimate of sensor 4. The estimates of other sensors overlap each other (and so yellow alone is visible) as same initial position and velocity estimates have been assumed for all four sensors. The divergence in the estimated states of the IFF is visibly clear from Fig. 5.2(a). The same scenario was evaluated for the FIFF also. The sustained tracking from the scenario plot in Fig. 5.2(b) is marvelous, underscoring the effect of the modifications carried out on the measurement covariance R (Eq. 5.14 and Eq. 5.15), which resulted in a better estimate of the information state ,y and Information matrix ,Y (Eq.5.16 and Eq.5.17).

The actual initial state vector of the target. Initial position assumed by the IFF and FIFF.	Target:X=[15000 m,15000 m,3 m/s,2 m/s]′ $X_{0/0}$ =[14900 m,14900 m,2.9 m/s,1.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 100\text{m}^2 & 0 & 0 & 0 \\ 0 & 100\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensors	$R1$=5 rad^2, $R2$=8 rad^2,$R3$=3 rad^2, $R4$=4 rad^2	Measurement error covariance R
Sensor positions	(-2500 m,-500 m) (-5000 m,-1000 m) (-4000 m,-1500 m) (-3000 m, 1500 m)	(x, y) position of the sensor.
Q matrix	$Q=\sigma^2\begin{bmatrix} 0.25 & 0 & 0.5 & 0 \\ 0 & 0.25 & 0 & 0.5 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 1 \end{bmatrix}$	Process noise covariance matrix, Q σ^2=0.0001
For the fuzzy membership function.	$-1\leq c_i$, $c_j\leq +1$ σ_i^2=2 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance of error and change of error for FIF

Table 5.3 Simulation scenario for FIFF- Case 1

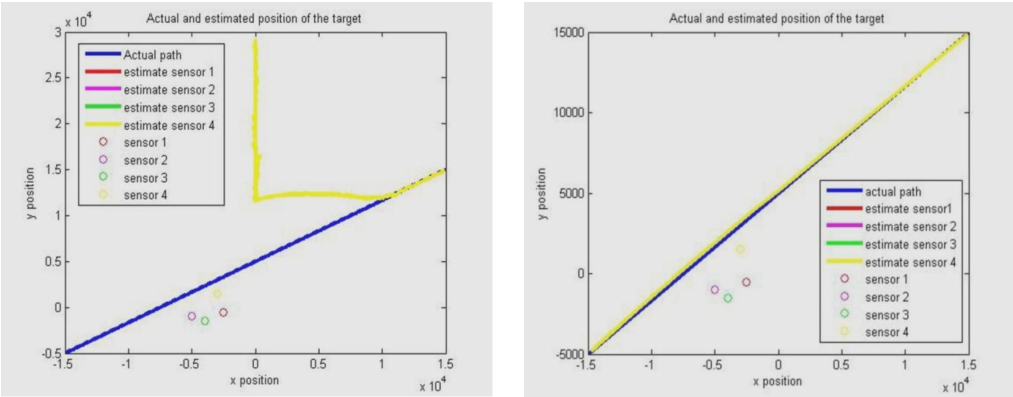


Fig.5.2 (a) Fusion and tracking with IFF **Fig.5.2 (b) Fusion and tracking with FIFF**

Fig.5.2 Actual and estimated track of the target using an FIF and FIFF

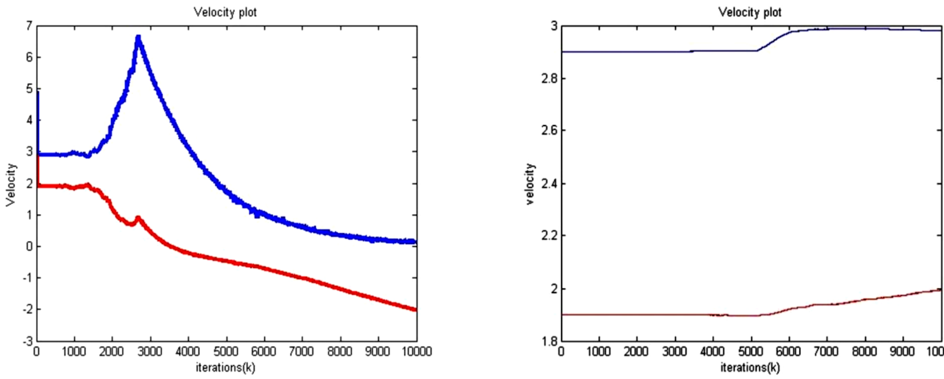


Fig. 5.3 (a) Velocity estimate totally flawed in IFF **Fig. 5.3(b) The estimate converges to 3m/s and 2 m/s in the case of FIFF**

Fig. 5.3 Velocity estimate of the target

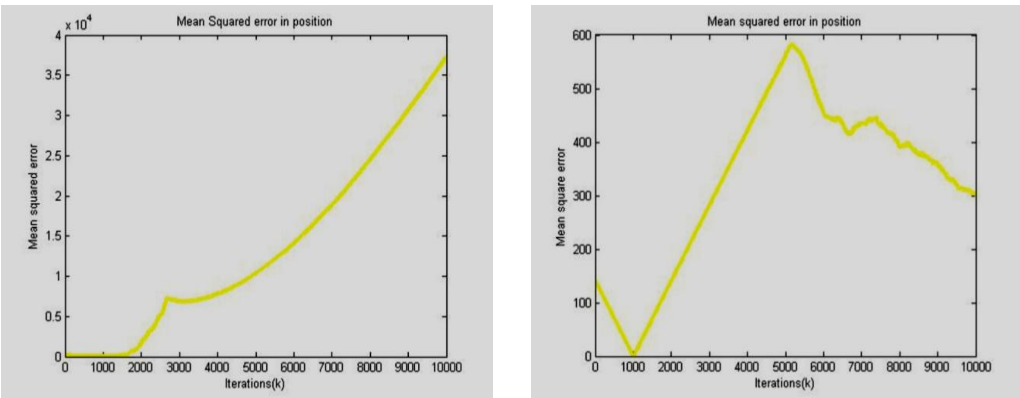


Fig 5.4 (a) Totally diverging error for the IFF **Fig 5.4 (b) FIFF recovers from the divergence in error**

Fig 5.4 MSE in estimating position

The estimates in velocity is totally blemished after a few tracking steps in the case of IFF (Fig. 5.3(a)), while the estimates converge correctly to v_x=3m/s and v_y=2 m/s, in the case of FIFF (Fig. 5.3(b)). The MSE in position estimate is calculated as $\sqrt{(x-x)^2+(y-y)^2}$.

5.3.2 Performance of Fusion filters (IFF vis-à-vis FIFF) – Case 2

The simulation scenario for Case 2 is as depicted in Table 5.4.

The actual initial state vector of the target. Initial position assumed by IFF and FIFF.	Target :X=[15000 m,15000 m,3 m/s,2 m/s]´ $X_{0/0}$ =[14900 m,14900 m,2.9 m/s,1.9 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix}1 & 0 & -T & 0\\ 0 & 1 & 0 & -T\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$	*A* is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix}100m^2 & 0 & 0 & 0\\ 0 & 100m^2 & 0 & 1\\ 0 & 0 & 0.001\frac{m^2}{s^2} & 0\\ 0 & 0 & 0 & 0.001\frac{m^2}{s^2}\end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensors	$R1$=1 rad^2, $R2$=2 rad^2, $R3$=3 rad^2, $R4$=4 rad^2	Measurement error covariance *R*
Sensor positions	(-2500 m,-500 m) (-5000 m,-1000 m) (-4000 m,-1500 m) (-3000 m, 1500 m)	(*x*, *y*) position of the sensor.
Q matrix	$Q=\sigma^2\begin{bmatrix}0.25 & 0 & 0.5 & 0\\ 0 & 0.25 & 0 & 0.5\\ 0.5 & 0 & 1 & 0\\ 0 & 0.5 & 0 & 1\end{bmatrix}$	Process noise covariance matrix, Q σ^2=0.0001
For the fuzzy membership function.	$-1\le c_i, c_j\le +1$ σ_i^2=1 rad^2 (for e) ; σ_j^2=3 rad^2 (for Δe)	Variance of error and change of error for FIF

Table 5.4 Simulation scenario for FIFF- Case 2

The influence of different measurement errors in the target fusion is simulated here. The performance of both IFF and FIFF for this scenario is illustrated in this section. The filter is run for 10000 iterations, and the estimated tracks, MSE in position estimation and velocity estimate of IFF and FIFF are plotted in Fig. 5.5, Fig. 5.6 and Fig. 5.7 respectively. The IFF tracks well initially and then begins to diverge as is observed from Fig. 5.5(a). For the same scenario, the proposed FIFF is able to able track well, controlling divergence as is observed from Fig. 5.5(b)

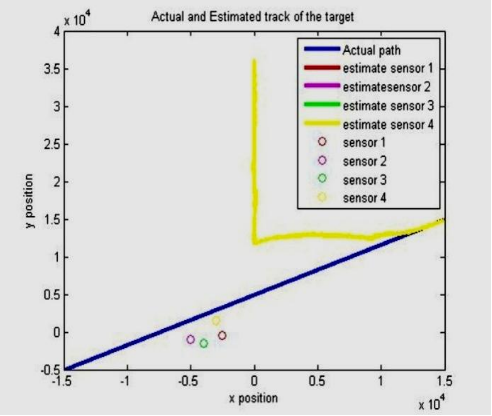


Fig 5.5 (a) Fusion and tracking with IFF

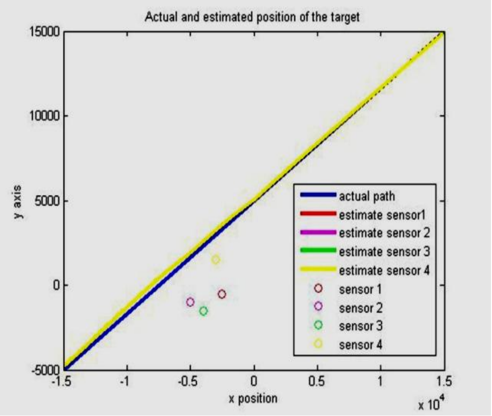


Fig 5.5 (b) Fusion and tracking with FIFF

Fig 5.5 Actual and estimated track of the target using an IFF and FIFF

Although a CV model has been assumed, it is observed that the divergence causes drastic distortion in velocity estimate of IFF as seen in Fig.5.6 (a) , but the proposed fuzzy technique in FIFF has been able to estimate the velocities correctly as observed in Fig. 5.6 (b). The velocity plot in Fig. 5.6(b), is seen to converge in 6000 iterations to velocities v_x=3m/s and v_y=2m/s, which are the actual velocities of the target.

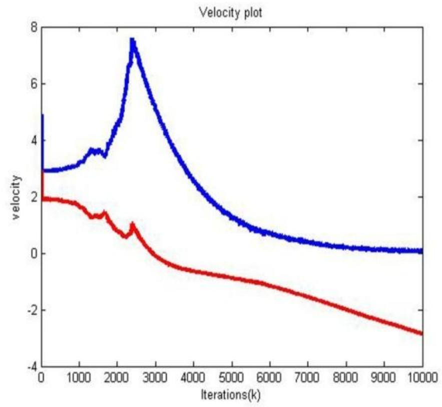


Fig. 5.6 (a) Velocity estimate totally flawed in IFF

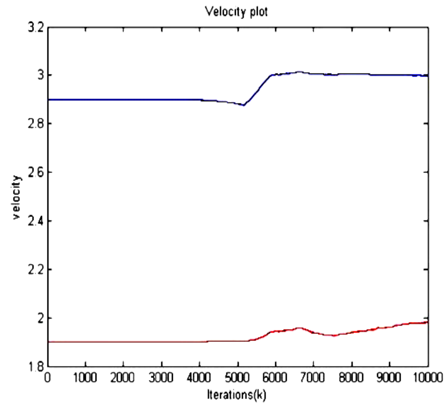


Fig. 5.6 (b) The estimate converges to 3m/s and 2 m/s in the case of FIFF

Fig. 5.6 Velocity estimate of the target

The divergence in IFF is clearly visible from the MSE in position estimate shown in Fig. 5.7(a). The divergence first increases gradually for some time, after which the increase in estimation error is drastic and the track does not show any sign of recovery after divergence. The effect of controlling divergence by FIFF is visible from the MSE plot in Fig. 5.7 (b). The FIFF recovers after an initial tendency to diverge. It is observed that in 6800 iterations the MSE reduces to 300 m in FIFF, while with higher measurement variance as in Case 1, it took 10000 iterations to reach this level as seen in Fig. 5.4(b).

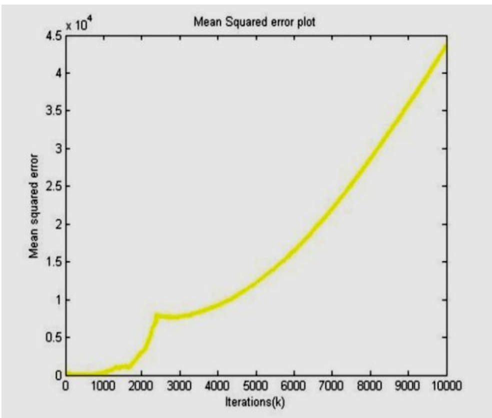


Fig 5.7 (a) Totally diverging error for the IFF

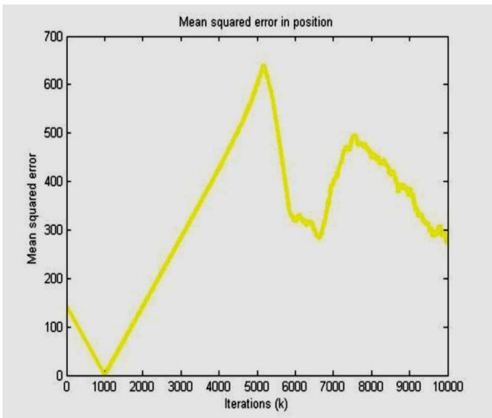


Fig 5.7 (b) FIFF recovers from the divergence in error

Fig 5.7 MSE in estimating position

5.3.3 Performance of fusion filters (IFF vis-à-vis FIFF) for maneuvering target - Case 3

It may be noted that the Cases 1 and 2, portrayed the relative performance of IFF and FIFF for tracking a target following a CV model. The present case, the scenario was designed to make the target maneuver halfway, and to assess the reaction of both the fusion filters. The simulation scenario is as described in Table 5.5.

