STOCHASTIC DEMAND FORECAST OF NOVEL AND SHORT LIFE PRODUCTS BY USING MARKOV BASED ALGORITHM.

A thesis

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BIJESH PAUL

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Dr. Jayadas N.H. (Guide)

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KOCHI-682022

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Declaration

I hereby declare that the thesis entitled "STOCHASTIC DEMAND FORECAST OF NOVEL AND SHORT LIFE PRODUCTS BY USING MARKOV BASED ALGORITHM" submitted by me to the Cochin University of Science and Technology, Cochin in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy have not been submitted and will not be submitted to any other University or Institute for the award of any degree, diploma, associate ship, fellowship or other similar title of recognition.

Cochin,
22-09-2015

Bijesh Paul
PhD Research Scholar (Part Time)
School of Engineering
Cochin University of Science and Technology
Cochin 682022

Certificate

This is to certify that the thesis entitled " STOCHASTIC DEMAND FORECAST OF NOVEL AND SHORT LIFE PRODUCTS BY USING MARKOV BASED ALGORITHM" submitted by Bijesh Paul to the Cochin University of Science and Technology, Cochin in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy is a bonafide record of research work carried out by him under my supervision. The contents of this thesis have not been submitted and will not be submitted to any other University or Institute for the award of any degree/diploma.

Cochin,
22-09-2015

Dr.Jayadas.N.H
Associate Professor (Research Guide)
School of Engineering
Cochin University of Science and Technology
Cochin 682022

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Abstract

In case of novel products with short shelf life, sales data was either unavailable or scarcely available. The available methods for the estimation of demand of such products were direct survey methods, collection of opinion or indirect survey methods, comparison with established products and limited market trial. From literature review it was concluded that existing literature for predicting the demand of novel and short life products were scarce. This led to identification of problem namely demand forecast of relatively novel and short life products. Initially conventional methods like naive, exponential smoothing and moving average methods were used to predict the demand. Markov based model was then applied to forecast errors of the conventional methods. This model or algorithm requires only demand data of two consecutive months and hence is suited for demand forecast of novel products.

This algorithm was then applied to two novel baked products, one of relatively large quantity and another of relatively small quantity. Naive, exponential smoothing and moving average methods were applied to this data and the forecasts as well as error for all the working days of two consecutive months were estimated. Markov based algorithm was then applied for these errors and the steady state probability was determined for each state of demand. A state of a system is where the system was at a point of time. The demand corresponding to the state with maximum probability was selected and the corresponding profit was estimated. The obtained profits were then compared and the combination with maximum profit was identified and the method is validated by estimating the annual savings that this method will bring to the firm when compared to existing methods in case of products A and B.

The suitability of the model was validated by the fact that its implementation on product A and product B fetched more annual savings when compared to existing practice. Return on investment increased for product A and product B when compared to existing methods.

Thus it was concluded that a firm can further enhance its profit by implementing this model or algorithm for more number of products. Further the model can be generalized by applying it to more types of novel products with short shelf life. The forecasting of novel and short life products was not much explored in previous research works. This model can act as the benchmark for future researches in forecasting of novel and short life products.

Key words: Novel, Short Life, Markov, Forecasts, Baked products and Prediction.

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List of Abbreviations

ADI	Advance Demand Information
AR	Auto Regressive
ARIMA	Auto Regressive Interactive Moving Average Method
ARMA	Auto Regressive Moving Average
BBD	Best Before Date
BES	Basic Exponential Smoothing.
DMM	Delinquency Movement Matrix
ES	Exponential Smoothing
GNP	Gross National Product
LCL	Lower Control Limit
MA	Moving Average
MAD	Mean Absolute Deviation
MAPE	Mean Absolute Percentage Error
MCMC	Markov Chain Monte Carlo
MMFE	Martingale Model of Forecast Evolution
MSE	Mean Squared Error
QR	Quick Response
SOM	Self Organizing Map
TPM	Transition Probability Matrix
UCL	Upper Control Limit

Chapter

1 INTRODUCTION

• Contents •

1.1 Overview of forecasting
1.2 Relevance of novel and short life product forecasting
1.3 Problem statement
1.4 Research goal
1.5 Research objectives
1.6 Scope and opportunities
1.7 Methodology
1.8 Actual Time series data
1.9 Structure of the thesis

1.1 Overview of forecasting

A forecast is a measure of an event which will happen in future. The event can be demand of a product, rainfall at a particular place, population of a country and growth of technology.

Forecasting is the process of gathering data and developing assumptions about the future. Demand forecasting is the key to success of any organization as this data is critical for planning at different departments of the organization. Based on the demand forecasts the finance department can estimate the cost, profit and capital requirements for each product. Demand forecasts provide a ready reckoner for sales department about the sales volume of each product based on which it can organize the sales force. It provides a blue print for the development of production and purchase plan for production and purchase department. Marketing department uses demand forecasts for formulating different marketing strategies to boost the demand of the products. Logistics utilizes these forecasts for developing appropriate logistics infrastructure and to meet specific logistics needs.

In case of novel products with short shelf life the data of demand is unavailable or scarcely available (Chen *et al.*, 2009) and any possibility of over stocking may lead to huge financial burden. An under stocking will lead to direct sales loss and loss of customer good will. Hence it is necessary to forecast the demand of these novel products with short shelf life. Demand estimation is considered as the first step in supply chain planning and proper demand estimation can effectively control phenomenon like bullwhip effect (Kahn, 1987).

1.2 Relevance of novel and short life product forecasting

After the entry of multinational companies in India, indigenous small and medium industries are finding it hard to match these multinational companies and are engaged in a battle of survival. This is particularly true in the retailing where indigenous single stores are finding it tough to withstand the onslaught of multinational and indigenous chains. Hence it is important to equip these small and medium enterprises with appropriate technology and modern management techniques to combat these retail chains. The first step in any production planning process is to identify a forecasting model for the products to be launched. Many forecasting models are in existence for products that can be stocked but for novel and short life products little literature exits. Hence this thesis focuses on such products and develops a Markov based model for such products. Reasons of products being converted into short life are

1. Growth of income level has increased buying capacity resulting in more demand growth thus converting life cycle of most products into short periods.
2. Fashion trends can lead to low shelf life (Kincade *et al.,* 2002).

3. The variations in consumer demand are caused by factors like price, promotions, changing consumer preferences and weather changes (Munson and Rosenblatt, 1998).

Problems encountered by short life products are

1. Out of stock rate is in the range, 10-40%.
2. Chances of over stocking resulting in over stocking costs.

The available methods for the estimation of demand of such novel and short life products are

1. Direct survey methods: A survey by sampling technique to know the responses of the customers about the product.
2. Collection of opinion or indirect survey methods: Prediction through the opinion of distributors, salesmen, selling agents and retailers.
3. Comparison with established products: Sales figures can be compared, if the product under consideration is comparable to existing products.
4. Limited market trial: Opinion of potential customers is collected to predict the demand.

The above methods do not give a scientific basis for the determination of future demand nor does it provide any financial basis. Hence an algorithm was developed based on Markov chain to predict the demand of a novel product with short shelf life. The algorithm requires only two consecutive period data to predict the demand for the subsequent period. The trend associated with demand data are analyzed by using a conventional method namely naive forecasting or basic exponential smoothing or moving average method and the errors of demand of such methods is modeled by Markov based method. Cycleness and seasonality are not considered as the data taken is only for two months.

1.3 Problem statement

To develop an algorithm by using Markov chain to predict the total sales volume as well as total revenue earned and total profits of novel and short life products for which sales data is unavailable or scarcely available. To estimate the financial savings by adopting the forecasts prescribed by the above algorithms.

1.4 Research objectives

1. To develop a model/algorithm for predicting the future demand of novel and short life products using Markov method in combination with naive, Exponential Smoothing (ES) and Moving Average (MA) methods.
2. To validate this model with real time case studies.
3. To estimate the annual savings that this method will bring to the firm when compared to existing practices.

1.5 Research goal

The goal of this work is to forecast the demand of a novel product with short shelf life and to equip the supply chain of small scale retailers to face the challenges posed by indigenous and multinational retail chains. The improvement in demand forecast provides potential cost reductions and assists a decision manager to determine the best demand planning for a novel product.

1.6 Scope and opportunities

Accurate measures of demand uncertainty and its financial implications are very important in many applications. The forecasting model developed in this research can be used for any novel product with minimum historical data of demand and with short life cycle. The models are specifically applicable to forecast the sales of baked products as well as newspapers with single day shelf life.

1.7 Methodology

1. Identification of the research problem based on literature review.
2. Formulation of the model or algorithm.
3. Collection of actual demand data of two novel baked products, one of relatively large quantity and other of relatively small quantity.
4. Application of the developed model or algorithm to the collected data of two novel baked products.
5. Usage of the research findings from the case studies stated above to validate the developed model.

1.8 Actual time series data

The actual data is collected from two baking firms of two novel products with short life, one of relatively large baked quantity and another of relatively small baked quantity. The developed algorithm is applied to this collected data.

1.9 Structure of the thesis

The thesis is organized in the following manner: Chapter one presents motivation for the research, the statement of the research problem, research objectives, procedure involved and scope as well as opportunities. The second chapter is devoted to review of literature. The third chapter deals with model or algorithm formulation for the research problem. In fourth chapter, the model developed is applied to baked product A of relatively large quantity. Fifth chapter discusses the application of model to a second novel baked product B of relatively small quantity. Sixth chapter of the thesis presents the results, research findings, limitations of the study, the summary and the scope for further research. The references and publications are listed at the end.

Chapter

2 LITERATURE SURVEY

Contents

2.1 Introduction
2.2 Demand forecasting techniques
2.3 Literature of demand forecasting
2.4 Demand forecasting of short life products
2.5 Markov chains in forecasting
2.6 Conclusion

2.1 Introduction

Initially the definition, objectives and advantages of forecasting are discussed. Various demand forecasting techniques such as objective and subjective methods are then explained. The existing literature for demand forecasting including novel products are listed thereafter. Demand forecasting of short life products is then reviewed. The literature on Markov chains is also discussed. The findings of the review are then furnished at the end.

The success of any firm depends on the accuracy of prediction of future demand of its products. The process of determining future demand in advance is called as demand forecasting. Thus demand forecasting is the process of measuring the status of demand before it occurs. It is the process of assigning a value to future demand.

The objective of operations management is to arrive at the minimum difference between supply and demand. For this purpose, a forecast of demand is vital to determine the amount of supply needed to meet demand. Two aspects of demand should be taken into consideration: One is the required level of demand and the other is the magnitude of forecast error (forecast accuracy). The required level of demand depends on trend or seasonality whereas forecast accuracy depends on the ability of forecasters to correctly model demand, random variations, and unforeseen events. Forecasting is done for a specific time period. Short term forecasts are done for an hour, day, week, month and year. Long term forecasts pertain to new products or services, new equipment, new facilities or something else that will require a somewhat long lead time to develop, construct or otherwise implement. Forecasts act as the foundation for budgeting, planning capacity, sales, production and inventory, personnel, purchasing and more. Forecasts play a vital role in the planning process because they allow managers to predict the future so that they can plan accordingly. They play a key role in decision making and various activities of an organization.

2.2 Demand forecasting techniques

The economic literature by Chendroyaperumal (2009) provides a number of techniques for demand forecasting which is broadly classified as objective methods using statistical or mathematical approach and subjective methods using intuition based on experience, intelligence and judgment. Some of the popular objective methods used are: (1) extrapolation methods, (2) regression models, (3) leading indicator methods, (4) econometric methods and (5) end-user or input-output models etc. Some of the popular subjective methods are: (1) consumer surveys, (2) expert opinion surveys and (3) test market methods etc.

2.2.1 Objective methods of demand forecasting

According to Chendroyaperumal (2009) the extrapolation methods extend the behaviour of the variable, from time series data into the future. The trend fitting methods notices the direction (increase, decrease or the trend, cycles, seasonality and randomness) of the movement or behavior of the demand and assume that the behaviour will be exhibited in future also. This is done by assuming time as the cause of demand. Ultimately the trend fitting method gives the value of demand as a refined average of the past values. The family of the smoothing methods (simple smoothing method and exponential smoothing method) forecast or measure the demand based on averaging the past values and refining the averages in various degrees and stages so that the deviation of the actual values of the past from its mean is minimum. The basis of this method is logically to extend the average value into the future point of time.

The regression method determines the degree of influence (coefficients) of cause or causes (determinants) of demand, significance and reliability from the past values of the assumed causes or causes advised by the economic theories using a statistical method based on the least squares of the deviation of the observed value from its mean i.e. the least squares method. The degree of influence of these causes is in the form of an equation and it calculates the value of the demand variable. The essence of this method is measuring the future value of demand based on its causes, assuming the absence of problems such as autocorrelation, multicollinearity and the like errors.

The barometric methods such as leading indicator and lagging indicator or coincidental indicator use another variable other than the variable that is being forecasted. But those variables are indicative of the behaviour or direction of the movement of the demand. These indicator variables may be or may not be the factors influencing the demand. The essence of this method is that the behaviour of demand is not directly observed from the variable that is being forecast. Instead the behaviour of demand is observed indirectly through another variable (proxy variable) which is indicative of the behaviour of demand. There are three types of indicator variables. (i) The leading indicators are those variables which always lead the demand, i.e. these variables precede the demand variable in occurrence along the time scale. (ii) The lagging indicators are those variables that always lag behind the demand variable, i.e. the demand variable precedes these variables in occurrence along the time scale. (iii) The coincidental indicators are those variables that occur along with the demand variable at the same time along the time scale. The essence of this method is that it assumes that by studying the behaviour of an indicator variable (cause variable or non-cause variable), it studies the behaviour of the demand. Therefore it logically follows that forecasting the future value of the indicator variable is equivalent to forecasting the demand variable under study.

The econometric methods of demand forecasting are described as the estimation of the demand equation based on the degree of influence of the causes (determinants) on demand which, in turn, is measured by the degree of influence of the causes of the determinants of demand. The relationships across the demand and their determinants are established as an equation estimated from their past behaviour using the least squares method. The end user method is also used to forecast demand based on the end use to which the products are put to. Based on the quantum of end use the demand for the product is extrapolated.

2.2.2 Subjective methods of demand forecasting

Basically the subjective methods are designed to know the intentions or presence or absence of the willingness of the buyers given the determinants of demand like the price of the product, quality, quantity etc through intelligent questions, interviews or questionnaires (direct or mailed). The various subjective methods listed by Chendroyaperumal (2009) are furnished below.

Complete enumeration or census method questions (either face to face or through a mailed questionnaire) all the consumers, without excluding any one, to know their intentions of buying. Sample survey method questions, to know the intentions of only a selected representative sample of the whole population of consumers and logically extends the inferred findings to the whole population. These methods simply sum up what the consumers say or opine and assume that what they say is true and correct and that they will do the same. The basis of this method is that demand is based purely on consumers opinion about the quantity, quality etc and that demand are not based on the demand determinants or its end use.

Expert opinion survey method bases its subjective forecast on the opinion of the experts instead of the consumers. That is the demand of a product is set equal to the quantity pronounced by these experts. If the forecast quantity is reasoned through an interactive procedure justified by the experts, then this method is called as the reasoned expert survey or Delphi Technique. If the forecast is not reasoned out then it is known as the simple expert opinion poll.

The test market method is based on learning or knowing the demand for a product by actually selling or experimenting with the sales of the product. A product is simply introduced into a test market and its demand is observed and logically extending the behaviour of demand to markets similar to the test market.

2.3 Literature of demand forecasting

Cachon and Lariviere (2001) analyzed forecast sharing in a single product with two level supply chain. Aviv (2001) studied three settings under a two level supply chain. Zhao *et al.* (2002) introduced the impact of different forecasting models on the value of information sharing in a supply chain. The prediction of future demand forms the foundation for all strategic and planning decisions in a supply chain. Pourahmadi (2001) provides many techniques for time series prediction. Johnston and Boylan (1996) propose estimating the average interval between orders and the average size of an order when it occurs and to combine these statistics to give an unbiased estimate of the variability of the demand.