The actual initial state vector of the target. Initial position assumed by IFF and FIFF.	Target:X=[15000 m,15000 m,2 m/s,1 m/s]′ $X_{0/0}$ =[14900 m,14900 m,1.9 m/s,0.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix}1 & 0 & -T & 0\\ 0 & 1 & 0 & -T\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix}100\text{m}^2 & 0 & 0 & 0\\ 0 & 100\text{m}^2 & 0 & 1\\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0\\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2}\end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensors	$R1$=1 rad^2, $R2$=2 rad^2,$R3$=3 rad^2, $R4$=4 rad^2	Measurement error covariance R
Sensor positions	(-4000 m,-2000 m),(-2000 m, 8000 m), (0 m, 4000 m), (-3000 m, 1500 m)	(x, y) position of the sensor.
Q matrix	$Q=\sigma^2\begin{bmatrix}0.25 & 0 & 0.5 & 0\\ 0 & 0.25 & 0 & 0.5\\ 0.5 & 0 & 1 & 0\\ 0 & 0.5 & 0 & 1\end{bmatrix}$	Process noise covariance matrix, Q σ^2=0.0001
For the fuzzy membership function.	$-1\le c_i, c_j \le +1$ σ_i^2=1 rad^2 (for e) ; σ_j^2=3 rad^2 (for Δe)	Variance of error and change of error for FIF

Table 5.5 Simulation scenario for FIFF- Case 3

The track of the target follows a straight course with v_x =2m/s and v_y = 1m/s for 10699 steps. Thereafter, the target gets accelerated with a_x=0.03m/s^2 and a_y=0.01m/s^2 for 29 steps and then for the next 2372 steps, v_x and v_y remain constant at the velocity obtained at the end of acceleration. The sensor positions are different from the locations selected in the previous cases. The performance of IFF is independent of the sensor locations as, it is only the information from the measurements that are fused. The program was run for 13100 iterations and the actual and estimated track of the target, the MSE in position estimate, and the velocity estimates are plotted.

The actual track of the target and the estimated tracks are plotted in Fig 5.8. The performance of IFF in this case is similar to Section 5.3.2.It is observed that IFF does not show any sign of recovery after divergence (Fig. 5.8 (a)), whereas FIFF is able to catch track even after maneuver. The recovery of the track after maneuver, is seen in the Fig. 5.8(b).It is thus observed that the fuzzy information filter is able to track maneuvering targets also.

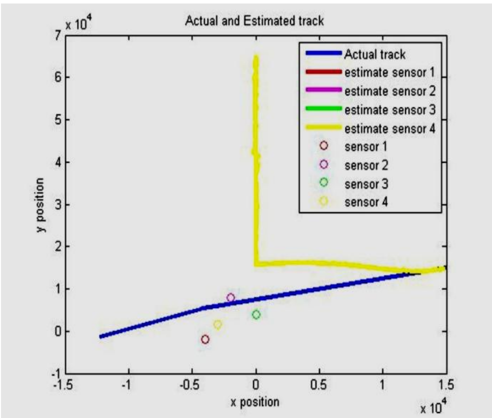


Fig.5.8 (a) Tracking with IFF

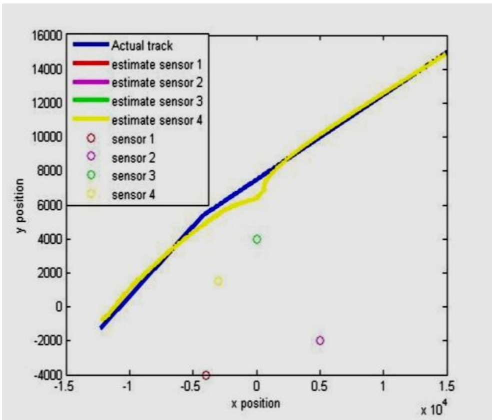


Fig.5.8 (b) Tracking with FIFF

Fig.5.8 Actual and estimated track of the maneuvering target

Both IFF and FIFF begin with an initial position error of 100 m in *x* and *y* directions. As the measurement error variances of the sensors are low, in nearly 1000 iterations, the error reduces to a very low value, and the filter catches the track in both IFF and FIFF tracking. But in IFF, the filter is seen to diverge after 500 runs and then does not show any sign of convergence as seen in Fig. 5.9(a). The error has almost doubled in 3000 iterations, compared to the previous cases of IFF. But in FIFF, the proposed fuzzy correction on *R*, reduces the error in position estimation, and thus the filter begins to track again after showing tendency to diverge (Fig. 5.9(b)). The MSE in position estimate reduces to 300m in 13100 steps.

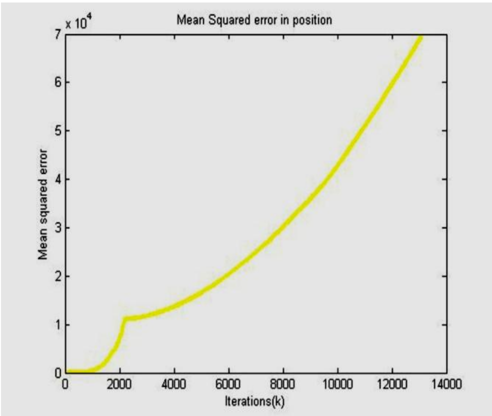


Fig 5.9 (a) Mean squared error for the IFF

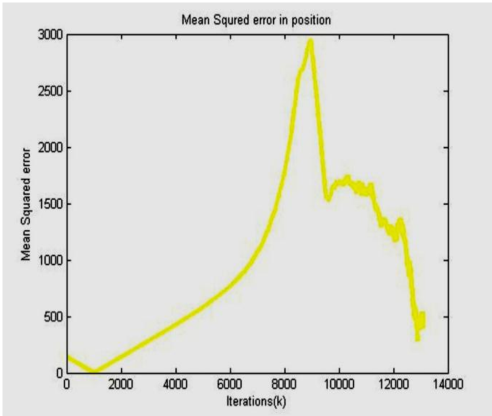


Fig 5.9 (b) FIFF recovers from the divergence in error

Fig 5.9 MSE in position estimate of maneuvering target

The velocity plot is also much distorted, similar to that observed in Cases 1 and 2 for IFF (Fig. 5.10(a)). It can be concluded that an IFF poorly tracks a maneuvering target. The recovery of the track after maneuver, in FIFF is seen in the Fig.5.10 (b).

It is thus observed that the FIFF is able to track maneuvering targets also. The disturbance in velocity estimate occurs during the period when the target maneuver is due to acceleration in velocity. The velocity also begins to stabilize after acceleration to 1.2m/s and 2.15m/s as seen in Fig. 5.10(b).

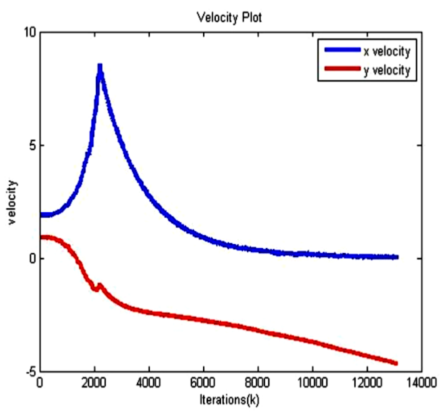


Fig. 5.10 (a)Velocity estimate in IFF

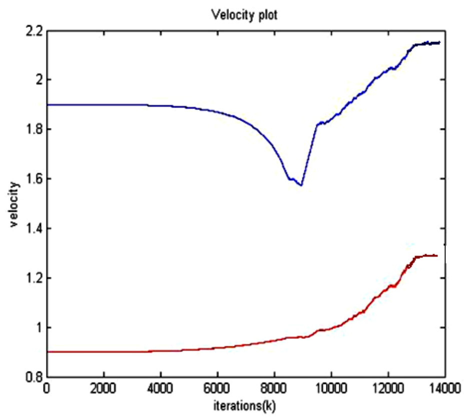


Fig. 5.10 (b)The estimate converges in the case of FIFF

Fig. 5.10 Velocity estimate of the maneuvering target

5.4 Performance of Fuzzy Information Filter (FIF) for single target

In order to highlight the superiority of multi sensor target tracking, the performance of fuzzy information filter (FIF) is investigated for single sensor tracking application. The algorithm of FIF is similar to FIFF shown in Figure 5.1 except for Eq. 5.16 and Eq. 5.17 which is modified as below since there is only a single sensor.

$$y_{k/k} = y_{k/k-1} + i_k \qquad 5.19$$

$$Y_{k/k} = Y_{k/k-1} + I_k \qquad 5.20$$

5.4.1 Performance of FIF- a second look

In order to ascertain once again the influence of four sensors fusing the information thereby leading to better tracking of steady target, the tracking of the steady target with only one sensor, the FIF was also tried out, for targets negotiating CV and CT model, separately .

5.4.1.1 Case 1 – CV model

The actual initial state vector of the target. Initial position assumed by FIF	Target :X=[15000 m,15000 m,3 m/s,2 m/s]′ $X_{0/0}$ =[14900 m,14900 m,2.9 m/s,1.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	A is the process transition matrix. Sampling time T=1.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 100m^2 & 0 & 0 & 0 \\ 0 & 100m^2 & 0 & 1 \\ 0 & 0 & 0.001\frac{m^2}{s^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{m^2}{s^2} \end{bmatrix}$	Initial value of state co variances matrix, of the target with variances of state vector elements along the diagonal elements.
Observation noise variance of the sensors	$R1=1\ rad^2$	Measurement error covariance R
Sensor position	(-4000 m,-2000 m)	(x, y) position of the sensor.
Q matrix	$Q=\sigma^2\begin{bmatrix} 0.25 & 0 & 0.5 & 0 \\ 0 & 0.25 & 0 & 0.5 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 1 \end{bmatrix}$	Process noise covariance matrix, Q σ^2=0.0001
For the fuzzy membership function.	$-1\le c_i$, $c_j\le +1$ $\sigma_i^2=1\ rad^2$ (for e) ; $\sigma_j^2=3\ rad^2$ (for Δe)	Variance of error and change of error for FIF

Table 5.6 Simulation scenario for FIF- CV model

The FIF was run for 10000 iterations and the actual and estimated track, MSE in position and velocity estimates were plotted.

A similar target motion was tracked in Section 5.3.1, where 4 sensors were used and the information filter performs information fusion in estimating the track of the target. Here, only a single sensor for the same tracking problem has been considered. The actual and estimated track of the target is shown in Fig. 5.11(a). On observing the MSE in position, it is seen that the single sensor has an estimation error of 1050m after 10000 iterations as in Fig. 5.11(b), while senor fusion tracking (FIFF, using 4 sensors) could estimate position with an error of 300m after 10000 iterations as shown in Fig. 5.4(b). This underscores the advantage of FIFF as against single sensor tracking using FIF.

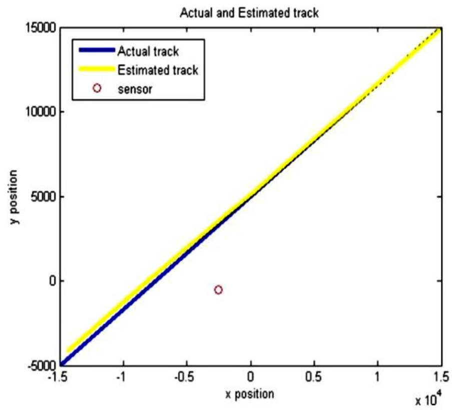


Fig. 5.11 (a) Actual and predicted path of FIF

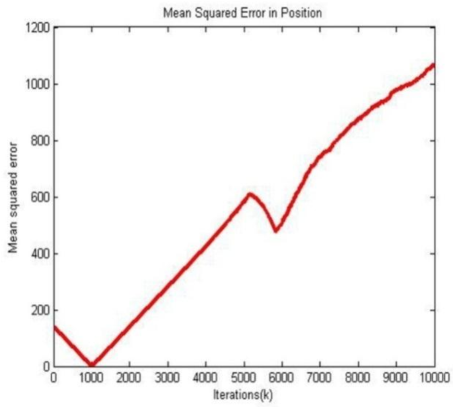


Fig. 5.11 (b) MSE in FIF

The velocity plot for the case is shown in Fig. 5.12.It is seen that the velocity is almost stable but does not converge to the actual velocities of v_x=3m/s and v_y =2m/s; but a bias in the final estimate in the velocity is obvious. On the other hand, the velocity convergence was achieved in section 5.3.1, when using multiple sensors fusing the information using the Fuzzy extensions proposed in this chapter.

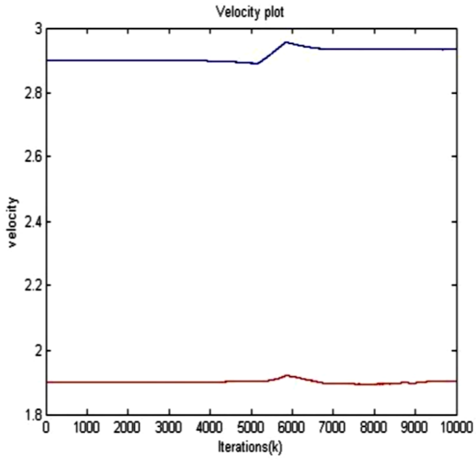


Fig. 5.12 Velocity estimate of fuzzy information filter (FIF)

5.4.1.2 Case 2 –CT model

In order to investigate the performance of a single sensor FIF for tracking maneuvering targets, the scenario as in Case 3 is considered, with the only difference that only one sensor is used for tracking the maneuvering target . The (*x*, *y*) position of the sensor is (-4000 m,-2000 m).The FIF was run for 13100 iterations and the actual and estimated track, the MSE in position plot and the velocity estimates are plotted.

The actual and estimated path of the FIF with single sensor for the maneuvering target case is shown in Fig. 5.13(a).The MSE plot in Fig. 5.13(b), shows that the filter shows less chances of recovery after maneuvering as against Fig. 5.9(b), the case of multiple sensors. It is also observed that the error in position in Fig. 5.13 (b) is ten times lesser compared to FIFF described in Section 5.3.3 (Fig. 5.9(a)). The velocity estimate in Fig. 5.13 (c) for FIF is not stable as that for FIFF using multiple sensors.