2.3.1 Demand forecasting based on probabilistic models

Sani and Kingsman (1997) picture the forecasting methods for items with low or intermittent demand. Other studies on forecasting intermittent demand include papers by Ghobbar and Friend (2003), Willemain *et al.* (2004), Kalchschmidt *et al*. (2006), Altay *et al*. (2008), Wallstrom and Segerstedt (2010) and Rahman and Sarker (2012). Heath and Jackson (1994) suggest a general probabilistic model for modeling the evolution of demand forecasts, referred to as the Martingale Model of Forecast Evolution (MMFE). Gullu (1996), using MMFE, studies the effects of the forecasts on the production/ inventory system comparing the optimal ordering policy and the expected costs of the model that keeps forecasts with that of a comparable standard inventory model. Toktay and Wein (2001) provides dynamically updated forecasts to determine production order releases for a capacitated system and propose an order release rule maintaining a forecast corrected base stock level. They attained approximate closed form expression for the optimal forecast corrected base stock level. Ozer and Wei (2004) also utilized forecast updating models to obtain value of Advance Demand Information (ADI) and approximate the optimal base stock levels.

2.3.2 Demand forecasts based on conventional methods

Woo and Nathaniel (2010) investigates different ways to perform data denoising in order to forecast demand in the retail setting inspired by and illustrating his point with the BYU Bookstore data. Gujarati and Damodar (2003) and Makridakis *et al.* (1998) explain the econometric skills of regression and statistical analysis. A linear demand model is used where the dependent variable is the sales of products and the independent variables are the price of the product and the prices of the other products related to that product.

Iyer and Bergen (1997) explore the effects of Quick Response (QR) on supply chain participants. They noticed that cutting lead time is always beneficial to the retailer while harmful to the vendor when the service level is more than 0.5.Wu (2005) stretched Iyer and Bergen's forecast updates mode from a single stage to multiple stages and proposes a flexible contract to coordinate the supply chain. Choi (2006) extended Iyer and Bergen's work to a scenario that new information can be used to update both the unknown mean and unknown variance for the short life cycle product's demand. Choi (2006) concludes that the QR policy is always beneficial to the retailer and not necessarily beneficial to the supplier when the service level is more than 0.5. Choi *et al.* (2006) takes dual updates and costs uncertainty and differences into account in his work, but there is no explicit relationship between the forecast error and lead time. Fisher and Raman (1996), Gurnani and Tang (1999), Donohue (2000), Sethi *et al.* (2005), Choi and Li (2003), Choi (2007), Chen *et al.* (2006) and Chen *et al.* (2010) analyzes the optimal ordering policy with two ordering opportunities. In these works, the retailer decides on the two sequent ordering quantities by evaluating the tradeoff between a more accurate forecast and a potentially higher unit cost at the second stage.

Steven Nahmias (1989) and William J. Stevenson (1989) compared naive forecasts, moving average, single exponential smoothing, double exponential smoothing (Holt's method) and linear regression using Mean Absolute Deviation (MAD), Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE), utilizing forecasted data and actual data from 1986 to 1991.Triple exponential smoothing is utilized to predict a seasonal time series with a trend by Steven Nahmias (1989) and William J. Stevenson (1989). John Neter *et al.* (1990) and Wayne L. Winston (1987) uses linear regression, which fits a curve to a set of data to forecast trend based time series data. Steven Nahmias (1989) and William J. Stevenson (1989) use the last period's forecast and the present demand to derive the next period's forecast in double exponential smoothing. Ultimately this model incorporates all prior data into a given forecast value.

Single exponential smoothing utilizes the last periods forecast and the present demand to derive the next period's forecast and is appropriate for a stable time series. A naive forecast for any period equals the previous period's actual value, according to William J. Stevenson (1989).

2.3.3 Demand forecasts of newspapers and electronic media

Zhang and Buongiorno (1997) explored the effect of electronic communication media on the demand of printing and publishing papers in the US. Hetemaki and Obersteiner (2001) as well as Bolkesjo *et al.* (2003) analyzed the classical paper demand model and its Bayesian variation. Hetemaki and Obersteiner (2001) studies forecasts concerning the US newsprint demand until the year 2020 and Bolkesjo *et al.* (2003) augmented the method for a panel data concerning Western Europe and Japan. Paper demand is the dependent variable; gross domestic product and the real price of paper are used as regressors. In the Bayesian variation of it, both Hetemaki and Obersteiner (2001) and Bolkesjo *et al.* (2003) added industry experts extra knowledge, for example, about the substitution of newspapers by electronic media in the estimation to produce forecasts. According to Hetemaki and Obersteiner (2001), the classical paper demand model could not explain or forecast the newsprint demand in the US. Their future forecasts of the Bayesian models also indicated only a slight decline in newsprint consumption compared to year 2000. Results of the Bolkesjo *et al*. (2003) indicated that the Bayesian approach may have an advantage in comparison with the classical model of paper demand.

Witt and Song (2000) and Li *et al.* (2005) deduced that the performance of the forecasting models varies according to the data frequencies used in the model estimation, destination, origin, country/region pairs under consideration of the demand and the length of horizons concerned.

2.3.4 Demand forecasts based on rental point of view.

Pasternack and Drezner (1999) concentrate on the purchasing problem from the rental point of view. Based on the demand pattern, they divide the lifetime of a movie into three phases (the first 30 days, the next t periods, and the remainder of time). Tang and Deo (2008) explore the impact of rental duration on the stocking level, rental price, and retailer's profit. There has been some recent related work on joint estimation and optimization of models like Liyanage and Shanthikumar (2005), Besbes *et al.* (2009), and Cooper *et al.* (2006). Liyanage and Shanthikumar (2005) deal with news vendor with a single unknown demand parameter. Besbes *et al.* (2009) takes into consideration a statistical test that incorporates decision performance into a measure of statistical validity in the context of fitting a demand curve.

2.3.5 Demand forecasts based on autoregressive models

Kahn (1987) and Lee *et al.* (1997) were among the first to exhibit the existence of variance amplification upstream in the chain (bullwhip) when the retailer follows a base stock policy and demand is positively correlated. Dejonckheere *et al.* (2003) extended this result by showing that an ES or MA forecast produces bullwhip for all demand processes including Auto Regressive (AR) and Auto Regressive Moving Average (ARMA). Zhang (2004) analyzes the role of forecasting for AR demands and concludes that the MSE forecasting method minimizes the variance of the forecasting error among all linear forecasting methods and therefore leads to the lowest inventory costs. Alwan *et al.* (2003) applied this optimal MSE forecasting scheme and determined the underlying time series model of the resulting ordering process. They demonstrate that when consumer demand is negatively correlated (AR demand), the variability in order quantities is dampened with respect to the observed demand, as opposed to the ES and MA forecasting methods, which always create bullwhip independent of the demand process.

Kurawarwala and Matsuo (1998) determined the seasonal variation of products demand using demand history of pre season products and validated the models by checking the forecast performance with respect to actual demand. Hyndman (2004) applied various relationships between trend and seasonality under seasonal Auto Regressive Integrated Moving Average (ARIMA) procedure. De Alba (1993) deduced an auto regressive model under Bayesian approach to forecast the quarterly Gross National Product (GNP) of Mexico and the quarterly unemployment rate for the United States. Huerta and West (1999) analysed AR models where Markov Chain Monte Carlo (MCMC) process is used to forecast from AR processes. McCoy and Stephens (2004) extended Huerta and West's work (1999) and suggested ARMA models in which a frequency domain approach is adopted to identify the periodic behaviour of time series.

2.4 Demand forecasting of short life products

2.4.1 Demand forecasts based on algebraic estimation and fitting procedure

Mahajan and Sharma (1986) suggest an algebraic estimation procedure that utilizes the estimates of some key information about the life cycle of a product, such as the timing of peak sales and the market potential. Fitting procedures on the other hand establish the parameters that best fit the data available at each time period. Various fitting procedures have been proposed by Bass (1969), including the ordinary least squares estimation. However the ordinary least squares procedure linearizes the original growth model and results in inferior parameter estimates as suggested by Mahajan *et al.* (1990) and the maximum likelihood estimation procedure underestimate the standard errors of the parameters.

2.4.2 Demand forecasts based growth curve models

Islam *et al.* (2002) propose the diffusion of an innovation in several countries, having different starting times but similar dynamics of the diffusion process and linearized growth curve models pool the data from several countries. In this study leading indicator products are the sources of advanced demand information, but they are used in a different manner. The available data set of a leading indicator defines the expected nominal growth curve for the product group forecasted. Parametric deviation such as a shift in the volume and skewness of the nominal curve produce alternative demand scenarios. The probability of each scenario is updated as data when the leading indicators become available. Another stream of the related research is the quantification of uncertainty in growth curve forecasting, which is not a well studied subject. Chatfield (1993) observes the difficulty in modelling the uncertainty in the estimates with linear models and describes several underlying reasons such as incorrect model identification, uncertainty in parameter estimates and changes in the data generating process. The intrinsic nonlinearity of growth models contributes to this difficulty and makes it a less appealing area to study. Heeler and Hustad (1980) and Islam *et al.* (2002) are among the studies that empirically show that as the number of actual data points used for estimation increases, estimates of the parameters stabilize (i.e., their estimation variance becomes close to zero), and the accuracy of the forecasts increases. An attempt in the general forecasting literature to reduce forecast variance is to combine several forecasts. There are several empirical studies showing that, in combination, forecasts with smaller variances are obtained as stated by Dickinson (1973).

Fisher and Raman (1996), Eppen and Iyer (1997) and Kim (2003), identifies that the sales records in the pre season test periods and the actual sales data in the early season are demand indicators for the actual sales in the selling season. Hause (1971) uses spectral methods to determine the phase shift relating these data series.

2.4.3 Demand forecasts of perishable products

Existing forecasting tools based on time series tools need large data and are not suitable for short life-span products. Soloman *et al.* (2000) and Henke & Lai (1997) have developed the models for this problem. However, the complexity of the problem has increased due to the fact that the short life span products do not have adequate data for the construction of the suitable model for forecasting. Wee (1993) stated perishability as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Although managing perishability presents some of the most important challenges in supply chain management as indicated by Shulman (2001), the literature had overlooked this important issue for a long time. In fact the major body of literature on perishability deals with inventory management. Some current trends seem to point towards an increasing interest in the management of supply chains of perishable goods. From a consumer point of view, nowadays buyers are worried about having as much information as possible, especially with products that may have an impact on health, such as food products which are highly perishable. These expectations results in demand increasing traceability and higher production standards for perishable products. To add such trends, a critical regard to the current practices and plans of these supply chain needs to be taken.

Reviews about perishability were done in the field of inventory management by Nahmias (1982), Raafat (1991), Goyal and Giri (2001) and Karaesmen *et al*. (2009). There is also the review performed by Pahl *et al*. (2007) in production planning and inventory. Ahumada and Villalobos (2009) focus on models for the agri food business where products may be perishable or not, but their focus is on procurement and harvesting planning and the only goods they are interested in are crops. Finally, Akkerman *et al*. (2010) reviews the literature in the field of food distribution where different characteristics are identified as key issues such as quality, safety and sustainability. In Ghare and Schrader (1963), the authors classify the deteriorating properties of inventory with three categories.

1. Direct spoilage, e.g., vegetables, flowers and fresh food, etc.
2. Physical depletion, e.g., gasoline and alcohol, etc.
3. Decay and obsolescence, for instance in radioactive products and with the loss of value in inventory, e.g., newspapers and uranium.

Nahmias (1982) differentiate two classifications of perishability.

1. Fixed life time: Items life time is specified before hand and therefore the impact of the deteriorating factors is taken into account when fixing it. In fact the utility of these items degrades during their lifetime until the goods perishes completely and has no value to the customer, e.g., milk, yoghurt and blood in inventory, etc.
2. Random lifetime: There is no specified life time for these items. Hence the life time of these goods can be modelled as a random variable according to a given probability distribution. Examples of products in this category are fruits, vegetables and flowers.

In another work, Raafat (1991) first defines decay or deterioration as any process that prevents an item from being used for its intended original use and names the examples of spoilage (e.g., food stuff), physical depletion (e.g., evaporation of volatile liquids) and decay (e.g., radioactive substances). These examples are strictly related to the ones mentioned by Ghare and Schrader (1963). Afterwards, Raafat (1991) gives a categorization of perishability depending on the relation between time and value of the inventory.

1. Utility constant: Its utility remains the same as times goes by until the end of the usage period, e.g., liquid medicine.
2. Utility increasing: Its utility increases as time passes, e.g., some cheeses or wines.

3. Utility decreasing: Its utility decreases as time passes, e.g., fruits, vegetables and other fresh foods, etc.

More recently, Lin *et al.* (2006) say that deterioration can be classified as:

1. Age dependent on going deterioration.
2. Age independent on going deterioration with the assumption that the aging process starts just after production.

Meat, vegetables and fruits are examples of goods subject to age dependent on going deterioration. Volatile liquids such as gasoline and alcohol, radioactive materials, and agri food products are examples of age independent ongoing deteriorating items. In these items it is hard to define a dependency between age and perishability since these products can be stored indefinitely though they suffer natural attrition while being held in inventory degrading its condition. Finally, Ferguson and Koenigsberg (2007) highlight the related utility loss and distinguish two kinds of products.

1. With functionality deteriorating over time, e.g., fruits, vegetables and milk.
2. Without functionality deteriorating, but customers perceived utility deteriorates overtime, e.g., fashion clothes, high technology products with a short life cycle and newspapers.

There is another concept very related with perishability and deterioration which is shelf life. Shelf life is stated as the period of time after the manufacture of a product during which it is of satisfactory quality by Kilcast and Subramaniam (2000). It is the length of time that a given item can remain in a saleable condition on a retailer's shelf. Shelf life does not necessarily reflect the physical state of a product, since many products deteriorate only a while after their shelf life finishes; however, it may reflect its marketable life as suggested by Xu and Sarker (2003). The enumeration of perishability's classifications shows that these categories overlap among each other and furthermore, they are highly tailored for a specific proposes. Either they are specially concerned about the customer and then reflecting the utility of the product as given in Raafat (1991) or concerned about the physical state of the product itself as specified by Ghare and Schrader (1963) or for example, looking more into the mathematical modeling point of view of perishability as stated by Nahmias (1982). In fact, these classifications were neither always used to classify the papers dealing with perishability nor to define the applicability of the proposed models. Different classifications based on the mathematical properties of the modeling approaches were used afterwards by Raafat (1991), thus neglecting the previous classifications concerned with perishability and loosing, consequently the linkage to the underlying perishability phenomenon expressed by the model. A perishable good that illustrates the difficulty of having a clear and univocal classification, with the existing classifications, is the yoghurt. Yoghurt has a fixed shelf life expressed by its Best Before Date (BBD), when it perishes (after the BBD), its value is close to zero and finally from the customer's point of view it has been proven empirically by Tsiros and Heilman (2005) that the willingness to pay for it decreases over the shelf life of the product. Hence although its utility decreases, it has a fixed life time and its functionality does not deteriorate over time. The work of Chen *et al.* (2009) acknowledges that papers discussing other areas of perishable supply chains are rather scarce. Nevertheless, it is widely accepted that perishability in real world may enforce special constraints and different objectives throughout different problems in supply chain planning models. In the sales functions, the importance of considering perishability can be found in determining where to locate the decoupling point that may not only depend on lead times and variability, but also on the shelf life of the products as given in Van Donk (2001). Further pricing models for deteriorating products is also a very active field of research as suggested by Abad (2003).

2.4.4 Demand forecasts of novel products and short life products

The sourcing problem for a new product facing stochastic demand has been investigated from different viewpoints. Earlier research work assumed that the procurement decision had to be made before the realization of demand. An example of this research could be the classical newsboy problem where the entire demand for a style product occurs in a single period. Bitran *et al.* (1986) and Matsuo (1990) proposed enhancements to this problem and computed production sequence and production volume of the style products over the multi period horizon in order to meet entire demand that occurred in the final period.

Cantamessa and Valentini (2000) devised a deterministic mathematical model to decide production plans for new products and investigated the benefits associated with implementing reactive backorder and lost sales strategies. However, the demand for the new product is highly uncertain and unpredictable at its launch; but, it becomes more predictable after analyzing an early demand pattern as suggested by Raman (1999). QR research stream used this more refined demand information and Fisher and Raman (1996), Bradford and Sugrue (1990), Fisher *et al*. (2004) and Choi (2007) suggested some sophisticated sourcing options.

This prior work that decides ordering quantities for such short life cycle products assume that the unit product cost and unit transportation does not change with the ordered or transported quantity. However, it is a common knowledge that a customer can receive a price discount after placing large orders and vendors offer discounts to get economies of scale in purchasing, manufacturing and transportation as given in Munson and Rosenblatt (1998). Particularly, in the fashion industry, the vendors are increasingly using monetary support such as quantity discounts to attract retailers under intense competition as stated by Kincade *et al*. (2002). In shipping business, fixed costs such as custom fees, container charges etc are incurred for a shipment. This can be considered to be quantity discounts because average shipping cost per unit decreases with increase in the shipped quantity as suggested by Popken (1994). In the road and rail transportation, transporters often provide discounts on full truck load and full wagon load as stated by Munson and Rosenblatt (1998). Though the traditional practice is to receive all merchandise before the season, nowadays, retailers are using different delivery windows in the season. For example, suit retailers use two to three delivery windows in a season as stated in Daily News Record (1993). Because the retail shelf space is increasingly becoming costly, Sen (2008) suggested that it is necessary to decide on an optimal delivery schedule for the procured products. Also, given that the demand for new products is stochastic in nature, instead of offering a pre defined markdown prices that may make customers strategic buyers, the markdowns offered at the end of the season needs to be rational determined by the unsold quantity at the end of the season, as proposed in Sen (2008). As a result, it should depend upon the number of unsold units at the end of the product lifecycle.

2.5 Markov chains in forecasting

The Markov chain forecasting model can be used to forecast a system with randomly varying time series. It is a dynamic system which forecasts the development of the system according to transition probabilities between states which reflect the influence of all random factors. The Markov chain forecasting model is applicable to problems with random variation, Min Huang *et al.* (2007) and Dong Li *et al.* (2007).

2.5.1 Markov models for electricity demand forecasting

Zhang and He (2001) have developed a grey Markov forecasting model for forecasting the total power requirement of agricultural machinery in Shangxi Province. Akay and Atak (2007) have formulated a grey prediction model with rolling mechanism for electricity demand forecasting of Turkey. A grey Markov forecasting model has been developed by Huang *et al.* (2007).

2.5.2 Multivariate Markov models

The following multivariate Markov chain model has been proposed in Fung *et al.* (2003). The model assumes that there are s categorical sequences and each has m possible states in, M = {1, 2 … m}. In Wada and Ryosuke (2005) the author uses a collection of Markov processes to model a brokered foreign exchange auction. The processes adapt depending on the arrival of buyers and sellers and the bidding transactions that occur. In Wang and Shi (2000) the author models online retail per transaction using a hidden Markov model.

2.5.3 Markov models for stochastic processes

The study of how a random variable changes over time includes stochastic processes. Winston (2004) defines a continuous time stochastic process as a stochastic process in which the state of the system can be viewed at any time. For example, the number of people in a supermarket some times after the store opens for business may be viewed as a continuous-time stochastic process. Chisman (1992) and Berg *et al.* (2003) describes about Markov chains, which are a special type of stochastic process and have been applied in many areas such as education, marketing, health services, finance, accounting, production and reliability analysis.

Ben Noble and James W. Daniel (1988) states Markov chain model is utilized because it is a model that describes, in probabilistic terms, the dynamic behavior of certain types of systems over time. This model described by Robert E. D. Woolsey and Huntington S. Swanson (1975) utilizes a state matrix which describes the state of the horse market in terms of percent controlled by each horse category and a transitional matrix which describes by percentage the category of horse purchase by each type of owner. The main assumption in a Markov chain model is that knowledge of the current state occupied by the process is sufficient to describe the future probabilistic behaviour of the process. Hans G. Daellenbach and John A. George (1978) present another unique property of this Markov chain model namely the existence of a steady state matrix.

Although evaluations of probabilistic forecasts are common in applied statistics fields, forecast evaluation exercises published by central banks and other policymaking institutions restrict attention to point forecasting accuracy. Some recent papers considering probabilistic forecast evaluation include Garratt *et al.* (2003), Adolfson *et.al.* (2007), Lahiri and Wang (2007), Garratt *et al.* (2009), Kryshko *et al.* (2010), Berge and Jorda (2011), Clark (2011), Diks *et al.* (2011), Galbraith and Van Norden (2011, 2012), Gneiting and Ranjan (2011) and Mitchell and Wallis (2011). These papers typically utilize the forecast density relative to the outturn, or gauge performance in terms of predicting discrete events, such as a recession. In meteorology and other applied statistics fields, it is common to link forecast evaluation explicitly to the relevant economic decision. Berrocal *et al.* (2010) provide a recent example for a road maintenance problem. Granger and Pesaran (2000) propose applications in economics.

2.5.4 Markov based macroeconomic models

Most policymaking macroeconomic models are (approximately) linear Gaussian with features that are difficult to reconcile with the theory and data; see, for example, the discussion by Robertson *et al*. (2005). A long tradition in macro econometrics has emphasized the importance of nonlinearities in macroeconomics. Morley (2009) provides a review of the literature; and recent examples include Segers and Van Dijk (2009), Hamilton (2011), Arora *et al.* (2013), De Livera *et al.* (2011) and Koop *et al.* (2011). Methods for handling fat and asymmetric tails are common in financial econometrics; see, for example, Patton (2006). Copula models are widely exploited in other applied statistics fields as flexible tools to allow for non-linear dependence and non Gaussian error distributions. Examples include Clayton (1978), Li (2000), Lambert and Vandenhende (2002) and Danaher and Smith (2011). A number of recent papers in macroeconomics have proposed using mixtures or forecast density combinations to enhance performance by approximating non-linear and non Gaussian processes. Key contributions with forecast density combinations include (among others) Geweke and Amisano (2011), Jore *et al*. (2010), Gneiting and Thorarinsdottir (2010), Waggoner and Zha (2010), Billio *et al.* (2011), Bjornland (2011) and Garratt *et al.* (2011). These papers build on earlier macroeconomic research on forecast combinations by, for example, Hendry and Clements (2004), Wallis (2005), Mitchell and Hall (2005) and Kapetanios *et al.* (2008). Timmermann (2006) provides a review of forecast combination and Clements and Harvey (2011) discuss combining probabilistic forecasts. Aastveit *et al.* (2011) consider intra month probability forecasts, generalizing the more traditional point forecasting approach of, for example, Giannone *et al.* (2008), Lombadi and Maier (2011) and Kuzin *et al.* (2011). Faust and Wright (2011) and Kozicki and Tinsley (2012) discuss the scope for survey evidence to improve timely forecasting. Giordani *et al.* (2007) and Maheu and Gordon (2008) provide examples based on mixtures.

2.5.5 Discrete time Markov chain models

Cyert *et al.* (1962) proposed a discrete time Markov chain model for estimating loss on accounts receivable. The intuition and appeal behind a Markov chain model for accounts receivable is that an account moves through different delinquency states each month. For example, an account in the current state this month will be in the current state next month if a payment is made by the due date and will be in the 30 days past due state if no payment is received. Another valuable feature is that the Markov chain model maintains the progression and timing of events in the path from current to loss. For example, an account in the current state doesn't suddenly become a loss. Instead, an account must progress monthly from the current state to the 30 days past due state to the 90 days past due state and so on until foreclosure activities are completed and the collateral assets are sold to pay the outstanding debt. The transition matrix in the Markov chain represents the month by month movement of loans between delinquency classifications or states. Barkman (1977) observes the transition matrix is often of interest as an accounting summary that evaluates loan quality or loan collection practice. The matrix elements are commonly referred to as roll rates since they denote the probability that an account will move from one state to another in one month. The transition matrix is sometimes referred to as the roll rate matrix or the Delinquency Movement Matrix (DMM). Another application of the Markov chain model in credit risk is introduced in Jarrow *et al.* (1997). Institutional investors can use a continuous time Markov chain model that incorporates credit ratings to assess the risk of structured finance securities. The states of the Markov chain are the bond rating and the transition matrix reflects the likelihood of maintaining a rating or migrating to another rating level. The transition matrix is called the migration matrix in Gupton *et al.* (1997) for Credit Metrics. The estimation of the continuous time Markov chain transition probabilities is introduced in Fleming (1978) and more recently in Monteiro *et al.* (2006). While the issue of correlation between issuers discussed in that research may be applicable to a portfolio of mortgages, a mortgage is a simple accounts receivable discrete-time Markov chain model with no arbitrage or hedging opportunities that require more complicated model features. The statistical problem of interest is to estimate the transition matrix using a sample of observed monthly loan movements between delinquency states. The estimation is complicated by the frequent observation in studies that the Markov chain is neither homogeneous nor stationary. Betancourt (1999) concluded repayment for Freddie Mac data on prime mortgages was neither homogeneous nor stationary. Cyert and Thompson (1968) propose segments based on a credit score. However, there may not be enough observed transitions within each segment to provide accurate estimates of all transition probabilities. Second, it is reasonable to incorporate covariates that may change a few of the transition probabilities for a given loan as it progresses through the repayment period.

2.5.6 Markov based miscellaneous models

Berman and LeBlanc (1984) uses a scenario approach with Markovian transitions in modelling probabilistic links to study the optimal location relocation of a single and multiple mobile servers respectively, Song and Zipkin (1992) considers a multi echelon system with a Markov modulated Poisson demand process. They assume that each stage controls its inventory with a base stock policy that is independent of the state of the demand process. They develop an exact procedure to characterize the steady-state performance of the inventory system. In their second paper, Song and Zipkin (1996) assume the same demand process for a two echelon system consisting of a warehouse serving multiple retail sites. The retail sites again operate with an independent base stock policy, while the warehouse employs a state dependent base-stock policy. They again develop a procedure to characterize the steady state performance of the inventory system. Chen and Song (1997) consider a multi-stage serial system for which the demand distribution in each period depends upon the state of a Markov chain. They assume linear holding and backorder costs, and show that a state dependent echelon base stock policy is optimal for each stage. Categorical data sequences can be modelled by using Markov chains see for instance, Raftery and Tavare (1994) and Fung *et al.* (2002). In many occasions, one has to consider multiple Markov chains (categorical sequences) together at the same time, i.e., to study the chains in a holistic manner rather than individually. The reason is that the chains (data sequences) can be correlated and therefore the information of other chains can contribute to explain the captured chain (data sequence). Thus by exploring these relationships, one can develop better models. The conventional Markov chain model for s categorical data sequences of m states has ms states. The number of parameters, (transition probabilities) increases exponentially with respect to the number of categorical sequences. Because of this large number of parameters, people seldom use such kind of Markov chain models directly. In view of this, Ching *et al.* (2003) proposed a first order multivariate Markov chain model in for modelling the sales demands of multiple products in a soft drink company. Their model involves only O $(s^2m^2 + s^2)$ number of parameters where s is the number of sequences and m is the number of possible states. The model can capture both the intra and inter transition probabilities among the sequences. They also developed efficient estimation methods for solving the model parameters and applied the model to a problem of sales demand data sequences. A simplified multivariate Markov model has also been proposed in Zhang *et al.* (2007) where the number of parameters is only O ($sm^2 + s^2$). A Markov model represents a stochastic process, usually with discrete states and continuous time in which the system is modelled on observable parameters. The proposal then is that this model can be used to perform analysis of the evolution of states over time. There is a principle that earlier states are irrelevant for predicting the following states, since the current state is known, Billinton and Allan (1996). Self Organizing Map (SOM) is a good tool, Sperandio and Coelho (2006). In this case, each neuron in the SOM represents a state of the Markov model, allowing for the transition probability of states (neurons) each time.

2.6 Conclusion

An extensive literature survey was conducted on demand forecasting and it pictures the progress in demand forecasting as well as the approaches in predicting future demand. After this review it was evident that even though the existing literatures for demand forecasting were plenty, for novel and short life products they were found to be negligible. It was noted that literature existing for modelling the random component of demand is scarce. The literature review identified the capacity of Markov chain to model random variable. Thus this review laid the foundation for filling the void in the demand forecast of novel and short life products.

Chapter

3 METHODOLOGY

Contents

3.1 Introduction
3.2 Conventional methods of forecasting
3.3 Markov chain
3.4 Research methodology
3.5 Model or algorithm
3.6 Conclusion

3.1 Introduction

This chapter presents the generalized model or algorithm for predicting the future demand of novel and short life products. Initially the conventional methods of forecasting are discussed. A detailed description of Markov chains is provided thereafter. Research methodology is presented in the next section. Generalized algorithm or model for predicting the future demand, developed by using Markov chain is described thereafter. The demand predicted by this algorithm will act as a benchmark for future production and will lead to huge annual savings for each product. The chapter concludes with description of the developed model in a nutshell.

3.2 Conventional methods of forecasting

3.2.1 Naive forecasting

Naive is the simplest statistical method for forecasting which assumes that the forecast for present period is the demand for the previous period. A naive forecast for any period equals the previous period's actual value, according to William (1989). It works best when data is random. It is suited for the short term, not so good in the long run. It assumes high volatility in a series which is equal to the n^{th} term plus a movement that has a symmetric, zero centered probability distribution. Such a series has no memory of the path it took to arrive at the n^{th} value, thus no seasonal or cyclical patterns exist, except by pure chance. The disadvantage of such methods is that they are purely statistical in nature and they doesn't account for the profit measures obtained by conducting forecasts.

3.2.2 Basic exponential smoothing

For a weighted moving average thrust is on the most recent data. The weights should then get progressively smaller, the more periods one considers into the past. The exponentially decreasing weights of the Basic Exponential Smoothing (BES) forecast fit this situation (William, 1989). The forecast equation is given by:

$$F_{t+1} = \alpha \times D_t + (1 - \alpha) F_t \qquad (3.1)$$

Where α is a smoothing parameter between 0 and 1, D_t represents the demand in the t^{th} period, F_t and F_{t+1}represents the forecast in t^{th} and $(t+1)^{th}$ period respectively.

Also, Forecasting error, $E_t = (D_t - F_t)$ ....(3.2)

3.2.3 Moving average method

A MA forecast model is based on an artificially constructed time series in which the value for a given time period is replaced by the mean of that value and the values for some number of preceding and succeeding time periods (William, 1989). This model is best suited to time series data; i.e. data that changes over time. Since the forecast value for any given period is an average of the previous periods, then the forecast will always appear to lag behind, either increases or decreases in the observed (dependent) values. For example, if a data series has a noticeable upward trend then a moving average forecast will generally provide an underestimate of the value of the dependent variable.

The MA method has an advantage when compared to other forecasting models namely it does smooth out peaks and troughs in a set of observations. However, it also has several disadvantages. In particular this model does not produce an actual equation. Therefore, it is not all that useful as a medium and long range forecasting tool. It can only reliably be used to forecast one or two periods into the future.

The MA model is a special case of the more general weighted moving average. In the simple moving average, all weights are equal.

3.3 Markov chain

Markov chain is a stochastic process, with discrete states and time in which modeling is done on observable parameters (Aastveit *et al.*, 2011). This analysis is concerned with demand prediction based on a finite time interval, discrete time stationary Markov chains with a fixed number of states are used.

A state of a system is where the system is at a point of time. Transition probability is the probability of transforming from one state to another in a specific time period. A Markov model is described in terms of its transition probabilities, P_{ij} which can be represented in a transition probability matrix (TPM). A Markov chain is a stochastic process with the property that X_t at time t depends on its value at time $(t\text{-}1)$ and not the sequence of other values that the process passed through while arriving at X_{t-1}

$$P\left(\frac{X_t}{X_{t-1},X_{t-2}\ldots X_o}\right) = P\left(\frac{X_t}{X_{t-1}}\right) \quad \text{...} (3.3)$$

Such Markov chains are called single step Markov chains. Memory of the process is limited to only the previous state. Probability that say state *i* in time period *(t-1)* transforms into state *j* at time period *t* is called as transition probability matrix denoted by P_{ij} .

$$T.P.M\,(P_{ij}) = \begin{bmatrix} P_{11} & P_{12} & P_{13} & & P_{1n} \\ P_{21} & P_{22} & P_{23} & & P_{2n} \\ & & & & \\ P_{n1} & P_{n2} & P_{n3} & & P_{nn} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

The conditional probability gives the probability that at time *t* process will be in state *j* given the process was in state *i* at time *(t-1)*.The conditional probability is independent of states occupied prior to *(t-1)*.This conditional probability is called as transition probability. If the transition probability remains a constant with respect to time then the associated Markov chain is said to be homogeneous or regular.