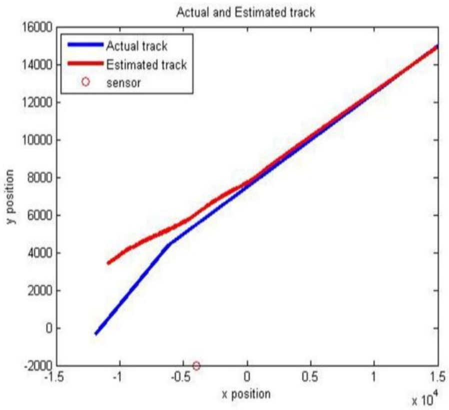


Fig.5.13 (a) Actual and estimated track FIF

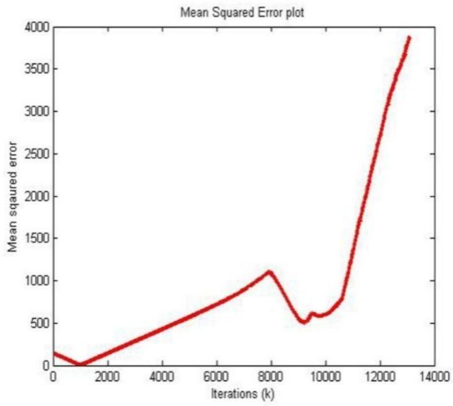


Fig. 5.13(b) MSE in FIF

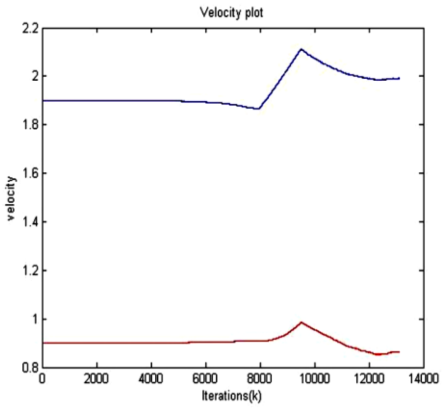


Fig.5.13(c) Velocity estimate of FIF

This chapter [125] addresses the information filter divergence for a bearing only target tracking problem. The FIFF presented in this chapter is shown to provide good control to this divergence problem, by correcting the variance of the measurement error *R*. It is seen that both the position and the velocity estimate show tendency to converge, though the time taken is influenced by the measurement variance. A comparison of the IFF reported in literature and the FIFF proposed in this chapter is presented in terms of the actual and predicted track, MSE in position of the track, and the velocities in the *x* and *y* directions. The work also brings out that the FIFF gives a better tracking performance than FIF, thus highlighting the advantage of multi sensor target tracking. The FIFF could also effectively track a maneuvering target with proper convergence in position and velocity after maneuver.

5.5 Performance of FIFF in tracking targets following switching models

The behavior of the FIFF, when a target switches between CV and CT models, is evaluated by simulating the scenario, where a moving object, is monitored by 4 tracking stations, which receive only bearing angles as the measurement. The scenario considered here (Table 5.7) encounters large maneuver of a single target, which are tracked by the FIFF.

The poor performance of the FIFF, which does not respond to the model switching is illustrated through the simulation, which considers a target, which moves in a linear path for first 1000 iterations and then switches to a CT path with a constant turn rate of 0.003 rad/s and continues in that path for the next 1330 steps. The actual and estimated track of the target, the MSE in position, and the velocity estimate were plotted. MSE in the track estimated is calculated as $\sqrt{(x-x)^2+(y-y)^2}$, where *x* and *y* refer to the actual *x* and *y* position of the target and x and y correspond to the estimated positions. The poor performance of FIFF, which assumes a CV process model alone (and which does not attempt to detect any maneuver) is evident from Fig. 5.14 (a), (b) and (c), which plots scenario, velocity estimate and MSE in position estimate respectively. As is expected, the estimate in red diverges away from the actual target movement in blue, as confirmed in Fig 5.14 (a), (b) and (c). (The initial states of the 4 sensors have been taken as same, and so as all the sensors perform information fusion, the estimated tracks of all the 4 sensors overlap. Hence while plotting the estimated tracks of all the 4 sensors, the fourth sensor’s track are only visible in the plot).

The actual initial state vector of the target. Initial position assumed by the filter	X =[10000 m ,10000 m, 3 m/s, 2 m/s]′ $X_{0/0}$=[9900 m,9900 m, 2.9 m/s.1.9 m/s]′	Position in m and velocity in m/s
State covariance $P(0,0)$	$P(0,0)=\begin{bmatrix} 1\text{m}^2 & 0 & 0 & 0 \\ 0 & 1\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.1\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.1\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state covariance matrix, with variances of state vector elements along the diagonal elements.
Observation noise of 4 sensors	1 rad^2, 2 rad^2, 3 rad^2 and 4 rad^2 respectively	Measurement error covariance R
Sensor positions	(-4000 m,-2000 m), (5000 m, -2000 m), (0 m,0 m), (-3000 m, -1000 m) respectively.	(x, y) position of the 4 sensors in m.
Q matrix	$Q=\sigma^2\begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix}$ $Q=\begin{bmatrix} 0.0014\text{m}^2 & 0 & 0.0027\frac{\text{m}^2}{\text{s}} & 0 \\ 0 & 0.0014\text{m}^2 & 0 & 0.0027\frac{\text{m}^2}{\text{s}} \\ 0.0027\frac{\text{m}^2}{\text{s}} & 0 & 0.0055\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0.0027\frac{\text{m}^2}{\text{s}} & 0 & 0.0055\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix $Q=\sigma^2 GG'$ σ^2 =0.0055 T=1
For the fuzzy membership function.	σ_i^2=3 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance of error and change of error

Table 5.7 Simulation scenario for FIFF- switching model

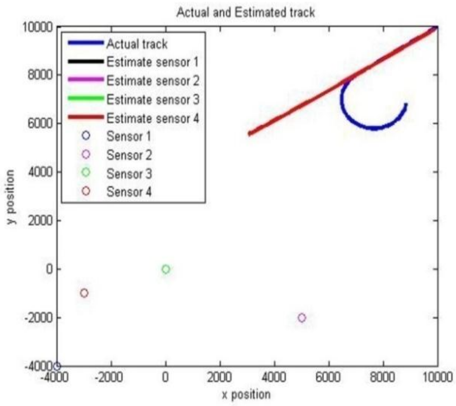


Fig. 5.14 (a)Scenario plot

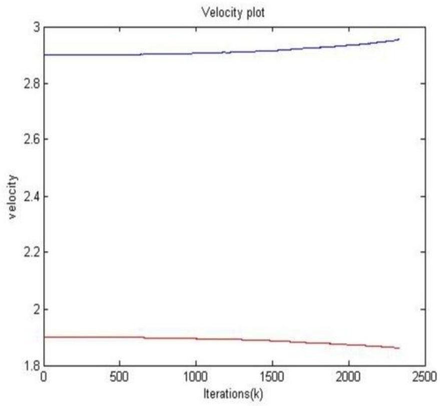


Fig. 5.14 (b) Velocity estimate

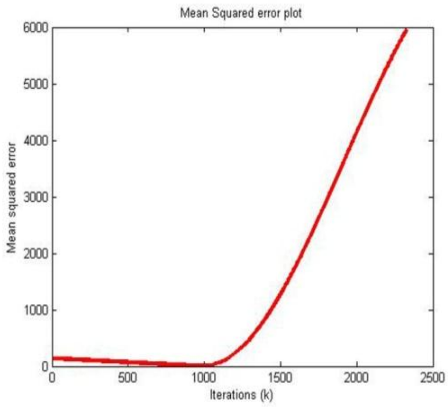


Fig. 5.14 (c) MSE plot

Fig. 5.14 FIFF in tracking switching models

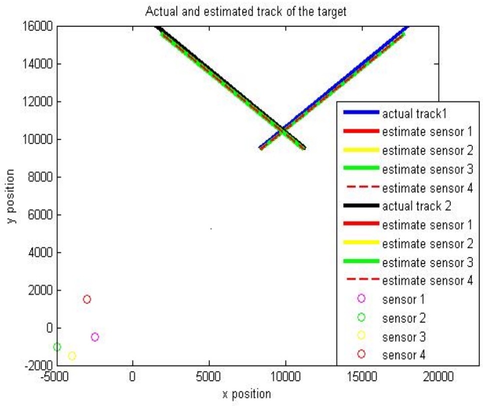
5.6 FIFF for tracking multiple targets following CV model (MTT)

Multi target tracking using FIFF is experimented in this section. The scenario depicts two objects flying at different directions and is being monitored by four tracking stations. The locations of the sensors are as in section 5.3.1. The sensors at all the observation points watch the relative bearing only.

The actual initial state vector of the targets. Initial position assumed by the filter	Target 1:$X1$ = [18000m,16000 m,3 m/s,2 m/s]′ Target 2: 2 = [1500 m,16000 m,3 m/s, -2 m/s]′ Target 1:$X1$= [17800m,15600 m,2.9 m/s,1.9m/s]′ Target 2:$X2$= [1800m,15600 m,2.9 m/s,-1.9 m/s]′	Position in m and velocity in m/s
Process transition matrix	$A1 = \begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A2 = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	*A1* for target 1 and *A2* for target 2. Sampling time T=5.
Initial State covariance matrix $P_{0/0}$	$P_{0/0} = \begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of target 1 and 2 , with variances of state vector elements along the diagonal elements.
Observation noise of 4 sensors	$R1$=1 rad^2 , $R2$=2 rad^2 , R3=3 rad^2 , R4=4 rad^2	Measurement error covariance R
Sensor position	(-2500 m,-500 m) (-5000 m,-1000 m) (-4000 m,-1500 m) (-3000 m, 1500 m)	(x, y) position of the 4 sensors.
Q matrix	$Q = \begin{bmatrix} 0.001\text{m}^2 & 0 & 0 & 0 \\ 0 & 0.001\text{m}^2 & 0 & 0 \\ 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.001\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix, Q
For the fuzzy membership function.	σ_i^2=2 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance of error and change of error for FIFF

Table 5.8 Simulation scenario for FIFF- multi target tracking

The FIFF was run for 650 iterations and scenario plot, mean square error in estimation and velocity estimate are plotted (Fig. 5.15). The velocity plot of target 2, was assumed to be -2 m/s, and hence is seen negative in the velocity estimate.



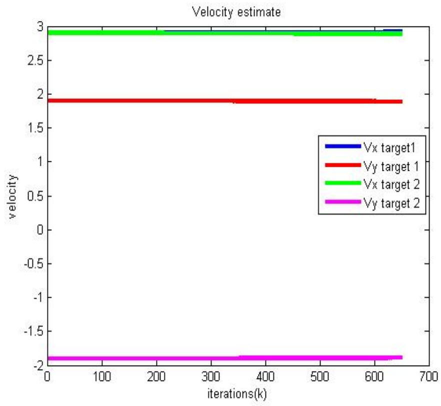


Fig. 5.15 (a)Scenario plot **Fig. 5.15 (b) Velocity estimate**

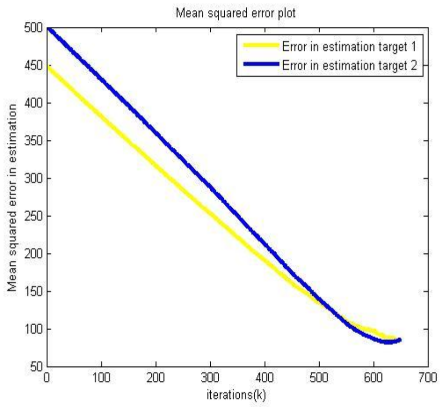


Fig. 5.15 (c) MSE plot

Fig. 5.15 FIFF in tracking multiple targets

It is observed that FIFF effectively tracks multiple targets, as seen in the simulation plots. The scenario plot (Fig. 5.15(a)) shows good tracking, which was further confirmed from the MSE plot in Fig. 5.15(c). It was observed that the instantaneous error in position estimate reduces from 500 m to 80 m in 650 iterations.

5.7 Conclusion

It has been summarized from the above simulations that a FIFF is capable of tracking targets following CV model and also slightly maneuvering targets (assuming CV model alone). The excellent tracking using FIFF was also confirmed with multiple targets. But, it is not effective as such in tracking targets following switching models by assuming CV model alone (case explained in Section 5.5). Chapter 6 proposes two techniques for tracking targets following switching models, using FIFF.

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Chapter - **6**

TRACKING OF MANEUVERING TARGETS USING THE FIFF

In the previous chapter, the admirable tracking performance of FIFF in tracking targets using the bearing only measurement was demonstrated. It was equally heartening to observe the commendable resilience of the FIFF to divergence, even when the target was maneuvering. In order to test and establish the hardiness of the FIFF in responding to more complex maneuver, the present chapter investigates the behavior of the FIFF when a target maneuvers from Constant Velocity model (CV) to Coordinated Turn model (CT), thereby giving exhibiting large maneuvers. It has been observed in Chapter 5; that FIFF fails to follow the track, when the target switches from CV to CT model. The present chapter outlines two methods to overcome the problem. While using the FIFF in tracking maneuvering targets,

i. Employing Chi square detector to assess the onset and termination of the maneuver, is a commonly used technique in maneuver detection. But this technique assumes the turning rate for the CT model and alternately

ii. Estimate the turning rate on line to detect the maneuver and use the estimated value in the CT model.

6.1 Constant Velocity (CV) and Coordinated Turn (CT)-Process and Observation Model

The state of the target is assumed to be a 4 dimensional vector representing *x* and *y* position of the target and the velocities in the *x* and *y* directions.
The state vector at any time *k* is defined as

$$X_k = [x_k, y_k, v_{xk}, v_{yk}]' \quad (6.1)$$

where, x_k, y_k, v_{xk} and v_{yk} are the *x* position ,*y* position, *x* velocity, *y* velocity respectively at time *k*. It is assumed that the maneuvering target follows a CV model or a CT model during different periods of time following the equation

$$X_k = A_{k-1}X_{k-1} + Gw_k \quad (6.2)$$

The state vector evolves in time according to the above equation, where the CV model followed was given in Eq. 3.27 and is repeated below for ready reference.

$$A=\begin{bmatrix} 1 & 0 & -T & 0 \\ 0 & 1 & 0 & -T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.3)$$

But when the target follows a coordinated turn model, the transition matrix, *A*, is defined as

$$A=\begin{bmatrix} 1 & 0 & \mathrm{Sin}\,\Omega T/\Omega & -(1-\mathrm{Cos}\,\Omega T)/\Omega \\ 0 & 1 & (1-\mathrm{Cos}\,\Omega T)/\Omega & \mathrm{Sin}\,\Omega T/\Omega \\ 0 & 0 & \mathrm{Cos}\,\Omega T & -\mathrm{Sin}\,\Omega T \\ 0 & 0 & \mathrm{Sin}\,\Omega T & \mathrm{Cos}\,\Omega T \end{bmatrix} \quad (6.4)$$

where, *T* is the sampling time and the turning rate Ω is assumed to be known. Positive values of Ω correspond to clock wise CT and negative values of Ω correspond to anti clock wise CT [95]. *G*, the process noise gain matrix is given by

$$G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} \tag{6.5}$$

The noise in the state vector is Gw_k, where, w_k is a random variable taken from a distribution $N(0,\sigma^2)$. Hence process noise covariance, $Q = E[\{Gw_k\}\{Gw_k\}']$, is calculated as $Q = \sigma^2 GG'$, where

$$GG' = \begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix}$$

Here again only the bearing angle measurements at any time *k*, are assumed to be available, designated by Z_k^i where *i* = 1, 2 3, 4 corresponding to the 4 sensors. The Jacobean of the measurement model, H_{k+1} earlier introduced in Eq. 3.30 is modified below, to take care of the variation w.r.t noise.

$$H_{k+1} = \begin{bmatrix} \frac{\partial h}{\partial x_{k+1}} & \frac{\partial h}{\partial y_{k+1}} & \frac{\partial h}{\partial v_{x_{k+1}}} & \frac{\partial h}{\partial v_{y_{k+1}}} \end{bmatrix} \tag{6.6}$$

The covariance of state matrix is defined as

$$P = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_{v_x}^2 & 0 \\ 0 & 0 & 0 & \sigma_{v_y}^2 \end{bmatrix} \tag{6.7}$$

where , $\sigma_x^2, \sigma_y^2, \sigma_{v_x}^2$ and $\sigma_{v_y}^2$ are the variances of the parameters in the state matrix.

6.2 Detection of maneuver onset using Chi-square test

1. Target state update is done in accordance with Eq. 6.2, where *A*, the state transition matrix, is made to switch between CV and CT model according to Chi square detector. The Chi-squared test on the measurement residual is a very simple technique that has been used in many tracking applications [145, 146,147]. Maneuver onset detection can be formulated as testing between two hypotheses:

   H_0: Maneuver not detected
   H_1: Maneuver detected

   Under the linear-Gaussian assumption and H_0, measurement residuals of a KF are zero mean and Gaussian variables, i.e. $e_k \sim N(0, S_k)$ where $e_k = Z_k - Z_k$ and $S_k = E[e_k e_k^T]$. Hence, $\varepsilon_k = e_k^T S_k e_k$ is a Chi square distributed variable, χ_{ne}^2 with $n_e = \dim(e_k)$ .This technique helps to check the goodness of fit so that it is possible to judge if e_k has assumed distribution under H_0. This gives the test for the maneuver detection as $\varepsilon_k > \chi_{n_e}^2(\alpha) \Rightarrow H_1$, where 1-$\alpha$ is the confidence level of the test. This means H_0 will be rejected with confidence 1-α if ε_k exceeds the corresponding threshold. The state transition matrix is switched between CV and CT model based on the outcome of the Chi square test.

2. As was explained in Chapter 5, the instantaneous error (e_k) and rate of change of error (Δe_k) are calculated using Eq. 5.7 and Eq.5.8, which are given as input to the fuzzy inference system, to compute the value of *q*, which is used to modify the information i_k and I_k that contribute to information state y_k and corresponding information matrix Y_k (computed using the Eq. 5.16 and Eq. 5.17).

6.3 Simulation and Results

6.3.1 Case 1: Performance of FIFF, which detects maneuver using chi-square test for a target that switches from CV to CT model

The behavior of the FIFF, when a target switches between CV and CT models, is evaluated by simulating the scenario, where a moving object, is monitored by 4 tracking stations, which receive only bearing angles as the measurement. The scenario considered here encounters large maneuver of a single target, which are tracked by switching the FIFF between CV and CT model. The performance of FIFF with chi square detector is demonstrated below. The scenario considered is the same as in section 5.5, where the target moves follows a CV model first and then switches to CT model. The proposed modification in FIFF, using the model switching triggered by the Chi-square test, is shown to track the target, even when it takes a turn showing a large maneuver. The actual and estimated track, the velocity estimate and MSE in position are plotted, in Fig.6 .1(a), Fig. 6.1(b) and Fig.6.1(c) respectively.

The estimated track in red (Fig.6.1 (a)), is very close to the actual track of the target, shown in blue, which is not visible due to overlapping of the tracks. The good tracking performance of the modified FIFF is confirmed from the velocity estimate in Fig. 6.1(b). It is seen that velocity is constant as assumed i.e. 3m/s and 2m/s respectively when moving along CV path and varies in a periodic fashion after switching to CT model. The steep fall in the MSE in position estimate (Fig. 6.1(c)) shows good tracking of the proposed algorithm initially along CV path and then it increases for some time, as the target switches its path through a new model. During this time, the algorithm is correcting itself after detecting the maneuver. The error again decreases as the algorithm switches over to the new model.

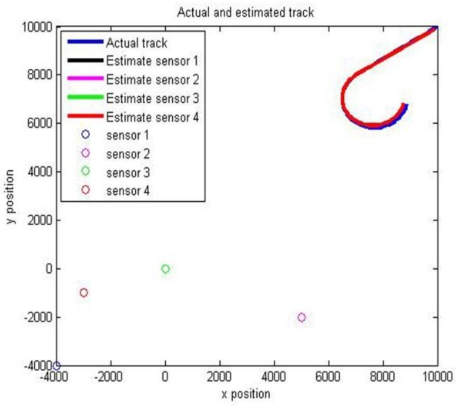

Fig. 6.1 (a) Scenario plot

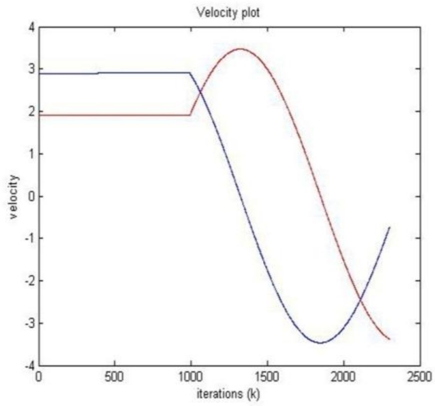

Fig.6.1 (b) Velocity estimate following chi-square test

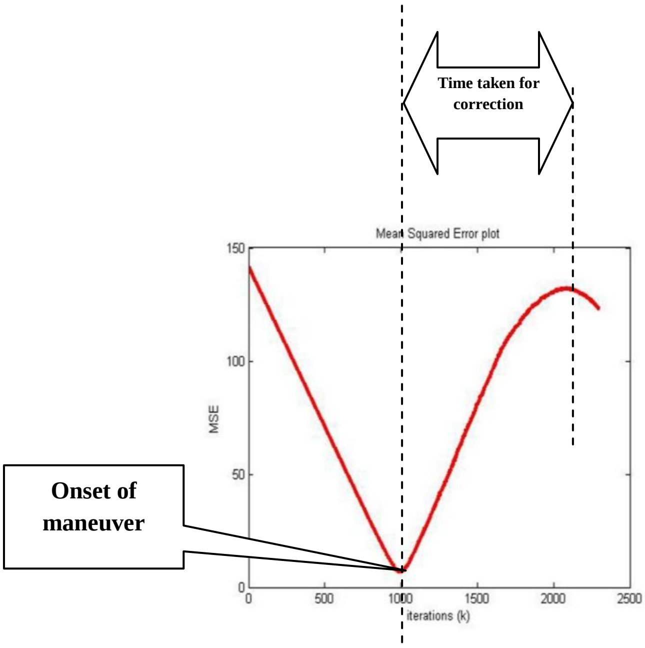

Fig.6.1(c) MSE plot, showing the onset of maneuver

Fig. 6.1 FIFF tracking the maneuvering target, using Chi-square detection

6.3.2 Case 2: Performance of FIFF, which detects maneuver using chi-square test for a target that switches from CT to CV model

The scenario considered here encounters large maneuver of a single target, which are tracked by switching the FIFF between CT and CV model. The target follows CT model for 550 time instants and then switches to CV mode (simulation scenario same as section 5.5). The performance of FIFF with chi square detector, run for 950 iterations is demonstrated in this section. Fig 6.2(a) gives the scenario plot of the target. The good tracking performance of the modified FIFF is confirmed from the velocity estimate in Fig. 6.2(b).It is seen that velocity attains a constant value as expected when moving along CT to CV model. The steep fall of the MSE (Fig. 6.2(c)) shows good tracking of the proposed algorithm initially along CV path and then the small increase after 500 iterations shows the switching of the target to a new model and after which the error again decreases.

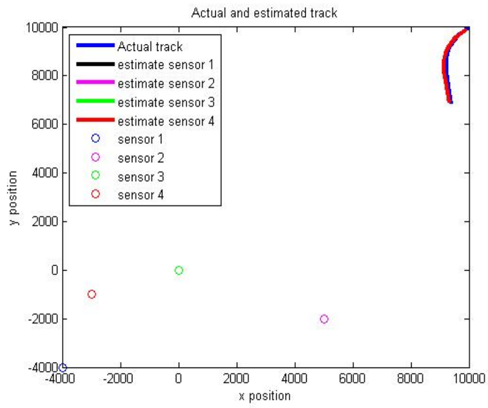

Fig. 6.2 (a) Scenario plot

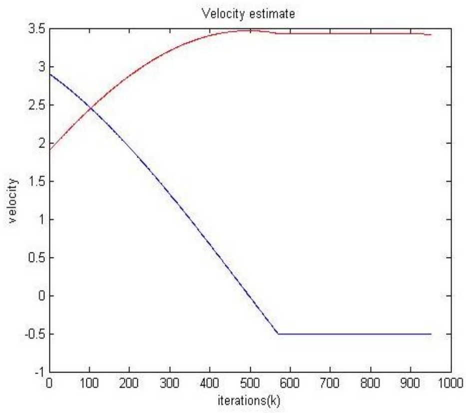

Fig. 6.2 (b) Velocity estimate following chi-square test

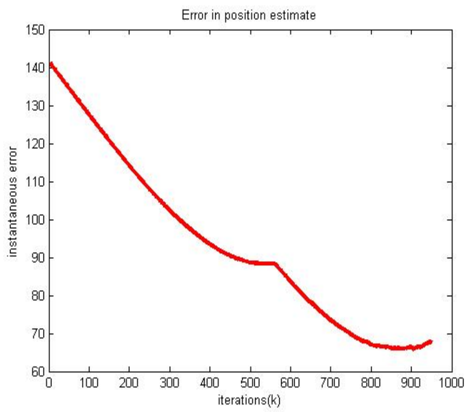

Fig. 6.2 (c) Plot of MSE showing onset of maneuver

Fig. 6.2 FIFF tracking the maneuvering target, using Chi-square detection (CT to CV mode)

6.4 Estimation of Turn Rate from range rate

The approach discussed in Sec.6.2 assumes that the target takes a known constant turning rate, every time it switches to CT model. In practical cases, such an assumption is not viable and would require that the turning rate is estimated online. Alternately, an adaptive turn rate model is utilized, where the turning rate of the moving target is estimated from the range rate to be used in the FIFF. However, the computation of turning rate does not assume range measurements. Matching to the bearing only tracking scenario, the range rate is also computed from bearing measurements only [148]. Accordingly, the detection of the onset of maneuver is on the basis of consistent change in the turning rate of the target. Following the results reported in [148], the turn rate of the moving target is estimated in the proposed work, using range rate, which in turn is calculated from bearing measurements only. All possible turn rates are calculated using range rate measurement computed and the minimum turn rate is chosen to be the acceptable turn rate [148].The efficacy of the approach has been demonstrated through a number of different scenarios involving maneuvering targets.

The present work blends the advantages of FIFF in tracking highly manoeuvring targets also, using only the bearing measurements from multiple sensors. On detection of a manoeuvre well in time, the filter switches models CV or CT as appropriate and continues to track, thereby giving stable tracks in terms of mean square error in position and stability in velocity. It then turns out that the success of the proposed approach depends on the detection of manoeuvre. The process and observation models are similar to what were discussed in Sec. 6.1.

6.5 Adaptive turn rate model based on range rate measurement

The manoeuvre model assumes that the target speed is constant between measurements. As outlined by several authors [148, 80], the turning rate calculated from the range rate is as detailed below. Referring to the definition of the state given at the beginning of the present chapter $X_k = \left[x_k, y_k, \mathrm{v}_{xk}, \ \mathrm{v}_{yk} \right]'$, The range rate is calculated from the target state using the relation

$$r_k^c = x_k v_{xk} + y_k v_{yk} / \sqrt{x_k^{\,2} + y_k^{\,2}} \qquad (6.8)$$

Assume s and α are the speed and heading of the target respectively, and γ is the difference between inverse bearing and target heading as illustrated in Fig. 6.3.

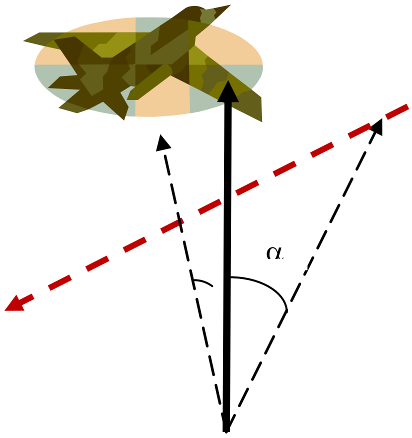

Fig. 6.3 Illustration of the heading angle w.r.t. to the reference and direction of flight

From the state estimate X_k, the target speed at time instant k ,is given by

$$s_k = \sqrt{v_{xk}^{\,2} + v_{yk}^{\,2}} \qquad (6.9)$$

γ_k is related to range rate and target speed using the relation,

$$\gamma_k = \arccos(-r_k^c / s_k) \ (6.10) \text{ and } \qquad \alpha_k = (Z_k + \pi) \pm \gamma_k \qquad (6.11)$$

Let τ_k be the change in heading angle from α_{k-1} to α_k,

$$\tau_{left} = (\alpha_{k-1} - \alpha_k) \bmod 2\pi \qquad (6.12)$$

$$\tau_{right} = (\alpha_{k-1} - \alpha_k) \bmod 2\pi \qquad (6.13)$$

The possible turn rates are calculated as

$$\tau_k = \begin{bmatrix} \tau_{\max left} & \tau_{\min left} & \tau_{\min right} & \tau_{\max right} \end{bmatrix} \qquad (6.14)$$

The magnitude of τ_k is less than 2π .The minimum value of τ_k , τ_{mink} is used to estimate the turn rate as

$$\Omega_k = \frac{\tau_{\mathrm{mink}}}{T} \qquad (6.15)$$

The CT model uses Ω_k so obtained to update the states using the transition matrix defined in Eq. 6.3 and Eq. 6.4. Here again, the detection of manoeuvre is also on the basis of consistent change in the turn rate of the target. If the change is beyond a pre-defined threshold, a manoeuvre is said to be detected. The decision making strategy proposed in the FIFF has been effectively combined in the simulations reported below, there by demonstrating the process of alleviating the problem of divergence on account of target maneuver.