Initial Probability Vector at time $t = 0$ is represented as

$$P^0 = [P_1^0, P_2^{0,} P_3^0, P_4^0 \ldots\ldots P_m^0] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

Probability vector or matrix at time $t = 1$ is represented as

$$P^1 = [P_1^1, P_2^1, P_3^1, P_4^1 \ldots\ldots P_m^1] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

Given the values of P^0 (initial probability matrix) and P_{ij} (transition probability matrix) we have

$$P^1 = [P_1^0, P_2^0, P_3^0, P_4^0 \ldots\ldots P_m^0]\,(P_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

$$P^2 = [P_1^1, P_2^1, P_3^1, P_4^1 \ldots\ldots P_m^1]\,(P_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

$$P^3 = [P_1^2, P_2^2, P_3^2, P_4^2 \ldots\ldots P_m^2]\,(P_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.9)$$

$$P^n = [P_1^{n-1}, P_2^{n-1}, P_3^{n-1}, P_4^{n-1} \ldots\ldots P_m^{n-1}]\,(P_{ij}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.10)$$

As time progresses P^n and P^{n-1} will converge as they are independent of previous states. Hence $P^n = P^{n-1}$, for large values of *n*. Then the process has reached a steady state and the corresponding probabilities are called as steady state probabilities. These probabilities will remain a constant.

3.4 Research methodology

1. Actual demand data for a novel short life cycle product is collected for any two successive months.
2. Apply moving average, naive and exponential smoothing methods to the collected data.
3. Estimate the errors in forecasting for all the relevant working days of two successive months after predicting its future demand.
4. Implement the generalized algorithm for the errors of the forecasted model.
5. Deduce the initial probability matrix and the transition probability matrix for the different states of errors of demand.
6. By utilizing the above two matrices the probability of different states of demand for any future period can be determined
7. The evolution of the system is determined by multiplying previous state vector (probability matrix), a stochastic vector representing the probabilities of the system being in any one of the given states, by the TPM.
8. Choose the state with maximum probability from the obtained current probability vector.
9. Determine the annual savings by adopting the demand of the state with maximum probability.
10. Validate the model by the computation of excess profit that the model brings to the firm when compared to existing practice.

3.5 Model or algorithm

1. Collect observed demand data for a short life cycle product for any two successive months
2. Apply naive, exponential smoothing and moving average methods to the collected data and determine the forecasts.
3. Determine the error of forecast for each day by taking the variation.
4. Apply the Markov based model to predict a single estimate of demand for random variation.
5. Determine the upper limit and lower limit of the collected sales data for the t^{th} month. Determine the range or band width of the collected data as the difference between upper limit and lower limit for the t^{th} month
6. Discretise the obtained range into states or class intervals with minimum possible no of sample size. Denote these states as $X_1, X_2, X_3 \ldots\ldots\ldots X_n$.
7. Determine the initial probability vector P^0 for the month t as per the following steps.
   a) List out all the days (m) in a month, say in the month t as the first column, in the ascending order of the table
   b) In second column enter the state of the observed sales data for all the days of t^{th} month listed in the first column
   c) Count the number of occurrence of each state in t^{th} month. (For e.g. say state X_i is occurring j times in the month t of m days, then initial probability of $X_i = \frac{j}{m}$)

d) Determine the initial probability of all states by using the formulae $X_i = \frac{j}{m}$ where j is the occurrence of i^{th} state in t^{th} month of m days and i= 1, 2, 3…….n.

e) Represent the obtained initial probabilities as a row vector (1 x n) with n no of entries and is called as initial probability vector denoted by P^0.This matrix gives the initial probability of all states say $X_1, X_2, X_3 \ldots\ldots X_n$ in t^{th} month.

8. Construct state occurrence table for t^{th} month and $(t+1)^{th}$ month as per the following steps.

a) List out all the days of t^{th} and $(t+1)^{th}$ month in the ascending order as the first column of the table. Assume the number of working days in both months as same.

b) In the second column of the table enter the state corresponding to sales data for all the days listed in t^{th} month.

c) In column three enter the state corresponding to sales data for all days listed in the $(t+1)^{th}$ month.

9. Deduce TPM from the event occurrence table as per the following steps.

a) Represent the probable transformation of current state X_i to other possible states $X_1, X_2, X_3 \ldots\ldots X_n$ as P_{11}, $P_{12 \ldots\ldots} P_{1n}$.

b) Form the TPM by representing all the current states as rows and succeeding states as columns. Now enter the probabilities as P_{11}, $P_{12 \ldots\ldots} P_{1n}$ in 1st row and repeat the same procedure for other rows. Any entry say P_{ij} is defined as the ratio of number of transformations of current state i of t^{th} month in a particular day to next state *j* of $(t+1)^{th}$ month in the same day to the total no of occurrence of current state *i* in the t^{th} month

10. Deduce the current probability vector for the succeeding months *(t+2), (t+3)*.... as

$P^1 = P^0 \times TPM$

$P^2 = P^1 \times TPM$

………

………

$P^m = P^{m-1} \times TPM$

11. Choose the state with maximum probability from the obtained current probability vector for say the m^{th} month which is a row matrix with probability of each state during say m^{th} month.

12. Determine the possible profit to firm by the adoption of this state of production.

13. Validate the model by determining the excess profit that the model brings to the firm when compared to existing practice

3.6 Conclusion

A generalized algorithm for predicting single point estimate of demand of a relatively novel and short life product is developed. The demand is initially predicted by conventional methods like moving average or naive or exponential smoothing. The Markov based model is then applied into errors of these forecasts to model the random component of demand. Further the demand corresponding to maximum probability is selected and the profit corresponding to this state of demand is determined. The model or algorithm is validated by the excess profit obtained by adopting this method when compared to the existing practice.

Chapter

4 APPLICATION OF MARKOV BASED MODEL TO THE BAKED PRODUCT A

Contents

4.1 Introduction
4.2 Forecasts based on naive method
4.3 Forecasts based on exponential smoothing
4.4 Forecasts based on moving average
4.5 Result
4.6 Conclusion

4.1 Introduction

This chapter deals with the application of the developed algorithm to a baked product A of relatively large quantity. The details of the collected data are furnished at first. Then the proposed algorithm is applied to the collected data. The demand corresponding to the state with maximum probability of errors of forecast is determined and the corresponding profit is estimated. Then the obtained profits are compared and the combination with maximum profit is identified. At last the method is validated by estimating the annual savings that this method will bring to the firm when compared to existing practice of estimating the profit corresponding to a daily production rate of 1300 items.

4.1.1 Implementation of the proposed algorithm for baked product A.

The actual data of a novel and short life product with a shelf life of single day, collected from a reputed baking firm is furnished below in Table 4.1. Holidays and Sundays are excluded resulting in data for twenty four days. The firm has been producing 1300 items per day and selling them at Rs.12. Any leftover item is sold at a rate of Rs.3 and thereby causing a possible loss of profit of Rs.5 per product for left over item. Cost of each item is Rs.7.

Table 4.1 Actual demand for April and May

Sl.No	Actual demand (April)	Actual demand (May)
1	1255	1267
2	1249	1269
3	1248	1268
4	1252	1271
5	1259	1266
6	1260	1268
7	1260	1271
8	1260	1268
9	1263	1271
10	1262	1265
11	1259	1262
12	1260	1269
13	1260	1271
14	1265	1266
15	1265	1271
16	1260	1267
17	1265	1269
18	1259	1265
19	1256	1268
20	1264	1267
21	1265	1264
22	1263	1270
23	1265	1268
24	1265	1267

4.2 Forecasts based on naive method

The naive method is applied to the collected data and forecasts for April and May are estimated as discussed in section 3.2.1. The error of forecast is also determined for each day of April and May and is provided in Table 4.2.

Table 4.2 Forecasts for April and May based on naive method

Actual demand April	Actual demand May	Forecasted demand April	Forecasted demand May	Error in demand forecast for each day of April	Error in demand forecast for each day of May
1255	1267				
1249	1269	1255	1267	-6	2
1248	1268	1249	1269	-1	-1
1252	1271	1248	1268	4	3
1259	1266	1252	1271	7	-5
1260	1268	1259	1266	1	2
1260	1271	1260	1268	0	3
1260	1268	1260	1271	0	-3
1263	1271	1260	1268	3	3
1262	1265	1263	1271	-1	-6
1259	1262	1262	1265	-3	-3
1260	1269	1259	1262	1	7
1260	1271	1260	1269	0	2
1265	1266	1260	1271	5	-5
1265	1271	1265	1266	0	5
1260	1267	1265	1271	-5	-4
1265	1269	1260	1267	5	2
1259	1265	1265	1269	-6	-4
1256	1268	1259	1265	-3	3
1264	1267	1256	1268	8	-1
1265	1264	1264	1267	1	-3
1263	1270	1265	1264	-2	6
1265	1268	1263	1270	2	-2
1265	1267	1265	1268	0	-1

Figure 4.1 provides actual demand for April and May whereas Figure 4.2 gives the forecasts of demand for April and May based on naive method. Figure 4.3 gives the errors in forecasted demand for April and May.

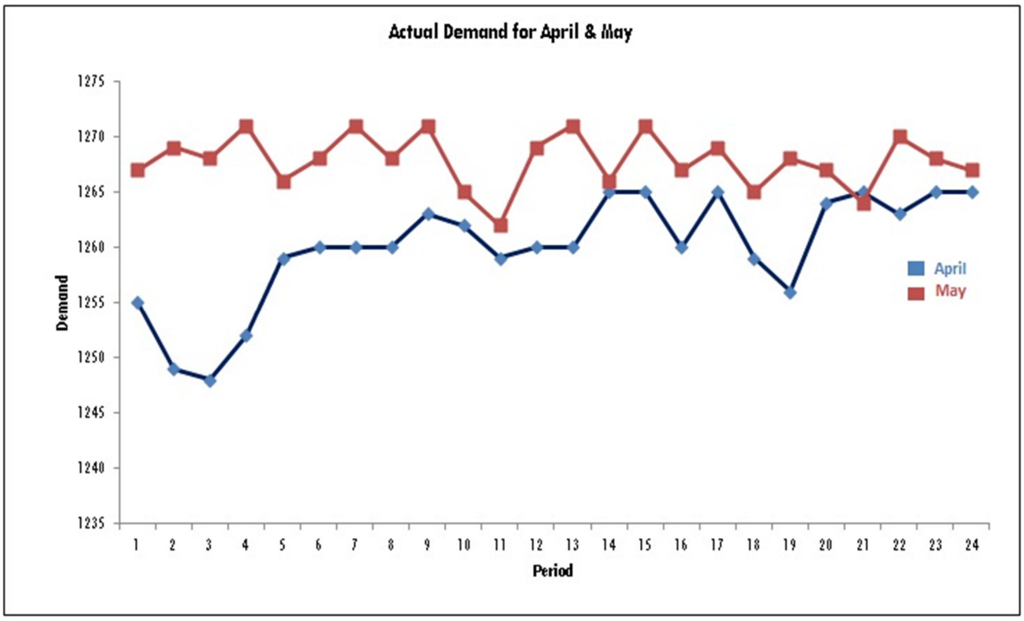

Figure 4.1 Actual demand for April and May

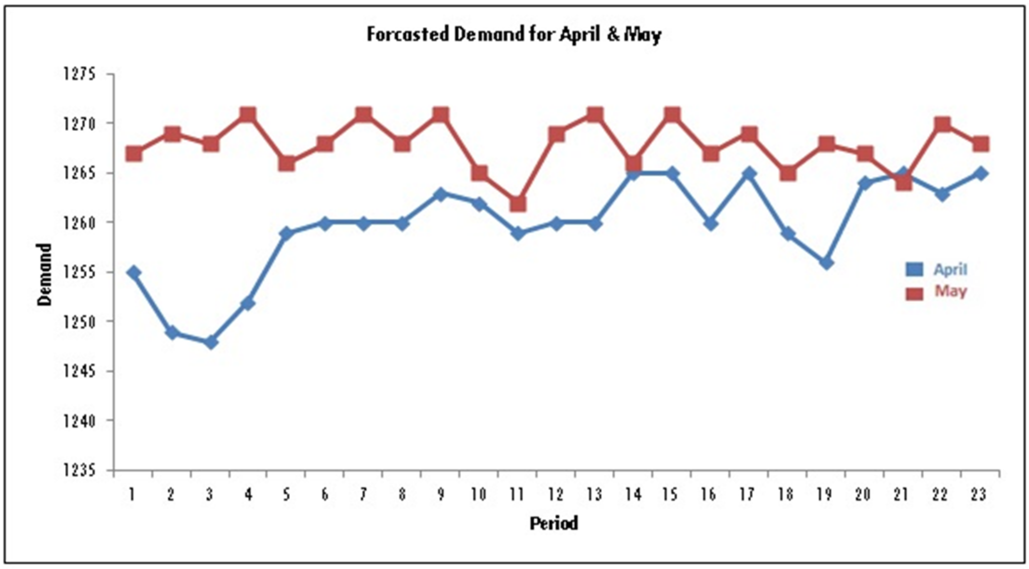

Figure 4.2 Forecasted demand for April and May

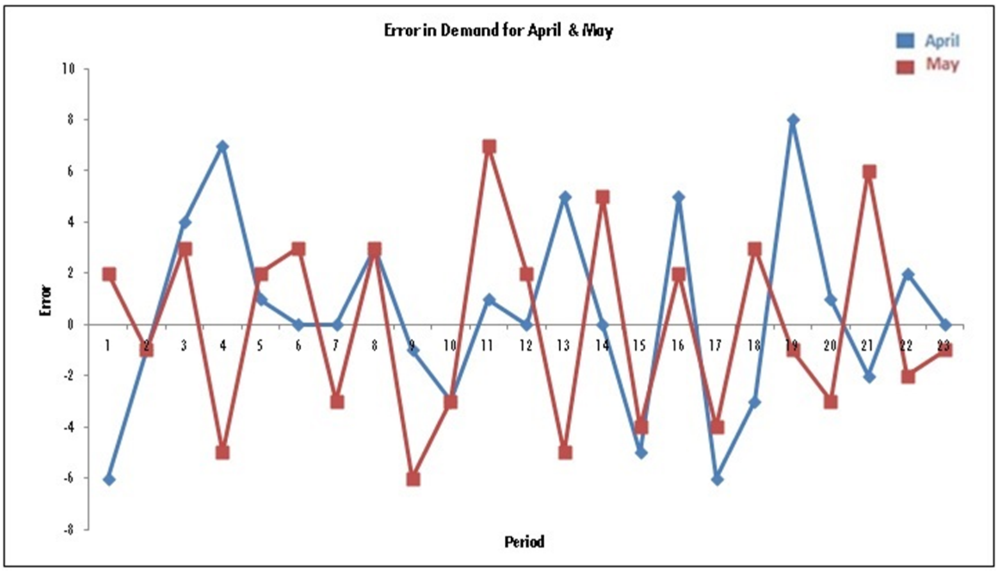

Figure 4.3 Error in forecasted demand for April and May

4.2.1 Control charts for errors based on naive method

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2S and +2S.

$$S = (MSE)^{.5} \quad \text{(4.1)}$$

$$\text{Upper control Limit (UCL)} = (0+2S) \quad \text{(4.2)}$$

$$\text{Lower control limit (LCL)} = (0-2S) \quad \text{(4.3)}$$

$$MSE = \{\sum e^2 / (n-1)\} \quad \text{(4.4)}$$

Where e is the error and n is the sample size.

For April, MSE = 15.21

$$S = 3.9$$

$$2S = 7.8$$

$$UCL = (0 + 7.8)$$

$$LCL = (0 – 7.8)$$

Figure 4.4 represents the control chart of errors in demand for April.