6.6 Simulations and Results- Adaptive turn rate model

As in the case of previous approaches, the performance of the FIFF with on line estimation of turning rate is simulated through, a scenario consisting of a moving target exhibiting manoeuvre, being observed from four different observation points. As in all the previous instances explained, the observing stations receive only the bearing angles as the measurement. To cite a practical example, a dived submarine often performs a CT motion, with a definite turning rate and the object has to be tracked from multiple points, which have the luxury to get only the bearing information. Three different tracks have been simulated to show the consistency in the convergence property of this technique. The simulation scenario is shown in detail in Table 6.1 below.

The actual initial state vector of the target. Initial position assumed by the filter	X =[10000 m ,10000 m, 3 m/s, 2 m/s]′ $X_{0/0}$=[9900 m,9900 m, 2.9 m/s.1.9 m/s]′	Position in m and velocity in m/s
State covariance $P_{0/0}$	$P_{0/0}=\begin{bmatrix} 1\text{m}^2 & 0 & 0 & 0 \\ 0 & 1\text{m}^2 & 0 & 1 \\ 0 & 0 & 0.1\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 0.1\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state covariance matrix, with variances of state vector elements along the diagonal elements.
Observation noise of 4 sensors	1 rad^2, 2 rad^2, 3 rad^2 and 4 rad^2 respectively	Measurement error covariance R
Sensor positions	(-4000 m,-2000 m), (5000 m, -2000 m), (0 m,0 m), (-3000 m, -1000 m) respectively.	(x, y) position of the 4 sensors in m.
Q matrix	$Q=\sigma^2\begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix}$ $Q=\begin{bmatrix} 0.0014\text{m}^2 & 0 & 0.0027\frac{\text{m}^2}{\text{s}} & 0 \\ 0 & 0.0014\text{m}^2 & 0 & 0.0027\frac{\text{m}^2}{\text{s}} \\ 0.0027\frac{\text{m}^2}{\text{s}} & 0 & 0.0055\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0.0027\frac{\text{m}^2}{\text{s}} & 0 & 0.0055\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix $Q=\sigma^2 GG'$ σ^2 =0.0055 T=1
For the fuzzy membership function.	σ_i^2=3 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance of error and change of error

Table 6.1 Simulation scenario in detail

In the plots given below, the MSE in the track is calculated as $\sqrt{(x_p - x_P)^2 + (y_p - y_p)^2}$, where x_p and y_p refer to the actual x and y position of the target and x_P and y_p correspond to the estimated positions. The different simulation results, confirming the promising and consistent performance of the proposed algorithm, are discussed in the Cases 1 to 3 below.

6.6.1 Case 1: Tracking of target switching from CV to CT model

In this case, the target, moves in a linear path for first 1000 s and then switches to a CT path with a constant turn rate of 0.003 rad/s and continues in that path for the next 1330s. As a result of the proposed modification in FIFF, using adaptive turn rate model to estimate turn rate, the filter detect the onset of manoeuvre and switch over to the CT model are illustrated in Fig. 6.4. It is seen that the track is stable, even when it takes a turn showing a large manoeuvre. The actual and estimated track of the target, the velocity estimate and mean squared error in position are plotted, respectively in Fig. 6.4(a), Fig. 6.4(b) and Fig. 6.4(c) respectively. It is worthwhile to note that the Ω used in the CT model is not assumed; but furnished on line by the adaptive turn rate model.

Fig. 6.4(b) shows the velocity along x and y directions. It is seen that velocity estimate is constant as assumed i.e. 3 m/s and 2 m/s respectively when moving along CV path and varies in a periodic fashion after switching to CT model. The steep fall of the mean square error (Fig. 6.4 (c)) brings out success of the proposed algorithm, in effectively holding the track, along linear path (CV model) and then subsequently during the CT model. It can be seen that the algorithm is correcting itself quickly, after detecting the maneuver. The error decreases as the algorithm stabilizes the CT model.

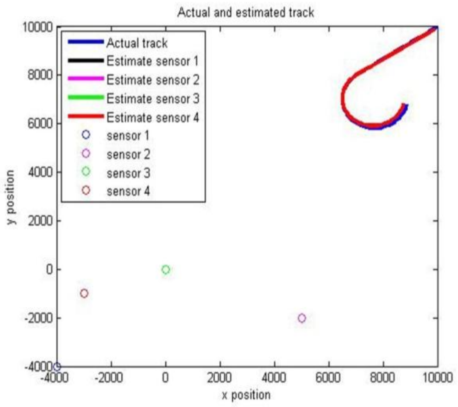

Fig. 6.4(a) Scenario plot showing excellent tracking

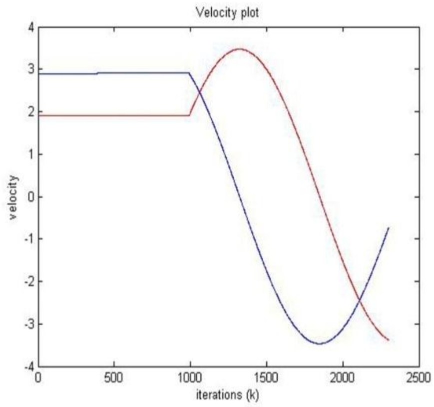

Fig. 6.4(b) Excellent estimate of velocity

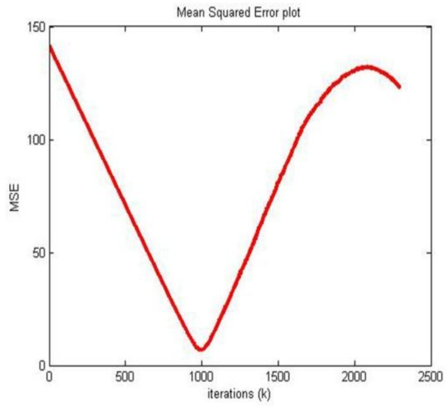

Fig. 6.4 (c) MSE showing excellent recovery after detecting maneuver

Fig. 6.4 Tracking of the target switching from CV to CT model; turn rate is estimated on line

6.6.2 Case 2: Tracking of target switching from CT to CV model

In this case the target initially follows a CT model for first 900 time instants, with a constant turning rate of 0.001 rad/s, and then switches to linear model. The program is run for 1200 iterations. The actual and estimated track, velocity estimate and MSE are plotted for the turn rate model furnishing Ω on line to the FIFF in Fig. 6.5(a), Fig. 6.5(b) and Fig. 6.5(c).The switching from CT to CV model, holding the track, is clearly seen in the velocity plot also. The MSE plot confirms the good performance of the proposed technique.

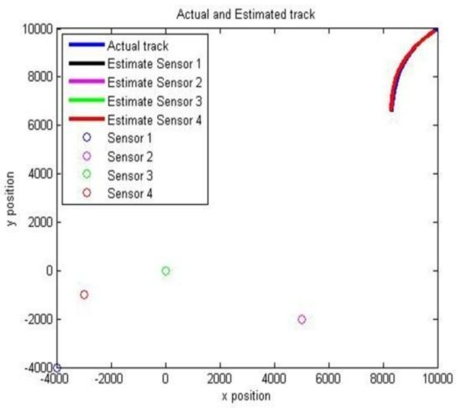

Fig. 6.5(a) Scenario plot

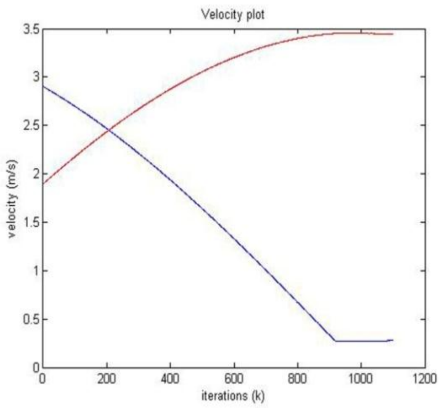

Fig. 6.5 (b) Velocity estimate

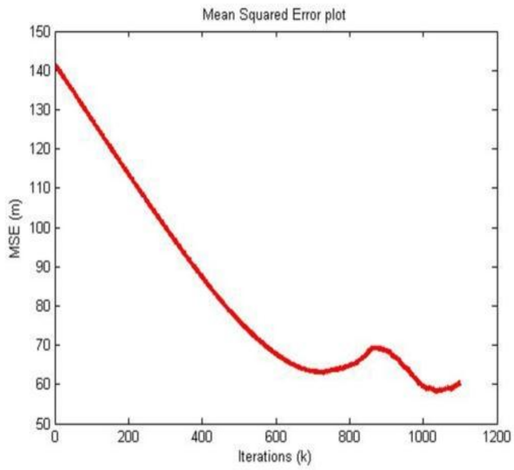

Fig. 6.5(c) Steady convergence of MSE

Fig. 6.5Tracking of the target switching from the CT model to CV model; the estimate of the velocities post switching is correct

6.6.3 Case 3: Tracking of the target switching from CV to CT and then to CV

In this case the target switches between CV and CT model during various time periods. The track considered here, initially follows a CV model with v_x =3 m/s and v_y=2 m/s for first 1000 time instants, then switches to CT model with a constant turning rate of 0.003 rad/s for next 1700 time instants and thereafter, switches again to CV with velocities v_x= - 3 m/s and v_y= 2 m/s and continues. The program is run for 4500 iterations. Fig. 6.6 (a), Fig. 6.6 (b) and Fig. 6.6(c) brings out the switching from one model to another and the unwavering tracking in a given mode is noticeably seen from the scenario plot in Fig. 6.6(a).

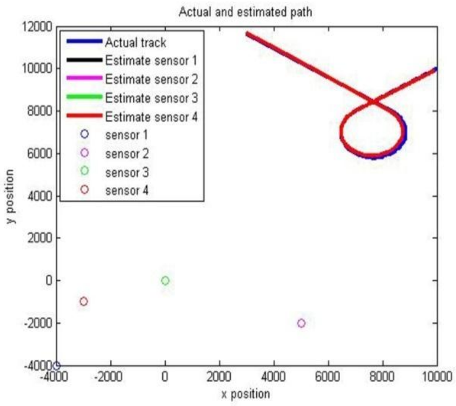

Fig. 6.6(a)Scenario plot

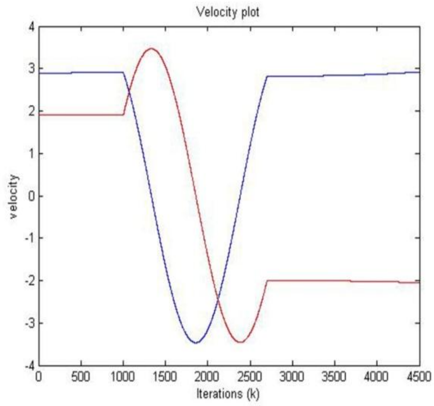

Fig. 6.6(b) Velocity plot

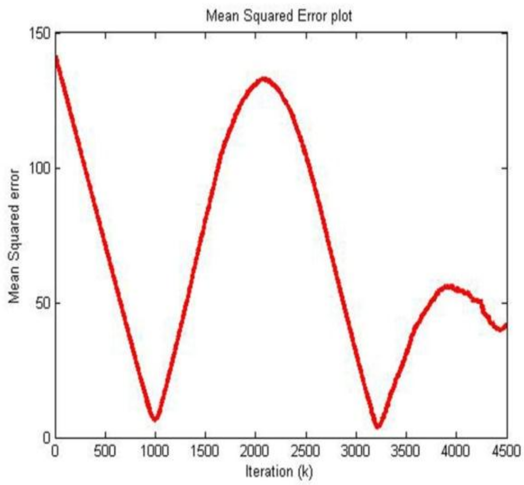

Fig. 6.6(c) MSE showing correct recovery after each maneuver

Fig. 6.6 Tracking of the target switching from CV to CT and then to CV; the velocity estimates are correct in magnitude and direction

While the correct estimate of the new velocity is evident in Fig. 6.6(b), the appreciable reduction in the MSE in position estimate in Fig. 6.6(c) reconfirms the efficacy of the proposed method.

	Track - CT followed by CV	**Track - CV followed by CT**	**Track- CV followed by CT followed by CV**
MSE sensor 1	53.9722 m	16.6 m	5.2163 m
Std. deviation (MSE sensor 1)	5.656 m	3.3 m	1.26 m
MSE sensor 2	53.9722 m	16.6 m	5.2163 m
Std. deviation (MSE sensor 2)	5.656 m	3.3 m	1.26 m
MSE sensor 3	53.9722 m	16.6 m	5.2163 m
Std. deviation (MSE sensor 3)	5.656 m	3.3 m	1.26 m
MSE sensor 4	53.9722 m	16.6 m	5.2163 m
Std. deviation (MSE sensor 4)	5.656 m	3.3 m	1.26 m
Velocity v_x	0.154 m/s	CT model-varying velocity	2.93 m/s
Std. deviation- v_x	0.137 m/s		0.0041 m/s
Velocity v_y	1.9 m/s	CT model-varying velocity	2.04 m/s
Std. deviation- v_y	0.000002 m/s		0.0023 m/s

Table 6.2 MSE in position and velocity estimate along with standard deviation in estimation

The performance of modified FIFF in tracking targets following switching models have been summarized in Table 6.2 by performing 1000 Monte Carlo runs. The commendable performance of FIFF is observed from the values of standard deviation of mean squared error in position estimate and velocity estimate.

6.7 FIFF for tracking multiple targets following CT model

Multi target tracking using FIFF is experimented in this section. The scenario depicts two object following CT model are being monitored by four tracking stations.

The actual initial state vector of the target. Initial position assumed by the filter	Target 1:$X1$ = [11000m,16000 m,3 m/s,2 m/s]´ Target 2:$X2$ = [7000 m,16000 m,3 m/s,-2 m/s]´ Target 1:$X1_{0/0}$=[11110m,15800 m,2.9 m/s,1.9 m/s]´ Target 2:$X2_{0/0}$= [6900 m,15800 m,2.9 m/s,-1.9 m/s]´	Position in m and velocity in m/s
Process transition matrix	$$A1 = A2 = \begin{bmatrix} 1 & 0 & -\sin(\Omega T)/\Omega & -(\cos(\Omega T - 1)/\Omega) \\ 0 & 1 & -(1-\cos\Omega T)/\Omega & -\sin(\Omega T)/\Omega \\ 0 & 0 & \cos(\Omega T) & -\sin(\Omega T) \\ 0 & 0 & \sin(\Omega T) & \cos(\Omega T) \end{bmatrix}$$	*A1* for target 1and *A2* for target 2. Sampling time T=5. Ω =0.001 rad/s for target1 and -0.001 rad/s for target 2.
State covariance $P_{0/0}$	$$P_{0/0} = \begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$$	Initial value of state co variances matrix, of target 1 and 2, with variances of state vector elements along the diagonal elements.
Observation noise of 4 co-located sensors	1 rad^2, 2 rad^2,3 rad^2,4 rad^2	Measurement error covariance *R*

Sensor positions	(4000 m,-500 m), (0 m,-1000 m), (1000 m,-1500 m), and (3000 m,1500 m)	(x, y) position of the 4 co-located sensors.
Q matrix	$Q=\sigma^2\begin{bmatrix}\frac{T^4}{4} & 0 & \frac{T^3}{2} & 0\\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2}\\ \frac{T^3}{2} & 0 & T^2 & 0\\ 0 & \frac{T^3}{2} & 0 & T^2\end{bmatrix}$ $Q=\begin{bmatrix}0.156\mathrm{m}^2 & 0 & 0.0625\frac{\mathrm{m}^2}{\mathrm{s}} & 0\\ 0 & 0.156\mathrm{m}^2 & 0 & 0.0625\frac{\mathrm{m}^2}{\mathrm{s}}\\ 0.0625\frac{\mathrm{m}^2}{\mathrm{s}} & 0 & 0.025\frac{\mathrm{m}^2}{\mathrm{s}^2} & 0\\ 0 & 0.0625\frac{\mathrm{m}^2}{\mathrm{s}} & 0 & 0.025\frac{\mathrm{m}^2}{\mathrm{s}^2}\end{bmatrix}$	Process noise covariance matrix $Q=\sigma^2GG'$ T=5
For the fuzzy membership function.	σ_i^2=2 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance in error and change of error for FIF

Table 6.3 Scenario of FIFF tracking multiple targets – CT model

The FIFF was run for 450 iterations and scenario plot, mean square error in estimation and velocity estimate are plotted in Fig. 6.7.

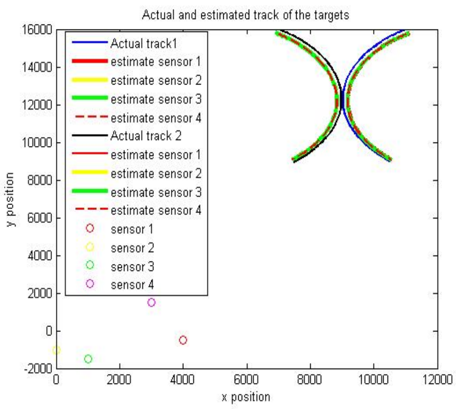

Fig. 6.7(a) Scenario plot

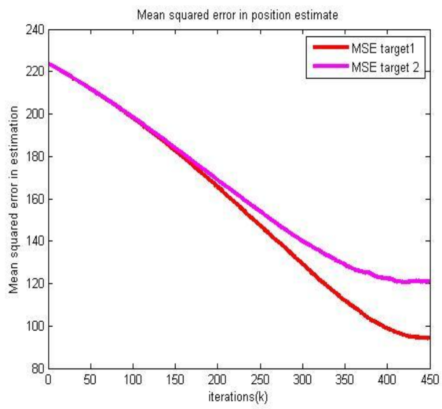

Fig. 6.7 (b) MSE plot

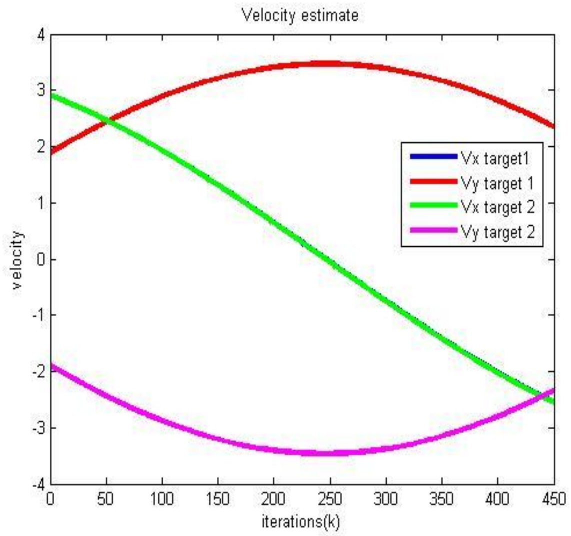

Fig. 6.7 (c) Velocity estimate

Fig. 6.7 FIFF for multi target tracking following CT model

Tracking results of multiple targets following CT model, are shown in Fig.6.7. The scenario plot in Fig. 6.7(a) shows good tracking. The MSE in position estimate reduces from 225 m to 120 m and 90 m respectively for the two targets in 450 iterations.

6.8 Conclusion

The problem of tracking a maneuvering target by fusing measurements from multiple sensors using the FIFF developed in Chapter 5 has been demonstrated in this chapter. The target which switches between the CV and CT model, makes the maneuver complex to be tracked using the bearing information only as measurements. The proposed method, has effectively utilized the advantages of FIFF, along with a switching technique construed from the estimates of the turning rate, for detecting the maneuver and consequently switching the model. A notable outcome of the work is that the estimate of Ω required in the CT model is furnished by the turn rate computed on line.

While underscoring the excellent performance of incorporating fuzzy corrections in the FIFF to track targets using only the bearing measurements, the work reported in the present chapter has demonstrated the improvement in performance while tracking targets, which switch between CT and CV models. The plots of actual and predicted track, the instantaneous error in the position of the track, and the estimated velocities in the *x* and *y* directions generated for the cases discussed here re-affirmed the efficacy of the proposed method.

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Chapter - 7

FUZZY INFORMATION FILTER FOR MULTI-TARGET TRACKING IN NON-CLUTTER ENVIRONMENT - A COMPARISON WITH JP DAF

Data association is crucial in tracking targets with low probability of detection in the presence of false alarms [128, 129, 130, 131]. Though these techniques work reasonably well for tracking targets in sparse scenarios, they begin to fail with targets of higher false alarm rate or which are maneuvering, with low observability [77]. A very effective technique in tracking a single target in clutter is the Probabilistic Data Association Filter (PDAF) [80]. In PDA, instead of using only one measurement among the received ones and discarding the others, all the validated measurements are combined using the likely hood function corresponding to each measurement [80]. Data association and tracking becomes trickier with multiple targets, as a single measurement itself can be corroborated by multiple tracks. Utilizing this concept, the Joint Probabilistic Data Association Filter (JPDAF), tracks multiple targets by evaluating the measurement to track association probabilities, and combining them to find the state estimate [130].

The Fuzzy Information Filter (FIF), which has been proposed in the thesis essentially to alleviate the problem of divergence in Information filter, is a re-cast of the Extended Kalman filter. The Information filter is seen to work well even without an initial estimate and is often preferred over EKF. Motivated by the success in controlling divergence, the present chapter, examines the applicability of FIF algorithm for multi target tracking and compares the performance with the JPDAF algorithm. The bearing only multi target tracking problem using FIF with a single fused measurement is demonstrated and compared with the JPDA in the present Chapter. While the JPDAF fuses the innovations from a set of co-located sensors, the proposed algorithm fuses the measurements directly before presenting to the FIF. The over view of both approaches are presented in the sections to follow and the performance is compared in Section 7.3, for different multi target tracking scenarios, including maneuver, in terms of estimated track of the target, MSE and velocity estimated. Monte Carlo simulations of 1000 runs were also performed for comparing the performance of FIF and JPDAF.

As outlined in the previous chapters, discrete time linear dynamic system, described by a vector difference equation with additive white Gaussian noise is assumed for modeling the targets. The target state at time instant *k* given by

$$X_k = \left[x_k , y_k , \mathrm{v}_{xk} , \mathrm{v}_{yk} \right]' \tag{7.1}$$

evolves in time according to the model

$$X_k = A_{k-1} X_{k-1} + w_k \tag{7.2}$$

The measurement is given by $Z_k = h\left(k, X_k\right) + v_k$

where *h* is the measurement function that relates the measurement to the target state. The process transition matrix A_k, process noise w_k. and the measurement noise v_k are as defined in section 5.1. For CT model, the target state is modeled as $X_k = A_{k-1} X_{k-1} + G w_k$, where *G* is as defined in Eq. 5.2.

7.1 The Joint Probabilistic Data Association filter (JPDAF)

The assumptions made in [80] are followed in total. The key feature of the JPDA is that the conditional probabilities of the joint association events are evaluated as

$$E_k = \bigcap_{j=1}^{m} E_{jt_j k} \tag{7.3}$$

For a current time *k*, $E_{jt_j k}$ is the event that measurement *j* at time *k* originated from target *t*, *j*=1, 2…*m*, where *m* is the number of measurements in the validation region of the target and t=0, 1...N_T. t_j is the index of the target to which measurement *j* is associated in the event under consideration, and N_T is the known number of targets.

Assuming the target tracking in a non-clutter environment, all measurements are assumed to originate only from targets of interest. In JPDAF [80], it is assumed that the states of the targets conditioned on the past observations are mutually independent. Hence marginal association probabilities are considered. These marginal probabilities β_{jt} are obtained from the joint probabilities, by summing over all joint events in which the marginal event of interest occurs.

$$\beta_{jt_k} = \sum_{E:E_{jt}\in A} P\{E_k / Z^k\} \tag{7.4}$$

where, Z^k is the cumulative set of measurements [80].

The state update equation for a given target is given by (subscript *t* is deliberately dropped)

$$X_{k/k} = X_{k/k\text{-}1} + K_k \upsilon_k \tag{7.5}$$

where ,the combined innovation[80] is

$$\upsilon_k = \sum_{i=1}^{m} \beta_{ik} \upsilon_{ik} \tag{7.6}$$

and K_k is the Kalman gain given by

$$K_k = P_{k/k-1} H_k^{'} S_k^{-1} \quad (7.7)$$

where, $$S_k = H_k P_{k/k-1} H_k^{'} + R_k \quad (7.8)$$

Here $\upsilon_{ik} = Z_k - H_k X_{k/k-1}$. It can be seen that $\upsilon_k = \sum_{i=1}^{m} \beta_{ik} \upsilon_{ik}$ combines the innovations from a set of measurements to a given target. The term β_{ik} accommodates only those measurements which are within the measurement gate. $P_{k/k-1}$ is the prediction covariance and H_k is the Jacobean of the measurement matrix. Assuming that there are no clutters, the covariance associated with the updated state is given by

$$P_{k/k} = P_{k/k-1} - K_k\, S_k\, K_k\,' + P_k \quad (7.9)$$

where, $$P_k = K_k [\sum_{i=1}^{m} \beta_i \upsilon_{ik} \upsilon_{ik}\,' - \upsilon_k \upsilon_k\,'] K_k\,' \quad (7.10)$$

7.2 Measurement fusion technique

The FIF, discussed in Chapter 5 is experimented for multi target tracking application. An observation Z_k contributes i_k to information state y and I_k to information matrix Y_k .The detailed algorithm and explanation are given in Eq. 5.13 to Eq. 5.16, Eq. 5.20 and Eq. 5.21 of chapter 5.The expression for i_k is as given below

$$i_k^{\,i} = H_k^{'} R_k^{-1} q Z_k \quad (7.11)$$

The problem of tracking multiple targets with a single fused measurement has been considered in this Chapter. This section proposes a technique to handle multiple measurements received from the targets. Since, it is not known as to which measurements come from which target, the technique proposed here, fuses all the measurements received at time *k*, on the basis of a likelihood function, and uses this fused measurement for updating the state of the target.

The FIF, here makes the same assumptions as JPDA algorithm. For m validated measurements and N_T targets at a time k, the expression for fused measurement is calculated as

$$Z_k^t = P_D^t \sum_{j=1}^{m} Z_{jk} \frac{f_{tj}[Z_{jk}]}{\sum_{t=1}^{N_T} f_{tj}[Z_{jk}]} \qquad (7.12)$$

where, the likelihood function is defined as

$$f_{t_j}[Z_{jk}] = N[Z_{jk}; Z^{t_j}{}_{k/k-1}, S^{t_j}{}_k] \qquad (7.13)$$

Here $Z^{t_j}{}_{k/k-1}$ is the predicted bearing angle for target t_j., $S^{t_j}{}_k$ is the associated innovation covariance and $P^t{}_D$ is the detection probability of target t. The innovation covariance $S^{t_j}{}_k$ is computed as in Eq.7.8, where H_k and $P_{k/k-1}$ is for target t_j and R_k is for sensor j.

The m measurements are the ones that fall in the validated region of the targets. Here Z_k^t is the fused measurement that is used for updating the state of the target. The FIF updates the information state y_k and information matrix Y_k using Eq. 5.20 and Eq. 5.21. Every fused measurement Z_k^t, as given in Eq.7.9 contributes to i_k (Eq. 7.14), which in turn is used for updating information state y [80] as given, in Eq.7.16.

The sensor estimates the information state y and information matrix Y, by adding the information received from the co located sensors.

$$y_{k/k} = y_{k/k-1} + i_k \qquad (7.14)$$

$$Y_{k/k} = Y_{k/k-1} + I_k \qquad (7.15)$$

where, I_k and i_k are the information matrix and information state contributions of the sensor, The posterior state estimate is obtained as

$$X_{k/k} = Y_{k/k}^{-1} y_{k/k} \qquad (7.16)$$

It is worthwhile to note that the FIF differs from the IF only in the Eq. 7.11. But the correction has helped to reduce the tendency of the filter to diverge. It is further demonstrated in the simulations below that the fused measurements are also handled equally well by the FIF, including those scenarios, where the targets maneuver and cross. The performance of FIF is compared with the JPDAF for tracking two targets, using bearing only measurements. Two different scenarios have been considered for simulation.