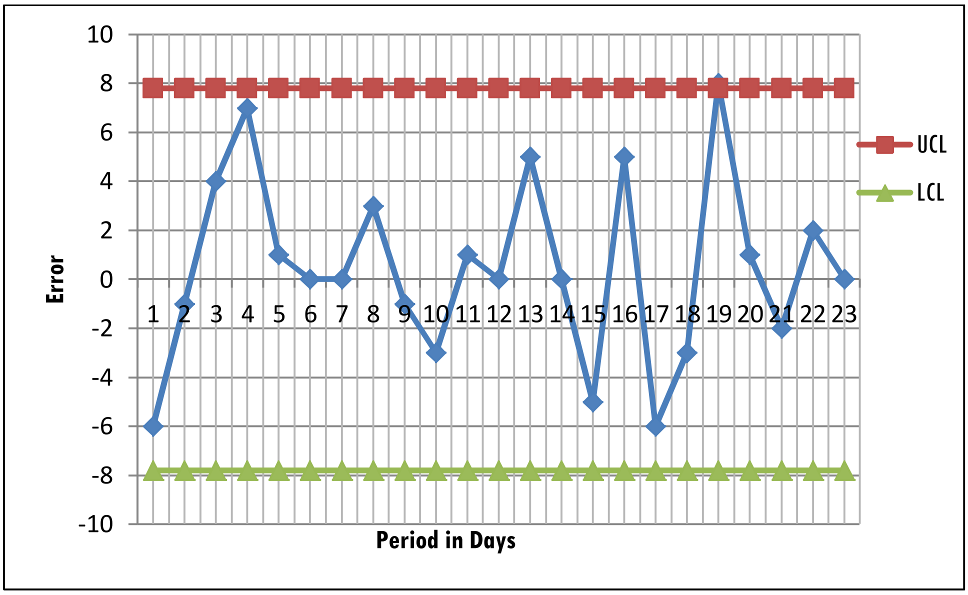

Figure 4.4 Control chart of error in demand for April

The error corresponding to 20th day is out of control.

For May, $MSE = 14.25$

$$S = 3.775$$

$$2S = 7.55$$

$$UCL = (0 + 7.55)$$

$$LCL = (0 – 7.55)$$

Figure 4.5 represents the control chart of errors in demand for May.

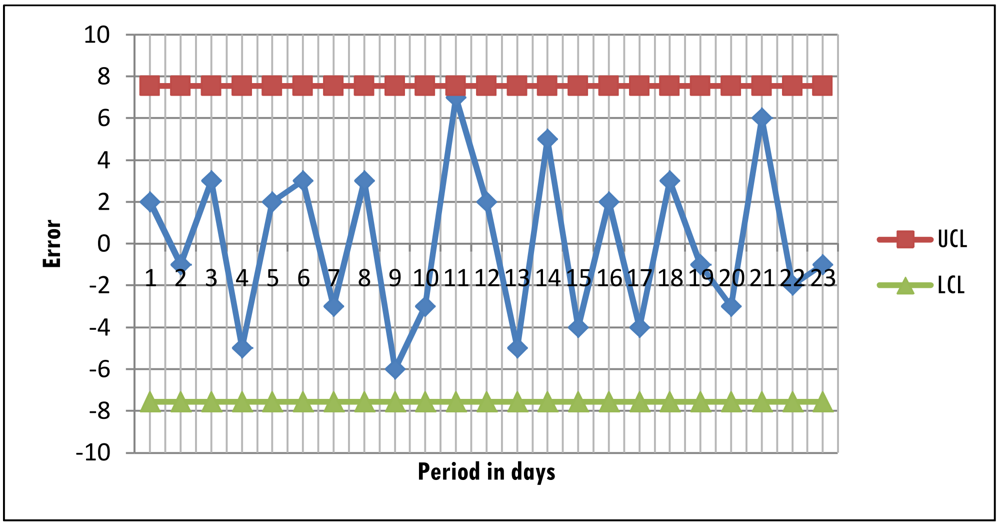

Figure 4.5 Control chart of error in demand for May

The errors for all the days lie within the control limits.

4.2.2 Initial probability matrix based on naive method

The initial probability matrix deduced by applying generalized algorithm to the collected data as per the step seven of algorithm given in section 3.5 is furnished below (Table 4.3).

Table 4.3 Initial probability matrix

Class interval of error of demand	State	No of occurrence	Probability
-6,-5	X_1	3	.125
-4,-3	X_2	2	.083
-2,-1	X_3	4	.167
0,1	X_4	8	.333
2,3	X_5	2	.083
4,5	X_6	3	.125
6,7,8	X_7	2	.083

The fourth column of the above table gives initial probability vector P^0 for the month April. This matrix gives the initial probability of all states say $X_{1,}$ $X_{2,}$ X_3……….X_8 in the month of April.

P^0 = [.125, .083, .167, .333, .083, .125, .083]

4.2.3 State transition table based on naive method

The state transition table obtained by applying the algorithm to collected data as discussed in the step eight of section 3.5 is provided below (Table 4.4).

Table 4.4 State transition table

Day	Current state (April)	Subsequent state (May)
1	3	5
2	1	5
3	3	3
4	6	5
5	7	1
6	4	5
7	4	5
8	4	2
9	5	5
10	3	1
11	2	2
12	4	7
13	4	5
14	6	1
15	4	6
16	1	2
17	6	5
18	1	2
19	2	5
20	7	3
21	4	2
22	3	7
23	5	3
24	4	3

4.2.4 Transition probability matrix based on naive method

The transition probability matrix deducted as per the step nine of the algorithm in section 3.5 is given below (Table 4.5).

Table 4.5 Transition probability matrix

	1	2	3	4	5	6	7
1	0	0.666	0	0	0.333	0	0
2	0	0.5	0	0	0.5	0	0
3	0.333	0	0.333	0	0	0	0.333
4	0	025	0.125	0	0.375	0.125	0.125
5	0	0	0.5	0	0.5	0	0
6	0.333	0	0	0	0.666	0	0
7	0.5	0	0.5	0	0	0	0

4.2.5 Probability matrix for the succeeding periods

The probabilities of different states for succeeding periods are calculated as discussed in step ten of section 3.5 and are shown below (Table 4.6).

Table 4.6 Probability table

P^1	0.1387	0.2080	0.1802	0	0.3327	0.0416	0.0972
P^2	0.1225	0.1964	0.2750	0	0.3443	0	0.0600
P^3	0.1216	0.1798	0.2937	0	0.3111	0	0.0916
P^4	0.1436	0.1709	0.2992	0	0.2859	0	0.0978
P^5	0.1485	0.1811	0.2915	0	0.2762	0	0.0996
P^6	0.1469	0.1895	0.2850	0	0.2781	0	0.0996
P^7	0.1434	0.1926	0.2825	0	0.2827	0	0.0949
P^8	0.1415	0.1918	0.2829	0	0.2854	0	0.0941
P^9	0.1412	0.1902	0.2839	0	0.2857	0	0.0942
P^{10}	0.1416	0.1891	0.2845	0	0.2850	0	0.0945
P^{11}	0.1420	0.1889	0.2845	0	0.2842	0	0.0947
P^{12}	0.1421	0.1890	0.2842	0	0.2839	0	0.0947

P^{13}	0.1420	0.1892	0.2839	0	0.2838	0	0.0946
P^{14}	0.1419	0.1892	0.2838	0	0.2838	0	0.0946
P^{15}	0.1418	0.1891	0.2836	0	0.2837	0	0.0945
P^{16}	0.1417	0.1890	0.2836	0	0.2836	0	0.0945
P^{17}	0.1417	0.1889	0.2835	0	0.2835	0	0.0944
P^{18}	0.1416	0.1888	0.2833	0	0.2833	0	0.0944
P^{19}	0.1415	0.1887	0.2832	0	0.2832	0	0.0943
P^{20}	0.1415	0.1886	0.2831	0	0.2831	0	0.0943
P^{21}	0.1414	0.1885	0.2830	0	0.2830	0	0.0943
P^{22}	0.1414	0.1885	0.2828	0	0.2828	0	0.0942
P^{23}	0.1413	0.1884	0.2827	0	0.2827	0	0.0942
P^{24}	0.1412	0.1883	0.2826	0	0.2826	0	0.0941
P^{25}	0.1412	0.1882	0.2825	0	0.2825	0	0.0941
P^{26}	0.1411	0.1881	0.2824	0	0.2824	0	0.0941
P^{27}	0.1411	0.1880	0.2822	0	0.2822	0	0.0940
P^{28}	0.1410	0.1880	0.2821	0	0.2821	0	0.0940
P^{29}	0.1409	0.1879	0.2820	0	0.2820	0	0.0939
P^{30}	0.1409	0.1878	0.2819	0	0.2819	0	0.0939
P^{31}	0.1408	0.1877	0.2817	0	0.2817	0	0.0939
P^{32}	0.1408	0.1876	0.2816	0	0.2816	0	0.0938
P^{33}	0.1407	0.1876	0.2815	0	0.2815	0	0.0938
P^{34}	0.1406	0.1875	0.2814	0	0.2814	0	0.0937
P^{35}	0.1406	0.1874	0.2813	0	0.2813	0	0.0937
P^{36}	0.1405	0.1873	0.2811	0	0.2811	0	0.0937
P^{37}	0.1405	0.1872	0.2810	0	0.2810	0	0.0936
P^{38}	0.1404	0.1872	0.2809	0	0.2809	0	0.0936
P^{39}	0.1403	0.1871	0.2808	0	0.2808	0	0.0935
P^{40}	0.1401	0.1870	0.2807	0	0.2807	0	0.0935

The maximum probability is for state 3 and 5 with errors -2,-1 and 2, 3. The corresponding demands are 1248, 1255, 1263 and 1265.

Figure 4.6 represents the probability time graph for the third state.

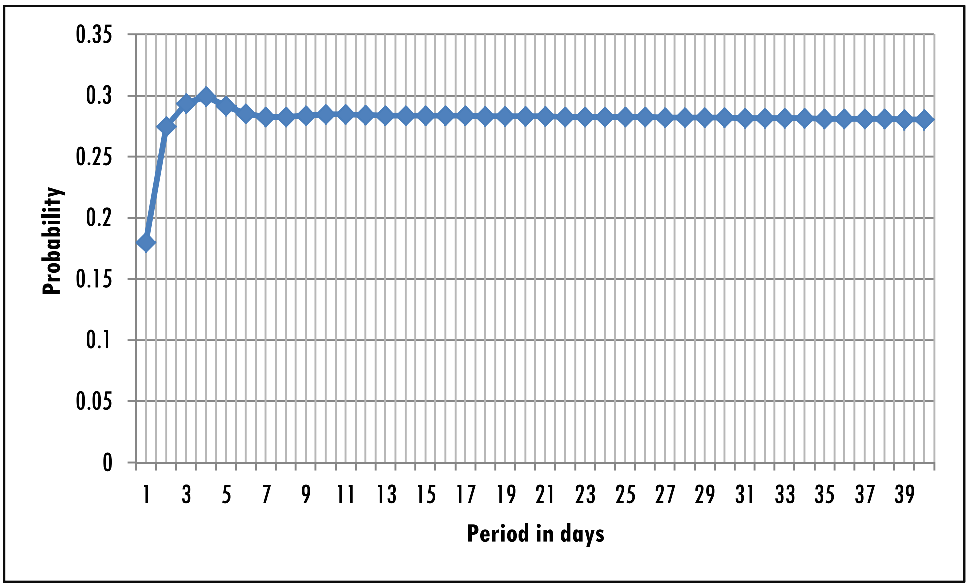

Figure 4.6 Probability time graph

4.2.6 Annual savings for various predicted demands based on naive method.

The annual savings by adopting the forecasted demand obtained as per the algorithm given in section 3.5 when compared to annual savings from the existing production of 1300 items per day is furnished below.

The parameters affecting annual savings are, P the existing daily production rate (1300 items), F the forecasted demand as per the algorithm, *C* the unit cost, *S* the selling price of left over item, *N* the no of working days in a month, C_u the total under stocking costs in that month, C_o the total overstocking costs in that month and N_0 the number of months in a year.

The term (P-F) provides the excess items of a day when existing production rate and predicted demand are considered. The value (C-S) gives the net loss of each excess item. The product of (P-F), (C-S) and N gives the gross profit of the month. The under stocking and over stocking costs for the month are then deducted from the gross profit to obtain the net profit for the month. This net profit can be used to determine the annual savings as shown.

Annual savings = $\{(P-F)\,(C-S)\,N - C_u - Co\}\ N_0$………………….. (4.5)

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1263 items are listed below in Table 4.7.

Table 4.7 Profit to the firm based on predicted demand of 1263 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1263	-8	32	
1249	1263	-14	56	
1248	1263	-15	60	
1252	1263	-11	44	
1259	1263	-4	16	
1260	1263	-3	12	
1260	1263	-3	12	
1260	1263	-3	12	
1263	1263	0	0	
1262	1263	-1	4	
1259	1263	-4	16	
1260	1263	-3	12	
1260	1263	-3	12	
1265	1263	2		10
1265	1263	2		10
1260	1263	-3	12	
1265	1263	2		10
1259	1263	-4	16	
1256	1263	-7	28	
1264	1263	1		5
1265	1263	2		10
1263	1263	0		
1265	1263	2		10
1265	1263	2		10
			Total 344	Total 65

Annual savings for a demand of 1263 items = {(1300- 1263) (7-3) 24 – 65-344}12 = Rs. 37716.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1255 items are listed below in Table 4.8.

Table 4.8 Profit to the firm based on demand of 1255 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1255	0		0
1249	1255	-6	24	
1248	1255	-7	28	
1252	1255	-3	12	
1259	1255	4		20
1260	1255	5		25
1260	1255	5		25
1260	1255	5		25
1263	1255	8		40
1262	1255	7		35
1259	1255	4		20
1260	1255	5		25
1260	1255	5		25
1265	1255	10		50
1265	1255	10		50
1260	1255	5		25
1265	1255	10		50
1259	1255	4		20
1256	1255	1		5
1264	1255	9		45
1265	1255	10		50
1263	1255	8		40
1265	1255	10		50
1265	1255	10		50
			Total 64	Total 675

Annual savings by adopting a demand of 1255 items per day = {(1300-1255) (7-3) 24 – 675-64}12 = Rs. 42972.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1248 items are listed below in Table 4.9.

Table 4.9 Profit to the firm based on demand of 1248 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1248	7		28
1249	1248	1		4
1248	1248	0		0
1252	1248	4		16
1259	1248	11		44
1260	1248	12		48
1260	1248	12		48
1260	1248	12		48
1263	1248	15		60
1262	1248	14		56
1259	1248	11		44
1260	1248	12		48
1260	1248	12		48
1265	1248	17		68
1265	1248	17		68
1260	1248	12		48
1265	1248	17		68
1259	1248	11		44
1256	1248	8		32
1264	1248	16		64
1265	1248	17		68
1263	1248	15		60
1265	1248	17		68
1265	1248	17		68
				Total 1148

Annual savings by adopting a demand of 1248 items per day = {(1300-1248) (7-3) 24 – 1148}12 = Rs. 46128.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1265 items are listed below in Table 4.10.

Table 4.10 Profit to the firm based on a demand of 1265 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1265	-10	40	
1249	1265	-16	64	
1248	1265	-17	68	
1252	1265	-13	52	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1260	1265	-5	20	
1263	1265	-2	8	
1262	1265	-3	12	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1265	1265	0		
1265	1265	0		
1260	1265	-5	20	
1265	1265	0		
1259	1265	-6	24	
1256	1265	-9	36	
1264	1265	-1	4	
1265	1265	0		
1263	1265	-2	8	
1265	1265	0		
1265	1265	0		
			484	

Annual savings by adopting a demand of 1265 items per day = {(1300-1265) (7-3) 24 – 484}12= Rs. 34512.

4.3 Forecasts based on exponential smoothing

The forecasts based on ES and the corresponding errors in forecast for each day of April and May as discussed in section 3.2.2 is furnished below (Table 4.11).

Table 4.11 Forecasts for April & May based on exponential smoothing.

Actual demand April	Actual demand May	Forecasted demand (alpha= .2) April	Forecasted demand (alpha= .2) May	Error in demand for each day of April	Error in demand for each day of May
1255	1267				
1249	1269	1255	1267	-6	2
1248	1268	1250	1269	-2	-1
1252	1271	1248	1268	4	3
1259	1266	1251	1270	8	-4
1260	1268	1257	1267	3	1
1260	1271	1259	1268	1	3
1260	1268	1260	1270	0	-2
1263	1271	1260	1268	3	3
1262	1265	1262	1270	0	-5
1259	1262	1262	1266	-3	-4
1260	1269	1260	1263	0	6
1260	1271	1260	1268	0	3
1265	1266	1260	1270	5	-4
1265	1271	1264	1267	1	4
1260	1267	1265	1270	-5	-3
1265	1269	1261	1268	4	1
1259	1265	1264	1269	-5	-4
1256	1268	1260	1266	-4	2
1264	1267	1257	1268	7	-1
1265	1264	1263	1267	2	-3
1263	1270	1265	1265	-2	5
1265	1268	1263	1269	2	-1
1265	1267	1265	1268	0	-1

Figure 4.7 provides actual demand for April and May whereas Figure 4.8 gives the forecasts of demand for April and May based on ES method. Figure 4.9 gives the errors in forecasted demand for April and May.

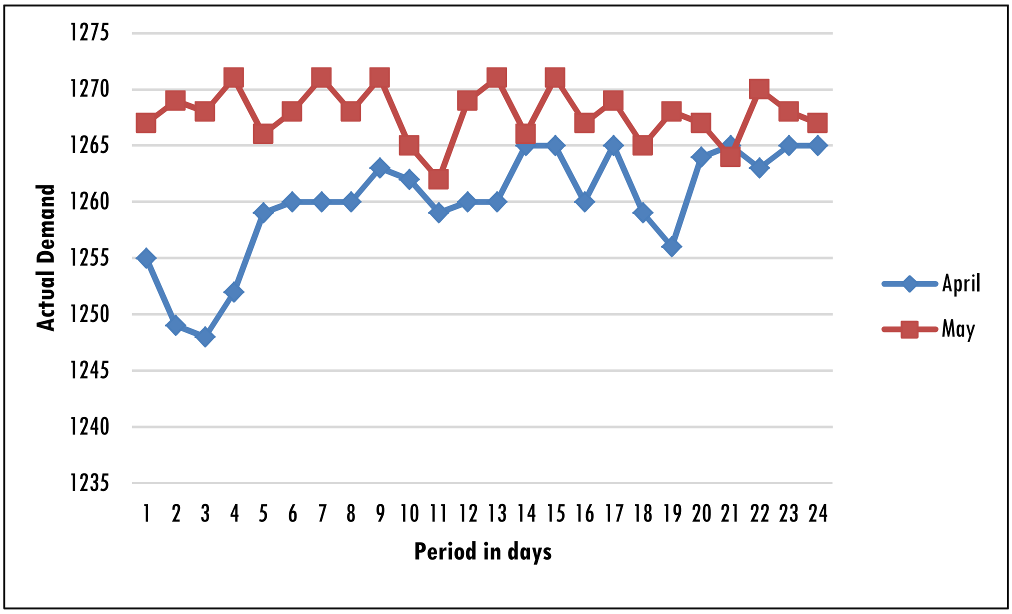

Figure 4.7 Actual demand for April and May

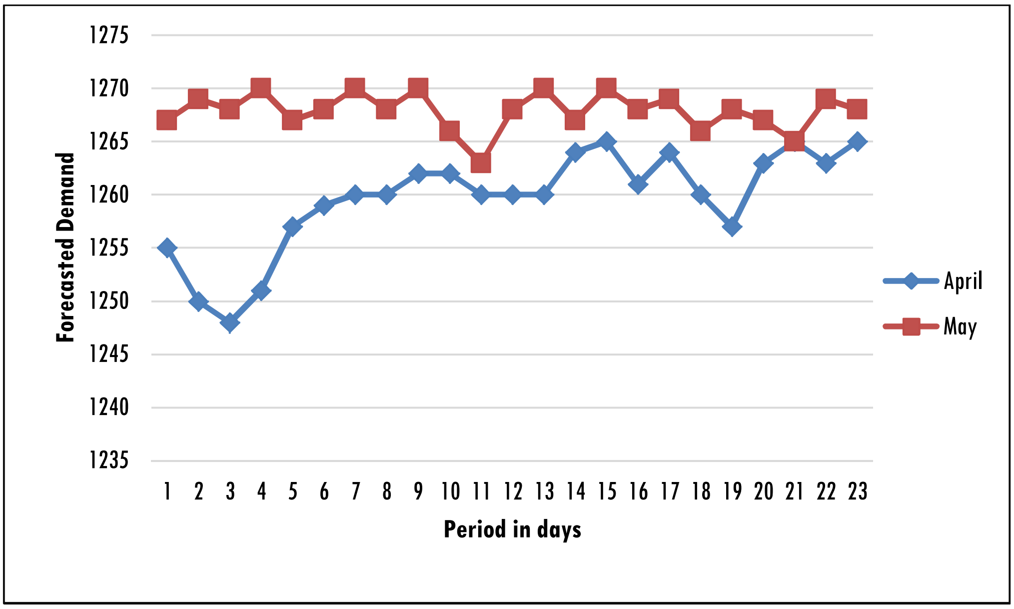

Figure 4.8 Forecasted demand for April and May

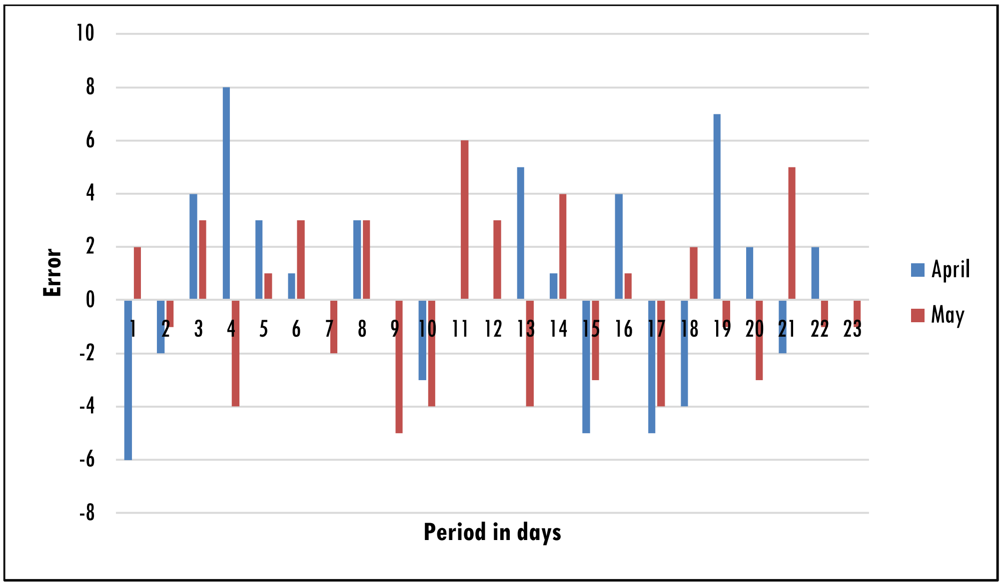

Figure 4.9 Error of forecasted demand for April & May

4.3.1 Control Charts for errors based on basic exponential smoothing

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 per cent of the values (errors in this case) can be expected to fall within limits -2S and +2S. For April,

MSE = 14.44

S = 3.8

2S = 7.6

UCL = (0 + 7.6)

LCL = (0 – 7.6)

Figure 4.10 represents the control chart of errors in demand based on ES for April.

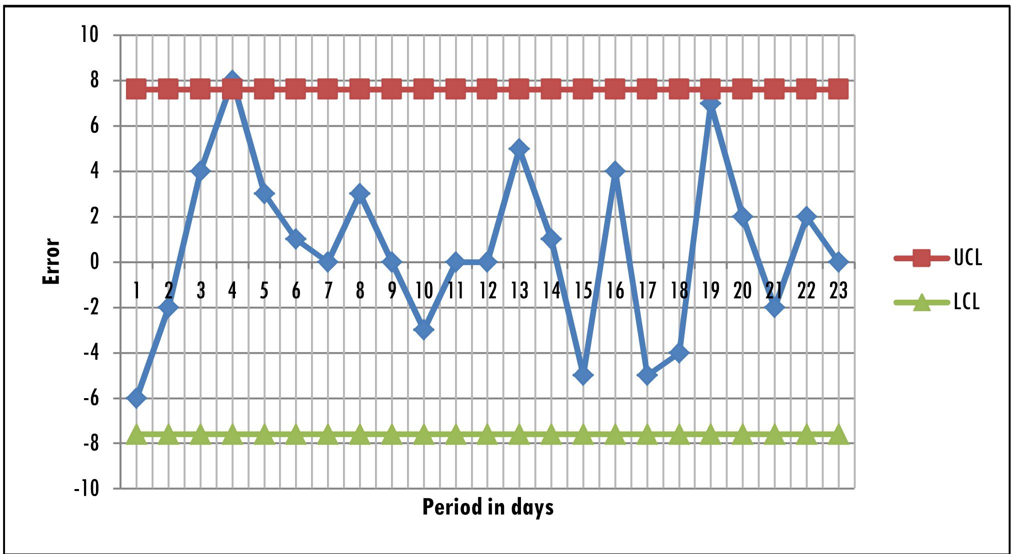

Figure 4.10 Control chart of error in demand based on exponential smoothing (April)

The error corresponding to 5th day is out of control.

For May,

MSE	=	10.89
S	=	3.3
2S	=	7.6
UCL	=	(0 + 6.6)
LCL	=	(0 – 6.6)

Figure 4.11 represents the control chart of errors in demand based on ES for May.

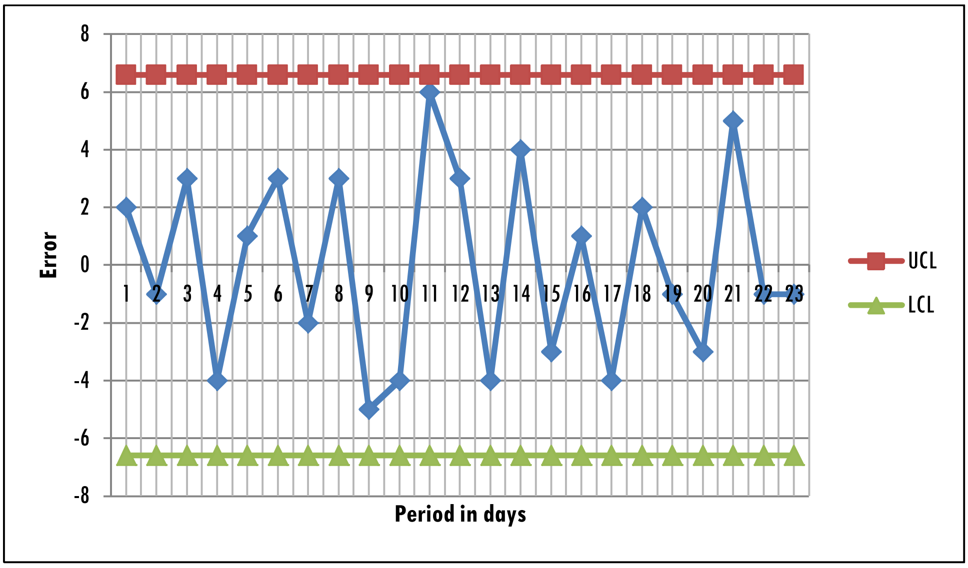

Figure 4.11 Control chart of error in demand based on exponential smoothing (May)

The errors for all the days lie within the control limits

4.3.2 Initial probability matrix based on B.E.S

The initial probability matrix deduced based on step seven of generalized algorithm mentioned in section 3.5 is given below (Table 4.12).

Table 4.12 Initial probability table

Class interval of error of demand	State	No of occurrence	Probability
-6,-5,-4	X_1	5	0.227273
-3,-2,-1	X_2	2	0.090909
0,1,2	X_3	6	0.272727
3,4,5	X_4	7	0.318182
6,7,8,9	X_5	2	0.090909

The fourth column of the above table gives initial probability vector P^0 for the month April. This matrix gives the initial probability of all states say X_1, X_2, X_3………..X_8 in the month of April.

$$P^0 = [0.227273, 0.090909, 0.272727, 0.318182, 0.090909]$$

4.3.3 State transition table based on B.E.S

The state transition table obtained by applying generalized algorithm to the collected data as discussed in the step eight of section 3.5 is given below (Table 4.13).

Table 4.13 State transition table

Day	Current state (April)	Subsequent state (May)
1	1	3
2	4	4
3	5	1
4	4	2
5	3	4
6	3	2
7	4	2
8	3	1
9	1	1
10	2	5
11	3	5
12	4	1
13	4	4
14	1	2
15	4	3
16	1	2
17	1	3
18	5	3
19	4	1
20	2	4
21	3	3
22	3	2

4.3.4 Transition probability matrix based on B.E.S

The transition probability matrix obtained in accordance with step nine of the generalized algorithm mentioned in section 3.5 is listed below in Table 4.14.

Table 4.14 Transition probability matrix

	1	2	3	4	5
1	0.2	0.4	0.4	0	0
2	0	0	0	0.5	0.5
3	0.167	0.333	0.167	0.167	0.167
4	0.286	0.286	0.143	0.286	0
5	0.5	0	0.5	0	0

4.3.5 Probability matrix for the succeeding periods based on B.E.S

The probability of different states for succeeding periods deduced as discussed in step ten of the algorithm stated in section 3.5 is given below (Table 4.15).

Table 4.15 Probability table

P^0	0.2273	0.0909	0.2727	0.3182	0.0909
P^1	0.2275	0.2727	0.2274	0.1820	0.0910
P^2	0.1810	0.2188	0.2005	0.2264	0.1743
P^3	0.2216	0.2039	0.2254	0.2076	0.1429
P^4	0.2128	0.2231	0.2274	0.1990	0.1396
P^5	0.2072	0.2177	0.2213	0.2064	0.1495
P^6	0.2122	0.2156	0.2241	0.2049	0.1458
P^7	0.2114	0.2181	0.2245	0.2038	0.1453
P^8	0.2107	0.2176	0.2238	0.2049	0.1466
P^9	0.2114	0.2174	0.2242	0.2048	0.1462
P^{10}	0.2114	0.2178	0.2244	0.2047	0.1461
P^{11}	0.2114	0.2178	0.2244	0.2049	0.1464
P^{12}	0.2115	0.2179	0.2245	0.2050	0.1464
P^{13}	0.2116	0.2180	0.2246	0.2051	0.1464
P^{14}	0.2117	0.2181	0.2247	0.2052	0.1465

The maximum probability is for state 3 with errors 0, 1 and 2. The corresponding demands are 1260, 1262 and 1265.

Figure 4.12 represents the probability time graph for the third state.

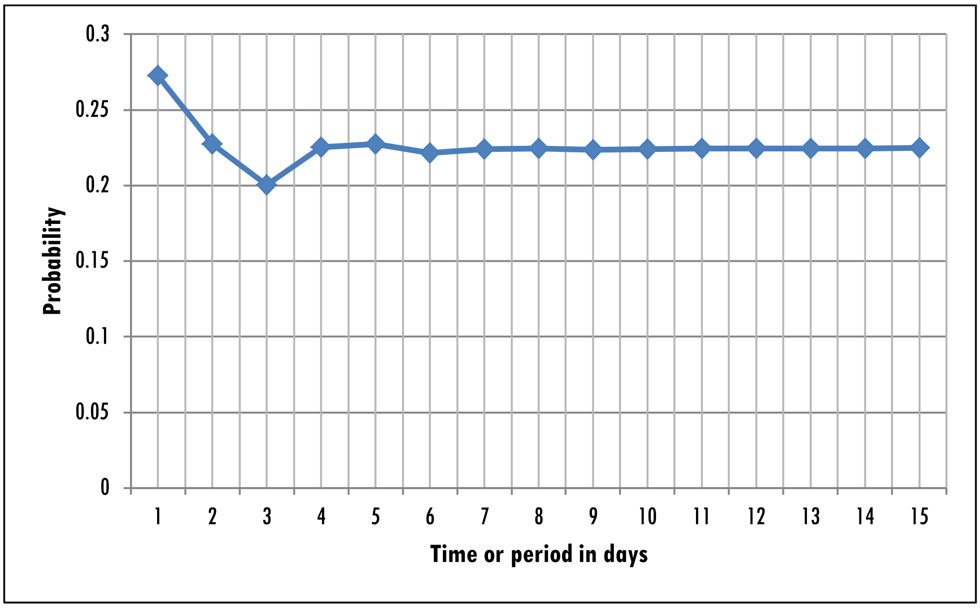

Figure 4.12 Probability time graph for the state with highest probability

4.3.6 Annual savings for various predicted demands based on B.E.S

The annual savings by adopting the forecasted demand obtained as per the algorithm given in section 3.5 when compared to annual savings from the existing production of 1300 items per day and by using equation 4.5 is furnished below.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1260 items are listed below in Table 4.16.