7.3 Simulation and Results

The scenario considered here, consists of two targets,(N_T=2) following CV model as described in Case 1, that cross over at a point and continues its motion following target model. Case 2 scenario considers two targets following CT model and at some point of time during its travel, they come very close together and then move apart, following the target model. The probability of detection of the targets, $P^t{}_D = 0.9$, for both the targets The only measurements that are available for tracking the targets are the bearing angles received at the co-located sensors. The sensor receives two measurements at every instant *k*, and has no knowledge as to which measurement originated from which target. In the cases discussed here, it is assumed that each target generates at most one measurement and there are no false alarms. Here, we take τ_j=1and δ_t=1, which are as defined in [80].

7.3.1 Targets following CV model - Case 1

This case simulates target tracking using FIF for tracking two targets following CV model, using measurements received from co-located sensors and compares its performance with JPDAF. The simulation scenario is shown in Table 7.1.

The actual initial state vector of the targets. Initial position assumed by the filter	Target 1:$X1$ = [18000m,16000 m,3 m/s,2 m/s]´ Target 2:$X2$ = [1500 m,16000 m,3 m/s,2 m/s]´ Target 1:$X1_{0/0}$ = [17800m,15600 m,2.9 m/s,1.9 m/s]´ Target 2:$X2_{0/0}$ = [1800 m,15600 m,2.9 m/s,1.9 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A1=\begin{bmatrix}1&0&-T&0\\0&1&0&-T\\0&0&1&0\\0&0&0&1\end{bmatrix}$ $A2=\begin{bmatrix}1&0&T&0\\0&1&0&-T\\0&0&1&0\\0&0&0&1\end{bmatrix}$	*A1* for target 1and *A2* for target 2. Sampling time T=5.
Initial State covariance matrix $P_{0/0}$	$P_{0/0}=\begin{bmatrix}10m^2&0&0&0\\0&10m^2&0&1\\0&0&1\frac{m^2}{s^2}&0\\0&0&0&1\frac{m^2}{s^2}\end{bmatrix}$	Initial value of state co variances matrix, of target 1 and 2, with variances of state vector elements along the diagonal elements.
Observation noise of 4 co-located sensors	4 rad^2	Measurement error covariance *R*
Sensor position	(-2500 m,-500m)	(*x*, *y*) position of the 4 co-located sensors.
Q matrix	$Q=\begin{bmatrix}0.001m^2&0&0&0\\0&0.001m^2&0&0\\0&0&0.001\frac{m^2}{s^2}&0\\0&0&0&0.001\frac{m^2}{s^2}\end{bmatrix}$	Process noise covariance matrix, Q
For the fuzzy membership function.	σ_i^2=2 rad^2 (for *e*) ; σ_j^2=2 rad^2 (for Δe)	Variance of error and change of error for FIF

Table 7.1 Simulation scenario of multi target tracking using FIF and JPDAF- Case 1

The MSE of the estimated track is calculated as $\sqrt{(x-x)^2+(y-y)^2}$, where *x* and *y* refer to the actual *x* and *y* position of the target and *x* and *y* correspond to the estimated positions. The filters are run for 680 iterations and their performances are compared. The actual and estimated tracks of the targets for FIF and JPDAF are shown in Fig.7.1 and Fig. 7.2.

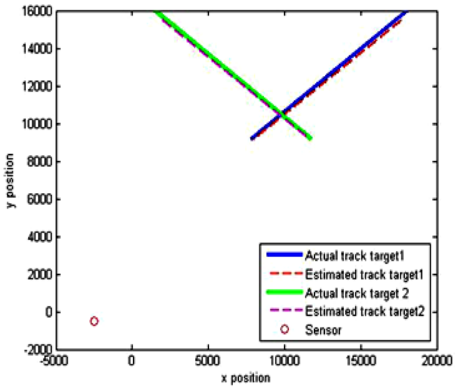

Fig. 7.1 Scenario plot (FIF)-CV model

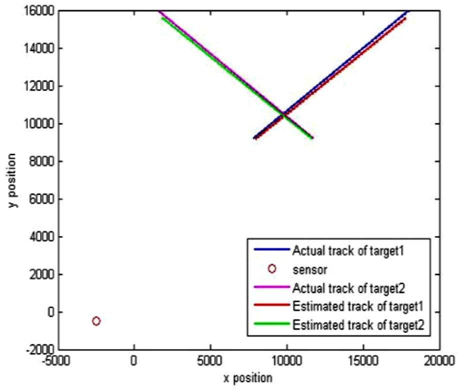

Fig. 7.2 Scenario plot (JPDAF)-CV model

It is observed from the scenario plots (Fig. 7.1 and Fig. 7.2) that both the filters track the target well. A better inference on the performance is obtained from the MSE plots and velocity estimates.

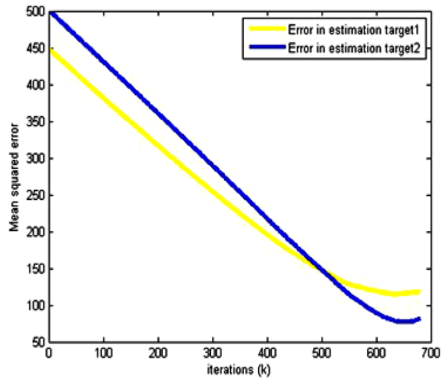

Fig. 7.3 MSE plot (FIF)- CV model

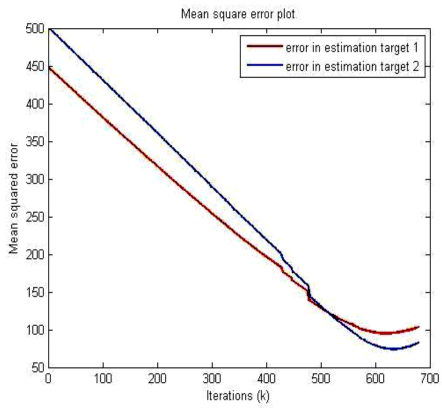

Fig.7.4 MSE plot (JPDAF) -CV model

It is observed that the JPDAF tracks, both the targets well, fusing multiple measurements, as can be seen from the mean squared error plot in Fig.7.4.The error in initial estimate for target 1 and target 2 is 450 m and 500 m respectively. In 680 iterations, the error is seen to settle to 120 m and 70 m respectively for the two targets. The JPDA filter also provides a stable velocity estimate as shown in Fig.7.6. The FIF track the targets well as can be seen from its MSE plot (Fig.

7.3).Considering the same error in initial estimate as in JPDAF, the error in position estimates settles to 120 m and 70 m respectively in 680 iterations, thereby attaining the same performance as JPDAF**.** The FIF also provides a stable velocity estimate as in JPDAF as shown in Fig.7.5. As the *x* velocities and y velocities for the two targets have been assumed to be same, the velocity estimates of the two targets overlap as observed in Fig. 7.5 and Fig. 7.6.The *x* and *y* velocity estimates converges to nearly 3 m/s and 2 m/s respectively showing good tracking in both FIF and JPDAF.

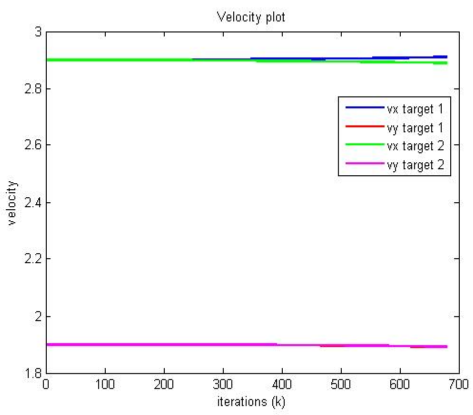

Fig. 7.5 Velocity estimate (FIF) -CV model

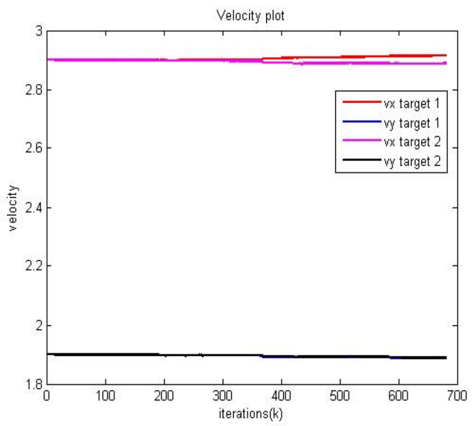

Fig. 7.6 Velocity estimate (JPDAF) -CV model

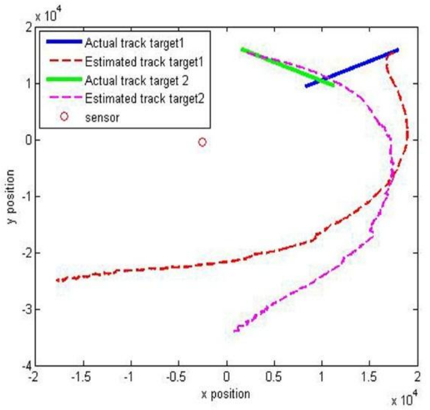

Fig. 7.7 Scenario plot (IF)

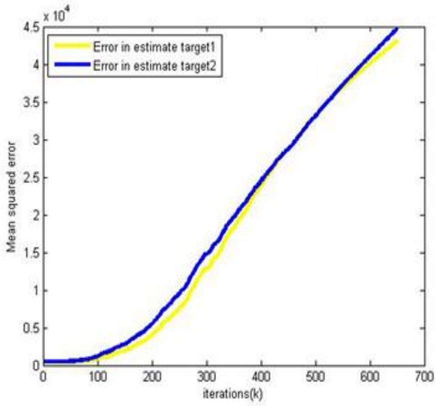

Fig.7.8 MSE plot (IF)

In order to underline the effect of fuzzy technique in the FIF employing measurement fusion, the performance of IF, without employing fuzzy correction is also demonstrated for the same scenario. As demonstrated in Chapter 4, the IF diverges after a small period of tracking, which can be clearly observed from the estimated tracks and MSE plot. The actual and estimated tracks of the targets and the MSE in estimation are plotted in Fig. 7.7 and Fig. 7.8.Thus the fuzzy technique is an essential ingredient of the FIF in multi target tracking.

7.3.2 Targets following CT model - Case 2

This case considers tracking of multiple targets maneuvering closely following CT model. The tracking performance of FIF is compared with JPDAF in terms of estimated track of the targets, MSE in position estimates and velocity estimates. Here, the turn rates of the targets are assumed to be known. The simulation scenario is described in Table 7.2.

The performance of FIF and JPDA are compared by running the filters for 500 iterations. The estimated track of the targets using FIF and JPDA are shown in Fig. 7.9 and Fig. 7.10.It is observed that both filters track the targets well. A clearer picture is obtained from the MSE plots (Fig.7.11 and 7.12).

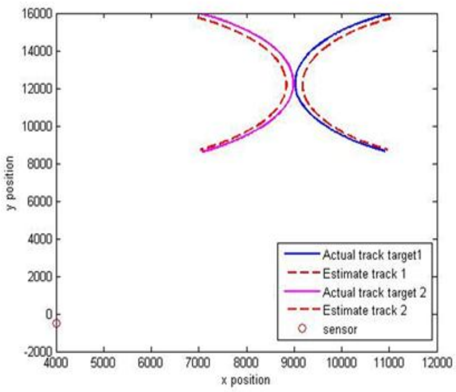

Fig 7.9 Scenario plot (FIF)- CT model

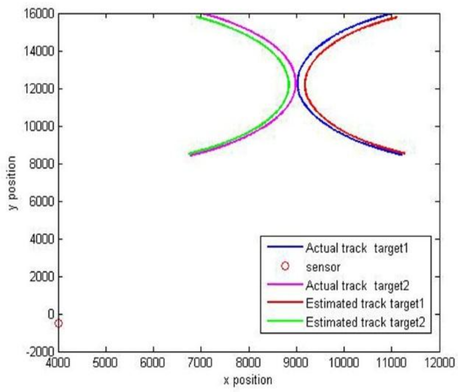

Fig 7.10 Scenario plot (JPDA)- CT model

The actual initial state vector of the target. Initial position assumed by the filter	Target 1: $X1$ = [11000m,16000 m,3 m/s,2 m/s]´ Target 2: $X2$ = [7000 m,16000 m,3 m/s,-2 m/s]´ Target 1: $X1_{0/0}$= [11110m,15800 m,2.9 m/s,1.9 m/s]´ Target 2: $X2_{0/0}$=[6900 m,15800 m,2.9 m/s,-1.9 m/s]´	Position in m and velocity in m/s
Process transition matrix	$A1 = A2 = \begin{bmatrix} 1 & 0 & -\sin(\Omega T)/\Omega & -(\cos(\Omega T-1)/\Omega) \\ 0 & 1 & -(1-\cos\Omega T)/\Omega & -\sin(\Omega T)/\Omega \\ 0 & 0 & \cos(\Omega T) & -\sin(\Omega T) \\ 0 & 0 & \sin(\Omega T) & \cos(\Omega T) \end{bmatrix}$	*A1* for target 1 and *A2* for target 2 Sampling time T=5 Ω =0.001 rad/s for target1 and -0.001 rad/s for target 2
State covariance $P_{0/0}$	$P_{0/0} = \begin{bmatrix} 10\text{m}^2 & 0 & 0 & 0 \\ 0 & 10\text{m}^2 & 0 & 1 \\ 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0 & 0 & 1\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Initial value of state co variances matrix, of target 1 and 2, with variances of state vector elements along the diagonal elements.
Observation noise of 4 co-located sensors	4 rad^2	Measurement error covariance *R*
Sensor position	(4000 m,-500m)	(x, y) position of the 4 co-located sensors.
Q matrix	$Q = \sigma^2 \begin{bmatrix} \frac{T^4}{4} & 0 & \frac{T^3}{2} & 0 \\ 0 & \frac{T^4}{4} & 0 & \frac{T^3}{2} \\ \frac{T^3}{2} & 0 & T^2 & 0 \\ 0 & \frac{T^3}{2} & 0 & T^2 \end{bmatrix}$ $Q = \begin{bmatrix} 0.156\text{m}^2 & 0 & 0.0625\frac{\text{m}^2}{\text{s}} & 0 \\ 0 & 0.156\text{m}^2 & 0 & 0.0625\frac{\text{m}^2}{\text{s}} \\ 0.0625\frac{\text{m}^2}{\text{s}} & 0 & 0.025\frac{\text{m}^2}{\text{s}^2} & 0 \\ 0 & 0.0625\frac{\text{m}^2}{\text{s}} & 0 & 0.025\frac{\text{m}^2}{\text{s}^2} \end{bmatrix}$	Process noise covariance matrix $Q = \sigma^2 GG'$ T=5
For the fuzzy membership function.	σ_i^2=2 rad^2 (for e) ; σ_j^2=2 rad^2 (for Δe)	Variance in error and change of error for FIF

Table 7.2 Simulation scenario of multi target tracking using FIF and JPDA- Case 2

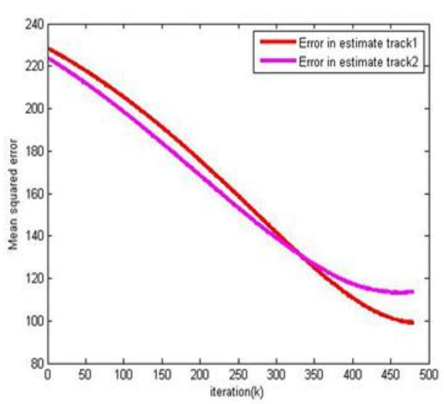

Fig 7.11 MSE plot (FIF) - CT model

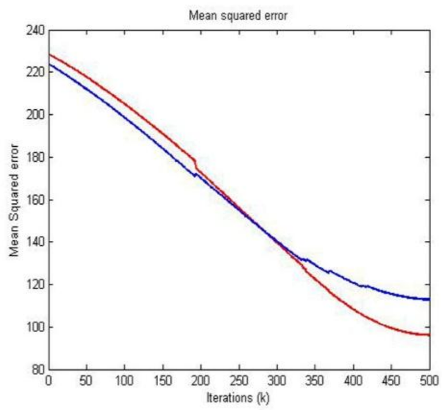

Fig 7.12 MSE plot (JPDA) - CT model

The JPDA filter tracks the target well. The initial error in position estimate is 230 m and 225 m respectively for targets 1 and 2.The error settles to 98 m and 112 m respectively in 500 iterations (Fig. 7.12). As observed in JPDA, MSE in FIF settles to 100 m and 112 m respectively for targets 1 and 2 (Fig. 7.11).The velocity estimate of JPDAF follows a particular pattern, corresponding to the CT model followed by the targets. As the *x* velocities of both targets have been assumed to be the same, they overlap. The *y* velocities for target 1 and 2 were assumed as 1.9 m/s and -1.9 m/s respectively, which is clear from the velocity plot. Thus the good performance of JPDA filter in tracking multiple targets is observed for CT model also. The velocity estimates of FIF also follow a similar pattern, thus underlining the comparable performance with JPDAF.

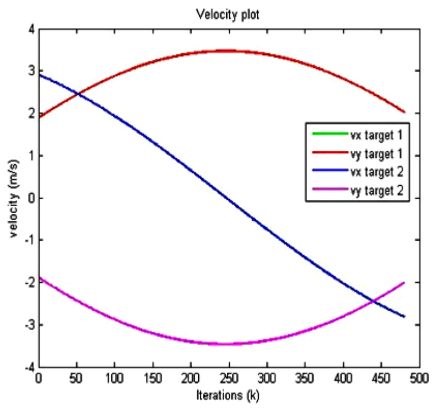

Fig. 7.13 Velocity estimate (FIF)

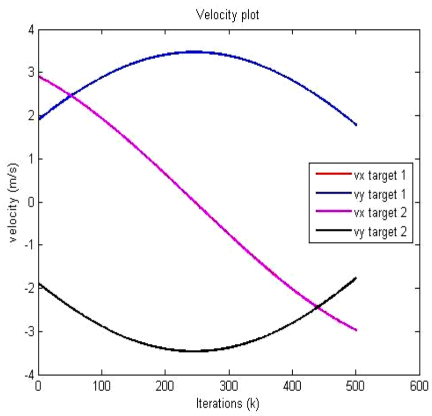

Fig. 7.14 Velocity estimate (JPDA)

The performance of FIF and JPDAF are compared by performing Monte Carlo simulation (1000 runs).

Filter	Case 1	Target 1	Target 2
JPDAF	MSE	95.5229 m	76.4659 m
	Std. deviation (MSE)	2.99 m	1.38 m
	v_x	2.9117 m/s	2.8826 m/s
	Std. deviation (Vx)	0.0476 m	0.0427 m
	v_y	1.8908 m/s	1.8837 m/s
	Std. deviation (Vy)	0.0441 m	0.0445 m
	Case 2		
	MSE	98.9124 m	104.3310 m
	Std. deviation (MSE)	5.38 m	9.405 m
	v_x	Varying (CT model)	Varying(CT model)
	v_y		
FIF	**Case1**		
	MSE	112.8821 m	77.4859 m
	Std. deviation (MSE)	1.94 m/s	0.403 m/s
	v_x	2.9086 m/s	2.8896 m/s
	Std. deviation (v_x)	0.00059 m/s	0.00059 m/s
	v_y	1.8917 m/s	1.8925 m/s
	Std. deviation (v_y)	0.00052 m/s	0.00055 m/s
	Case 2		
	MSE	99.1221 m	112.8831 m
	Std. deviation (MSE)	0.2165 m	0.8217 m
	v_x	Varying (CT model)	Varying (CT model)
	v_y		

Table 7.3Monte Carlo simulations

The performance of FIF and JPDAF are compared by performing Monte Carlo simulations (1000 runs).The comparable performance of FIF is observed from the Monte Carlo simulations for both cases, as shown in Table 7.2.

7.4 Conclusion

Multi target tracking is very common problem, dealt with air line traffic, both in the civilian and military scenario and also in under water scenario. The most difficult part of this tracking is data association. While there are a number of algorithms for multi target tracking based on Kalman filter algorithm, like the well established JPDA, the aim of exploring the possibility of using the FIF in tracking multiple targets was to assess the efficacy of the approach and evaluate both the approaches. The proposed technique of associating multiple measurements, in FIF for multi target tracking in non-clutter environment was developed and the performance of the new algorithm is demonstrated in terms of mean squared error in position estimate is also at par with the JPDAF. Hence this work has helped suggest an alternative to the JPDAF for multi target tracking in non-clutter environment, under scoring the effect of fuzzy correction in Information Filter.

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Chapter - 8

CONCLUSION AND FURTHER DIRECTIONS FOR RESEARCH

The present thesis has reported the development and evaluation of multi-sensor data fusion algorithms for bearing only target tracking (BOT problem). Literature provides a number of methods for multi sensor data fusion, based on EKF algorithm and its variants. Two commonly used techniques are the variance based fusion algorithm and the PDAF algorithm. The variance based fusion technique experimented in this work, assumed the availability of states estimated by independent EKF, along with its variance for each sensor suite. The variance based fusion technique was then implemented to fuse the states of the target being tracked and evaluated, considering various cases with different initial assumptions and measurement error variances. The performance of MSDF was also assessed in terms of mean squared error in position estimate of the target.

It was inferred from variance based fusion tracking that sensors with good initial assumption of states provide good estimation of the target state. While it was also observed that the fused estimate was always better than the poor estimates, the fused estimate was not as good as the best estimate from among all the inputs considered for fusion. The simulation studies have established the efficacy of the application of variance based fusion in target tracking. However it was interesting to note that poor initial estimates and large measurement variances often led to loss of track in due course even in spite of fusion. This led to a detailed investigation into the problem of divergence subsequently in thesis.

The PDAF algorithm for fusing measurements to generate the track also showed good performance for various simulation scenarios. However, all these algorithms suffered from the problem of divergence. Literature reveals that divergence is a central issue affecting the performance in all BOT problems.

The information fusion filter IFF, which is a recast of EKF, is an efficient algorithm based on a decentralized approach, where the information from different sensors is fused. The states are recovered from the fused information using the information matrix. Acknowledging the computational simplicity of IFF, this thesis further took up the evaluation of the performance of IFF in target tracking applications. A typical scenario consisting of 4 independent sensors, geographically well separated to monitor the movement of the target under track. Various cases were investigated with different values of Q, R and initial estimate X, and it was concluded that IFF was an effective alternative to EKF based fusion filter in target tracking. However IFF also shows tendency to diverge.

The thesis addressed the issue of divergence of IFF in greater detail with a view to alleviate the problem. As a first attempt, a new information fusion strategy called the modified IFF is introduced in the thesis. The measure of variance of the information of each of the state variable available as part of the IFF is used to combine the information in a manner similar to that of the variance based fusion. The approach resulted in commendable improvement in the performance of the IFF, which helped to retain the track for a longer period. However, the modification proposed could only delay the onset of divergence but was not effective in controlling the divergence.

Progressing further on the issue of controlling the divergence of IFF, the thesis proposed a new idea of regulating the divergence based on a fuzzy function computed on error and rate of change of error, which is a major contribution reported in the thesis. The resulting algorithm, FIFF alleviates divergence by controlling *R*, using generated defuzzifier output. The FIFF demonstrated promising results in alleviating the divergence problem. A notable observation on the proposed approach is that the FIFF is capable of tracking targets following CV model with mild maneuvers en-route. The excellent performance of FIFF was also confirmed with multiple targets following CV model. However, it was observed that the FIFF could not perform well when the target switches from CV to CT model.

Tracking of maneuvering targets only from bearing is a daunting task, because of observability issues and frequent tendency to diverge during tracking. The thesis has devoted a major portion to address this issue, utilizing the capability of FIFF in controlling divergence. Two approaches utilizing FIFF were implemented, where the FIFF switches between CV and CT models on the basis of a confirmatory test for maneuver.

- The first approach employs Chi- Square test to detect the onset of maneuver and the second approach estimates the turn rate adaptively. Both these techniques were convincingly demonstrated on a variety of maneuvering movement where the target switches between CV-CT, CT- CV and CV-CT-CV. In all the cases the FIFF follow the target very well detecting the maneuver using the Chi Square test and changing the model dynamically.

- In the second approach, the turn rates were estimated from the range rate computed out of estimates position and velocity only, (without depending on the range information which is not available in the BOT).The turning rate thus estimated was used to detect the onset of maneuver. The FIFF was successfully demonstrated to track the target continuously switching between the CV-CT, CT-CV and CV-CT-CV models. Monte Carlo runs on the simulation brought out that the variance in the estimate of the mean squared error was well within 20%, 10% and 24% respectively for CV-CT, CT- CV and CV-CT-CV for position. On the other hand for the standard deviation was well within 1% for similar instances in scenario. The simulation results of the various cases have reaffirmed the efficacy of using FIFF in tracking maneuvering targets continuously.

The thesis also has examined the application of FIF in multi target tracking, which a common problem is dealt with airline traffic, both in civilian and military scenario. Towards this, the present work has demonstrated a method for associating multiple measurements from collocated sensors, for tracking multiple targets in non-clutter environment. The excellent performance of the proposed technique confirmed through independent Monte Carlo runs was also compared with the well known JPDA algorithm and was found to be at par with the JPDAF. Thus the results suggest an alternative to JPDAF in multi target tracking, underscoring the effect of fuzzy correction in IF.

Consolidation of the work reported

The work reported in the thesis is a consolidation of the research work done to improve the performance of tracking algorithms in the context of bearing only tracking (BOT), fusing measurements/features from multiple sensors. Since it is an established fact that capitalizing the fusion of data from multiple sensors can help to improve the quality of the state estimates, the choice of the right algorithms for different applications, is of paramount significance. The basic PDA algorithm and its extension the JPDA is best suited for those cases where the sensors are co-located, as in the case of platforms with sensors in multiple bands. But often the sensors could be located at different positions, as in a battle field and the sensor suites have sufficient intelligence to extract the information from the measurements. The Information fusion filter will be the right choice, since the tracking can work even with a poor estimate of the initial position. But when the system has to track targets exhibiting a substantial maneuver, the uncertainty is more, especially when the observation is limited to only bearing. The fuzzy information filter with the on line estimation of the turning rate would be the right choice. Towards this, the thesis has proposed and convincingly demonstrated a few algorithms, effectively utilizing the fuzzy inference techniques. The work reported in the thesis has been supported by elaborate simulation on diverse scenarios, augmented by independent Monte Carlo runs to assess the performance. Five publications, as summarized in Appendix have come out of the thesis covering national and international conferences and refereed journals, with one publication appearing in an SCI indexed journal and another one in SCOPUS indexed journal

Directions for further research

The thesis has come up with some algorithms that can operate stably, while fusing the information from measurements to track phenomena. The notion of velocity and position as features extracted from the measured data can be extended to other parameters that characterize the dynamics of a situation, being monitored by a set of sensors. Fusion of data from sensors located to monitor the flow characteristics using the information gleaned out of the sensed data can help in evaluating the flow characteristics like velocity, formation of ripples and vortices and on set of turbulences

As one tries to derive further direction from the thesis, the algorithms proposed leads to one of the obvious avenues viz. the camera surveillance. Surveillance based on camera images, which are co-located could be used to enhance the quality of the image captured through fusion. Suitably designed measurement function from the images captured could be fused and the fused data could be subsequently used to recover the better quality image. The traditional problem of tracking objects through geographically located cameras suggests itself as a promising area to reap the rich dividends of the performance of the algorithms proposed in the thesis.

On the whole, the thesis has attempted to resolve some of the basic issues like divergence, while attempting to use multisensor data fusion in the context of BOT. The observability issues in BOT trigger a large number of problems that affects the convergence and continuous operation of the tracking algorithms. The thesis has effectively used the fuzzy set theory to alleviate the problem of divergence and demonstrated the continuous operation of filters tracking maneuvering and steady targets also.

While there could be many more approaches to overcome the problems discussed in the thesis, the present thesis has tried to develop and demonstrate a few effective techniques to assuage the problems of target tracking. To quote Sir Isaac Newton, “I know not how I may seem to others, but to myself, I am but a small child wandering upon the vast shores of knowledge, every now and then finding a small bright pebble from to content myself with, while the vast ocean of undiscovered truth lay before me”.

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PUBLICATIONS FROM THE THESIS

1. **Deepa Elizabeth George, A. Unnikrishnan and K. Poulose Jacob**, *"Experimental comparison of variance based fusion and information fusion in target tracking"*, International Journal of Latest Research in Science and Technology, Volume 2, Issue 1:Page No.567-572, January-February (2013).
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