Table 4.16 Profit to the firm based on predicted demand of 1260 items.

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1260	-5	20	
1249	1260	-11	44	
1248	1260	-12	48	
1252	1260	-8	32	
1259	1260	-1	4	
1260	1260	0	0	
1260	1260	0	0	
1260	1260	0	0	
1263	1260	3		15
1262	1260	2		10
1259	1260	-1	4	
1260	1260	0		
1260	1260	0		
1265	1260	5		25
1265	1260	5		25
1260	1260	0		
1265	1260	5		25
1259	1260	-1	4	
1256	1260	-4	16	
1264	1260	4		20
1265	1260	5		25
1263	1260	3		15
1265	1260	5		25
1265	1260	5		25
		Total	172	210

Annual savings by adopting a demand of 1260 items = {(1300- 1260) (7-3) 24 – 172-210}12 = Rs. 41496

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1262 items are listed below in Table 4.17.

Table 4.17 Profit to the firm based on demand of 1262 items

Actual Demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1262	-7	28	
1249	1262	-13	52	
1248	1262	-14	56	
1252	1262	-10	40	
1259	1262	-3	12	
1260	1262	-2	8	
1260	1262	-2	8	
1260	1262	-2	8	
1263	1262	1		5
1262	1262	0		
1259	1262	-3	12	
1260	1262	-2	8	
1260	1262	-2	8	
1265	1262	3		15
1265	1262	3		15
1260	1262	-2	8	
1265	1262	3		15
1259	1262	-3	12	
1256	1262	-6	24	
1264	1262	2		10
1265	1262	3		15
1263	1262	1		5
1265	1262	3		15
1265	1262	3		15
		Total	284	110

Annual savings by adopting a demand of 1262 items = {(1300- 1262) (7-3) 24 – 284-110}12 = Rs. 39048.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1265 items are listed below in Table 4.18.

Table 4.18 Profit to the firm based on demand of 1265 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1265	-10	40	
1249	1265	-16	64	
1248	1265	-17	68	
1252	1265	-13	52	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1260	1265	-5	20	
1263	1265	-2	8	
1262	1265	-3	12	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1265	1265	0		
1265	1265	0		
1260	1265	-5	20	
1265	1265	0		
1259	1265	-6	24	
1256	1265	-9	36	
1264	1265	-1	4	
1265	1265	0		
1263	1265	-2	8	
1265	1265	0		
1265	1265	0		
		Total	484	

Annual savings by adopting a demand of 1265 items = {(1300- 1265) (7-3) 24 – 484}12 = Rs. 34512.

4.4 Forecasts based on moving average method

The forecasts obtained by applying MA method to collected data as discussed in section 3.2.3 and the corresponding errors of forecast for each day of April and May are listed below (Table 4.19).

Table 4.19 Forecasts for April & May based on moving average method

Actual demand April	Actual demand May	Forecasted demand April	Forecasted demand May	Error in forecast for each day of April	Error in forecast for each day of May
1255	1267				
1249	1269	1252	1268	-3	1
1248	1268	1249	1269	-4	0
1252	1271	1250	1270	4	3
1259	1266	1256	1269	9	-4
1260	1268	1260	1267	5	-1
1260	1271	1260	1270	1	4
1260	1268	1260	1270	0	-2
1263	1271	1262	1270	3	2
1262	1265	1263	1268	1	-5
1259	1262	1261	1264	-4	-6
1260	1269	1260	1266	-1	6
1260	1271	1260	1270	1	6
1265	1266	1263	1269	5	-4
1265	1271	1265	1269	3	3
1260	1267	1263	1269	-5	-2
1265	1269	1263	1268	3	0
1259	1265	1262	1267	-4	-3
1256	1268	1258	1267	-6	1
1264	1267	1260	1268	7	1
1265	1264	1265	1266	5	-4
1263	1270	1264	1267	-2	5
1265	1268	1264	1269	1	1
1265	1267	1265	1268	0	-1

Figure 4.13 provides actual demand for April and May whereas Figure 4.14 gives the forecasts of demand for April and May based on MA method. Figure 4.15 gives the errors in forecasted demand for April and May.

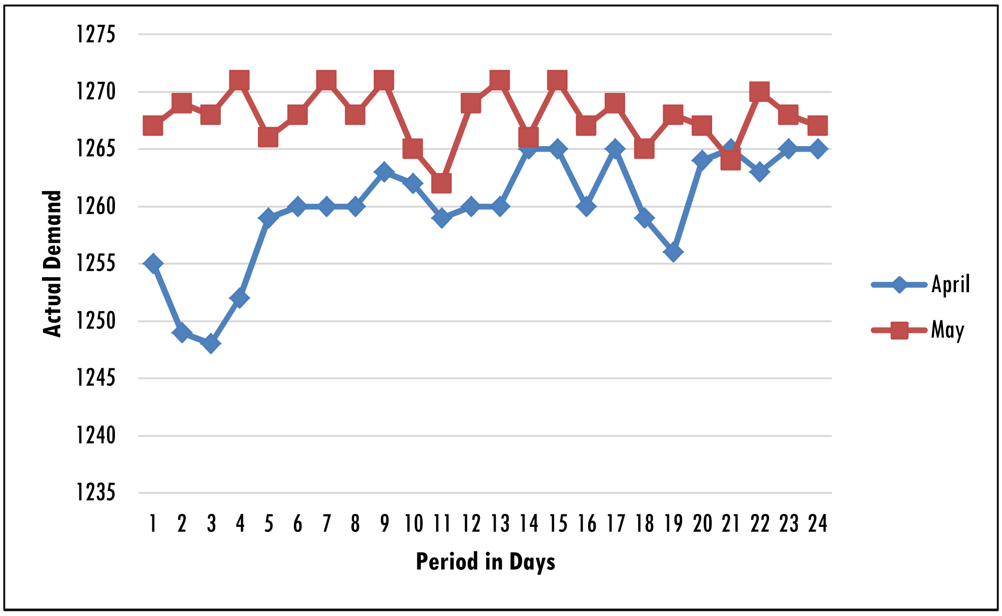

Figure 4.13 Actual demand for April and May

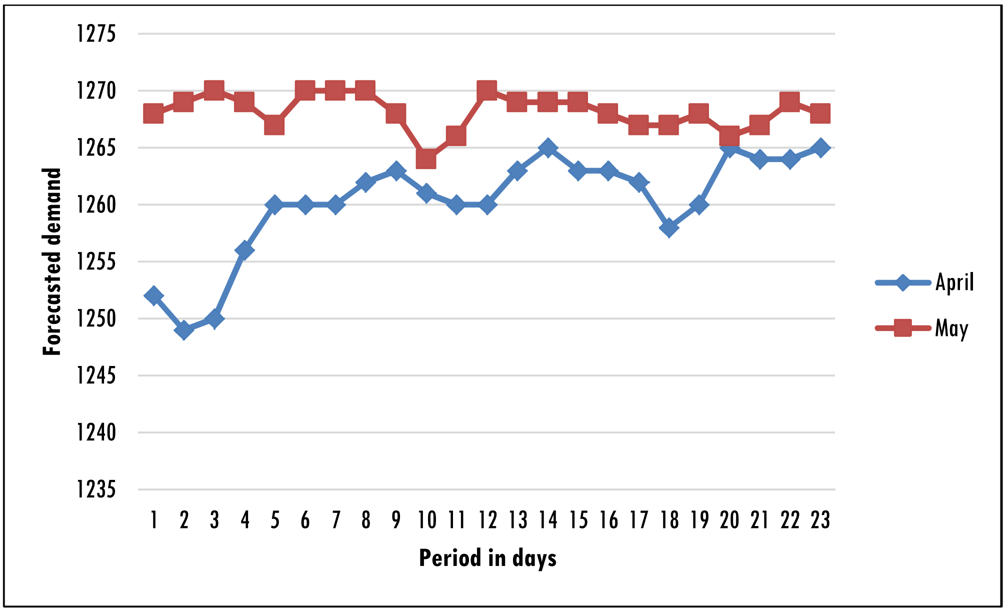

Figure 4.14 Forecasted demand for April and May

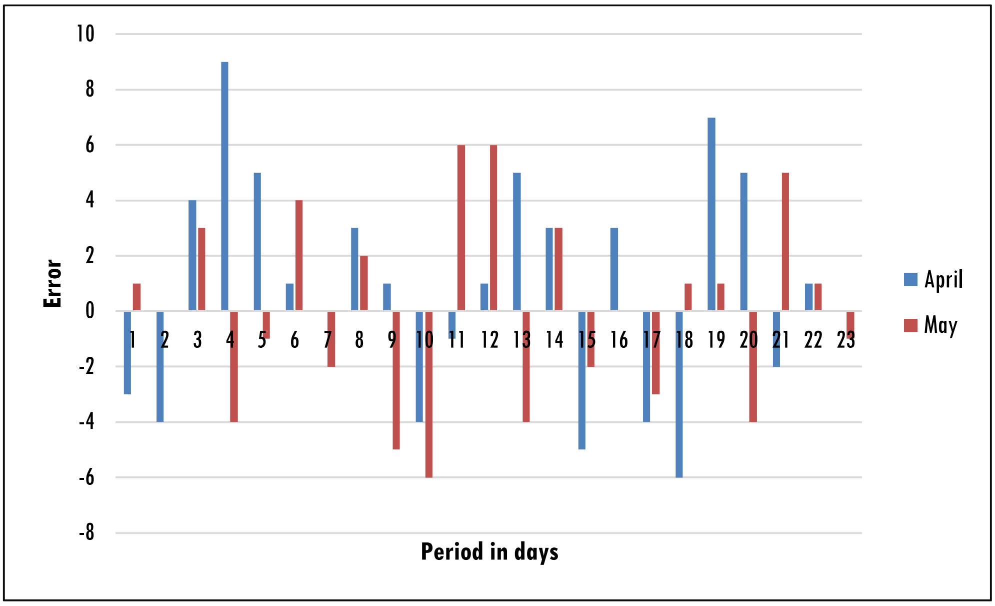

Figure 4.15 Error of forecasted demand for April & May

4.4.1 Control charts based on moving average method

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2S and +2S.

For April,

MSE = 17.05

S = 4.13

2S = 8.26

UCL = (0 + 8.26)

LCL = (0 – 8.26)

Figure 4.16 represents the control chart for errors of forecasted demand based on MA method for April.

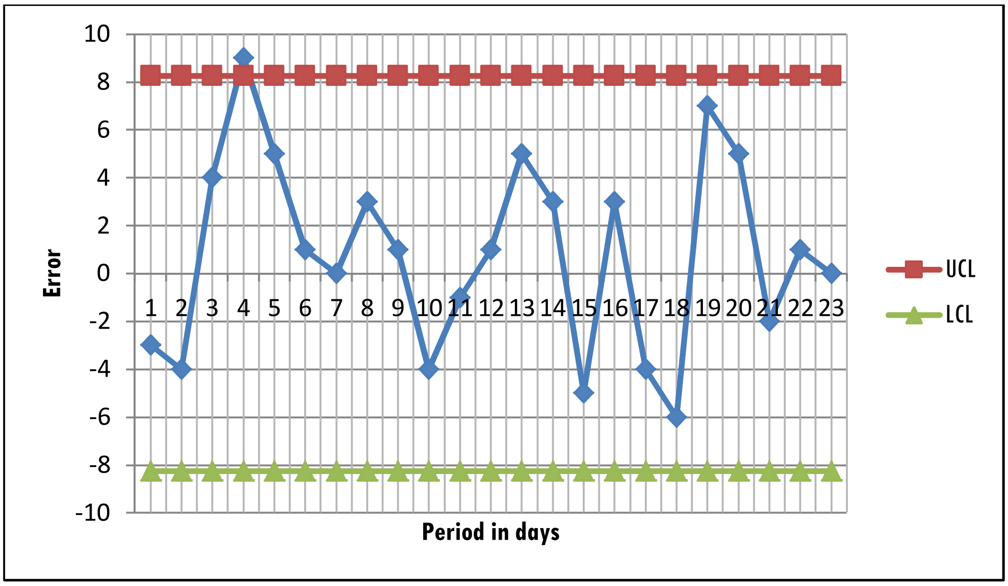

Figure 4.16 Control chart for error in demand for April.

The error of demand corresponding to the 5th day is out of control.

For May,

MSE = 12.14

S = 3.48

2S = 6.97

UCL = (0 + 6.97)

LCL = (0 – 6.97)

Figure 4.17 represents the control chart for errors of forecasted demand based on moving average method for May.

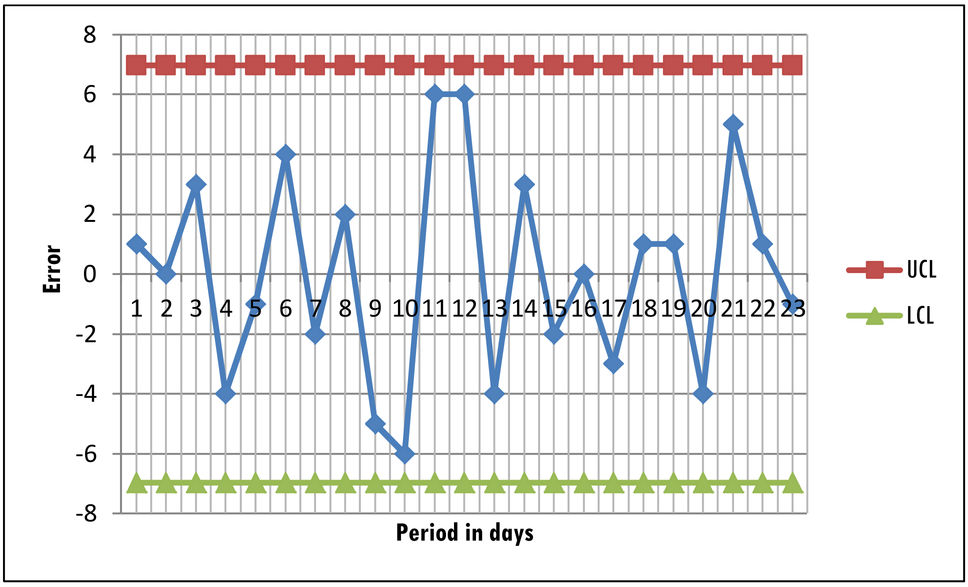

Figure 4.17 Control chart for error in demand for May

The errors for all the days lie within the control limits

4.4.2 Initial probability matrix based on moving average

The initial probability matrix obtained by applying algorithm to the collected data as discussed in step seven of section 3.5 is furnished below (Table 4.20).

Table 4.20 Initial probability matrix

Class interval of error of demand	State	No of occurrence	Probability
9,8,7,6	X_1	2	0.086957
5,4,3,2	X_2	7	0.304348
1,0-1,-2	X_3	8	0.347826
-3,-4,-5,-6	X_4	6	0.26087

The fourth column of the above table gives initial probability vector P^0 for the month April. This matrix gives the initial probability of all states say X_1, X_2, X_3..........X_8 in the month of April.

$P^0 = [0.086957, 0.304348, 0.347826, 0.26087]$

4.4.3 State transition table based on moving average

The state transition table deducted as per the step eight of the algorithm mentioned in section 3.5 is shown below (Table 4.21).

Table 4.21 State transition table

Day	Current state (April)	Subsequent state (May)
1	4	3
2	2	2
3	1	4
4	2	3
5	3	2
6	3	3
7	2	2
8	3	4
9	4	4
10	3	1
11	3	1
12	2	4
13	2	2
14	4	3
15	2	3
16	4	4
17	4	3
18	1	3
19	2	4
20	3	2
21	3	3
22	3	3
23	3	3

4.4.4 Transition probability matrix based on moving average

The obtained transition probability matrix based on MA method as discussed in ninth step of the algorithm given in section 3.5 is listed below (Table 4.22).

Table 4.22 Transition probability matrix

	1	2	3	4
1	0	0	0.5	0.5
2	0	0.4286	0.2857	0.285
3	0.2222	0.2222	0.4444	0.1111
4	0	0	0.6	0.4

4.4.5 Probability matrix for succeeding periods based on moving average

The probability of different states for succeeding months as discussed in step ten of the algorithm mentioned in section 3.5 is given below (Table 4.23).

Table 4.23 Probability matrix

P^0	0.0870	0.3043	0.3478	0.2609
P^1	0.0773	0.2077	0.4415	0.2732
P^2	0.0981	0.1871	0.4581	0.2562
P^3	0.1018	0.1820	0.4598	0.2558
P^4	0.1022	0.1802	0.4607	0.2562
P^5	0.1024	0.1796	0.4610	0.2561
P^6	0.1024	0.1794	0.4610	0.2560
P^7	0.1024	0.1793	0.4610	0.2560
P^8	0.1024	0.1793	0.4609	0.2559
P^9	0.1024	0.1793	0.4608	0.2559
P^{10}	0.1024	0.1792	0.4607	0.2558

The maximum probability is for state 3 with errors-2,-1, 0 and 1. The corresponding demands are 1260, 1262, 1263 and 1265.

Figure 4.18 represents the probability time graph for the third state.

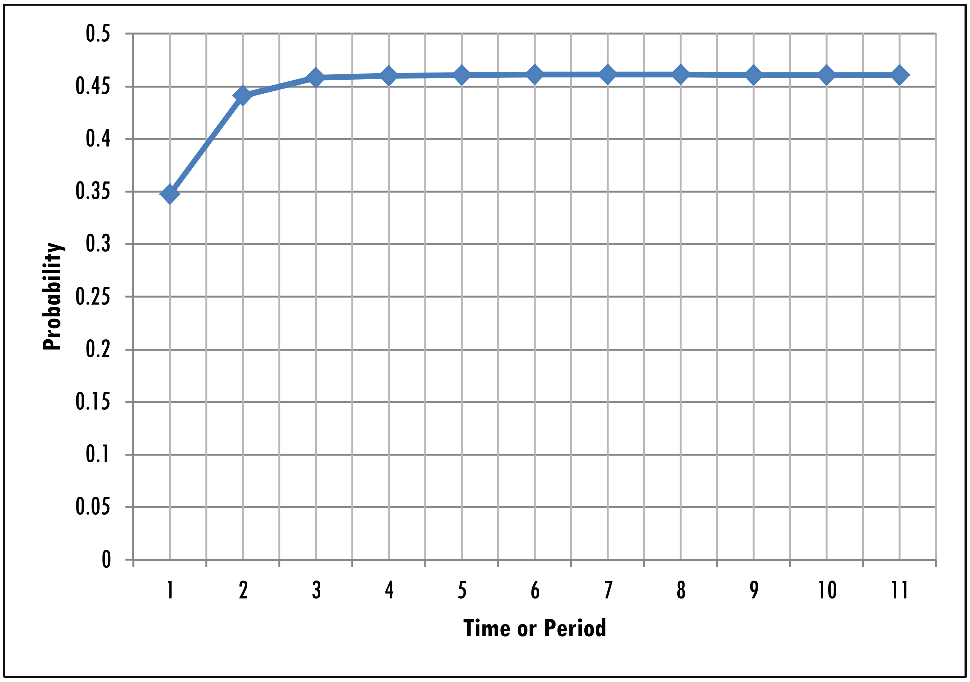

Figure 4.18 Probability time graph for the state with highest probability

4.4.6 Annual savings for various predicted demands based on moving average method.

The annual savings by adopting the forecasted demand obtained as per the algorithm given in section 3.5 when compared to annual savings from the existing production of 1300 items per day and by using equation 4.5 is furnished below.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1260 items are listed below in Table 4.24.

Table 4.24 Profit to the firm based on demand of 1260 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1260	-5	20	
1249	1260	-11	44	
1248	1260	-12	48	
1252	1260	-8	32	
1259	1260	-1	4	
1260	1260	0	0	
1260	1260	0	0	
1260	1260	0	0	
1263	1260	3		15
1262	1260	2		10
1259	1260	-1	4	
1260	1260	0		
1260	1260	0		
1265	1260	5		25
1265	1260	5		25
1260	1260	0		
1265	1260	5		25
1259	1260	-1	4	
1256	1260	-4	16	
1264	1260	4		20
1265	1260	5		25
1263	1260	3		15
1265	1260	5		25
1265	1260	5		25
		Total	172	210

Annual savings by adopting a demand of 1260 items = {(1300- 1260) (7-3) 24 – 172-210}12 = Rs. 41496.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1262 items are listed below in Table 4.25.

Table 4.25 Profit to the firm based on demand of 1262 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1262	-7	28	
1249	1262	-13	52	
1248	1262	-14	56	
1252	1262	-10	40	
1259	1262	-3	12	
1260	1262	-2	8	
1260	1262	-2	8	
1260	1262	-2	8	
1263	1262	1		5
1262	1262	0		
1259	1262	-3	12	
1260	1262	-2	8	
1260	1262	-2	8	
1265	1262	3		15
1265	1262	3		15
1260	1262	-2	8	
1265	1262	3		15
1259	1262	-3	12	
1256	1262	-6	24	
1264	1262	2		10
1265	1262	3		15
1263	1262	1		5
1265	1262	3		15
1265	1262	3		15
		Total	284	110

Annual savings by adopting a demand of 1262 items = {(1300- 1262) (7-3) 24 – 284-110}12 = Rs. 39048.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1263 items are listed below in Table 4.26.

Table 4.26 Profit to the firm based on demand of 1263 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1263	-8	32	
1249	1263	-14	56	
1248	1263	-15	60	
1252	1263	-11	44	
1259	1263	-4	16	
1260	1263	-3	12	
1260	1263	-3	12	
1260	1263	-3	12	
1263	1263	0	0	
1262	1263	-1	4	
1259	1263	-4	16	
1260	1263	-3	12	
1260	1263	-3	12	
1265	1263	2		10
1265	1263	2		10
1260	1263	-3	12	
1265	1263	2		10
1259	1263	-4	16	
1256	1263	-7	28	
1264	1263	1		5
1265	1263	2		10
1263	1263	0		
1265	1263	2		10
1265	1263	2		10
		Total	344	65

Annual savings by adopting a demand of 1263 items = {(1300- 1263) (7-3) 24 – 344-65}12 = Rs. 37716.

The errors of demand, over stocking costs and under stocking costs for a predicted demand of 1265 items are listed below in Table 4.27.

Table 4.27 Profit to the firm based on demand of 1265 items

Actual demand for April.	Predicted demand based on algorithm for April	Error for demand in April	Over stocking cost (Rs)	Under stocking cost (Rs)
1255	1265	-10	40	
1249	1265	-16	64	
1248	1265	-17	68	
1252	1265	-13	52	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1260	1265	-5	20	
1263	1265	-2	8	
1262	1265	-3	12	
1259	1265	-6	24	
1260	1265	-5	20	
1260	1265	-5	20	
1265	1265	0		
1265	1265	0		
1260	1265	-5	20	
1265	1265	0		
1259	1265	-6	24	
1256	1265	-9	36	
1264	1265	-1	4	
1265	1265	0		
1263	1265	-2	8	
1265	1265	0		
1265	1265	0		
		Total	484	

Annual savings by adopting a demand of 1265 items = {(1300- 1265) (7-3) 24 – 484}12 = Rs. 34512.

4.5 Result

The single point estimate of demand is predicted for the combination of naive and Markov method, ES and Markov method and MA with Markov method. For each combination the state with maximum probability is identified and the profits corresponding to various demands of this state are determined. The obtained results are furnished below.

4.5.1 Naive and Markov method

The maximum probability is for state 3 and 5 with errors -2,-1 and 2, 3. The corresponding demands are 1248, 1255, 1263 and 1265. Predicted demand and corresponding annual savings are provided in the Table 4.28.

Table 4.28 Predicted demand & annual savings for naive & Markov method

Demand	Annual savings compared to existing method
1248	Rs. 46128
1255	Rs. 42972
1263	Rs. 37716
1265	Rs. 34512

4.5.2 Exponential and Markov method

The maximum probability is for state 3 with errors 0, 1 and 2. The corresponding demands are 1260, 1262 and 1265. Predicted demand and corresponding annual savings are provided in the Table 4.29.

Table 4.29 Predicted demand & annual savings for exponential & Markov method

Demand	Annual savings compared to existing method
1260	Rs. 41496
1262	Rs. 39048
1265	Rs. 34512

4.5.3 Moving average and Markov method

The maximum probability is for state 3 with errors -2,-1, 0 and 1. The corresponding demands are 1260, 1262, 1263 and 1265. Predicted demand and corresponding annual savings are provided in the Table 4.30.

Table 4.30 Predicted demand & annual savings for moving average & Markov method

Demand	Annual savings compared to existing method
1260	Rs. 41496
1262	Rs. 39048
1263	Rs. 37716
1265	Rs. 34512

4.6 Conclusion

The single point estimate of demand is determined for the combination of naive and Markov method, exponential and Markov method and moving average with Markov method for the baked product A of relatively large quantity. For each combination the state with maximum probability is identified and the profits corresponding to various demand of this state are determined and are compared. The maximum profit is for naive and Markov combination which predicts a demand of 1248 items for which annual savings when compared to existing method is Rs 46128. The above concept can be extended to more products and annual savings to the firm can be multiplied.

Chapter

5 APPLICATION OF MARKOV BASED MODEL TO THE BAKED PRODUCT B

Contents

5.1 Introduction
5.2 Forecasts based on naive method
5.3 Forecasts based on basic exponential smoothing
5.4 Forecasts based on moving average method
5.5 Results
5.6 Conclusion

5.1 Introduction

This chapter deals with the application of the developed algorithm to a baked product B of relatively small quantity. The details of the collected data are discussed in the beginning. Then the proposed algorithm is applied to the collected data. This is done by applying naive & Markov method, exponential smoothing & Markov method and moving average & Markov method to the data collected. Thereafter the demand corresponding to the state with maximum probability is determined and the corresponding profit is estimated. These obtained profits are compared and the combination with maximum profit is then identified and is listed. The chapter is concluded with the validation of the model.

5.1.1 Implementation of the proposed algorithm for baked product B

The sales data of a second novel product from a reputed firm was collected for two months. The firm is selling this item at Rs. 30. Any leftover item is discarded. Cost of each item is Rs. 16. Markov based algorithm is then applied for these errors. This actual data collected from the reputed baking firm is furnished below (Table 5.1). Holidays and Sundays are excluded resulting in data for twenty one days

Table 5.1 Demand data for October and November

Sl.No	Sales(Oct)	Discarded	Production	Sales(Nov)	Discarded	Production
1	34	11	45	35	12	47
2	37	8	45	38	6	44
3	39	6	45	38	5	43
4	29	16	45	35	11	46
5	36	10	46	37	9	46
6	11	14	25	17	6	23
7	15	10	25	13	9	22
8	37	10	47	33	10	43
9	39	6	45	33	13	46
10	38	11	49	36	11	47
11	34	14	48	36	11	47
12	37	8	45	37	9	46
13	16	8	24	34	10	44
14	12	13	25	22	3	25
15	41	5	46	30	6	36
16	37	11	48	35	8	43
17	39	9	48	34	10	44
18	32	16	48	33	12	45
19	38	12	50	18	6	24
20	15	9	24	19	7	26
21	14	8	22	16	6	22

5.2 Forecasts based on naive method

The forecasts obtained by applying naive method to collected data as discussed in section 3.2.1 and the corresponding errors of demand are listed below (Table 5.2).

Table 5.2 Forecasts for October and November based on naive method

SL.NO	Demand October	Demand November	Forecast October	Forecast November	Error October	Error November
1	34	35				
2	37	38	34	35	3	3
3	39	38	37	38	2	0
4	29	35	39	38	-10	-3
5	36	37	29	35	7	2
6	11	17	36	37	-25	-20
7	15	13	11	17	4	-4
8	37	33	15	13	22	20
9	39	33	37	33	2	0
10	38	36	39	33	-1	3
11	34	36	38	36	-4	0
12	37	37	34	36	3	1
13	16	34	37	37	-21	-3
14	12	22	16	34	-4	-12
15	41	30	12	22	29	8
16	37	35	41	30	-4	5
17	39	34	37	35	2	-1
18	32	33	39	34	-7	-1
19	38	18	32	33	6	-15
20	15	19	38	18	-23	1
21	14	16	15	19	-1	-3

Figure 5.1 provides actual demand for October and November whereas Figure 5.2 gives the forecasts of demand for October and November based on naive method. Figure 5.3 gives the errors in forecasted demand for October and November.

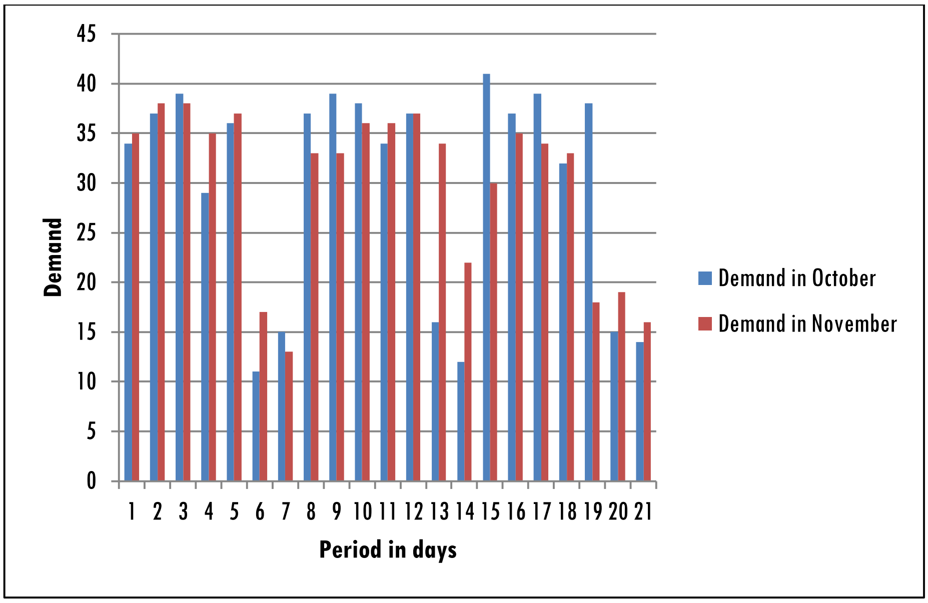

Figure 5.1 Demand plot for October and November

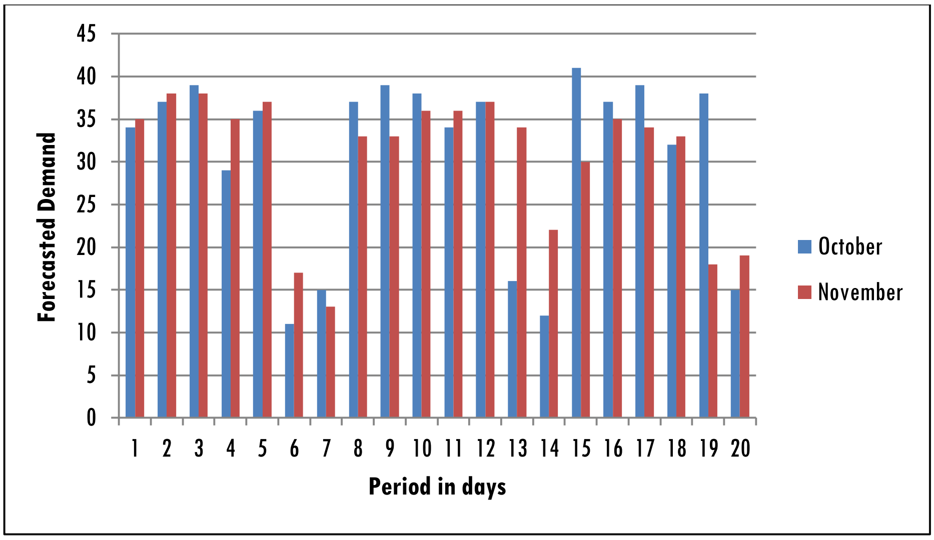

Figure 5.2 Forecast chart for October and November

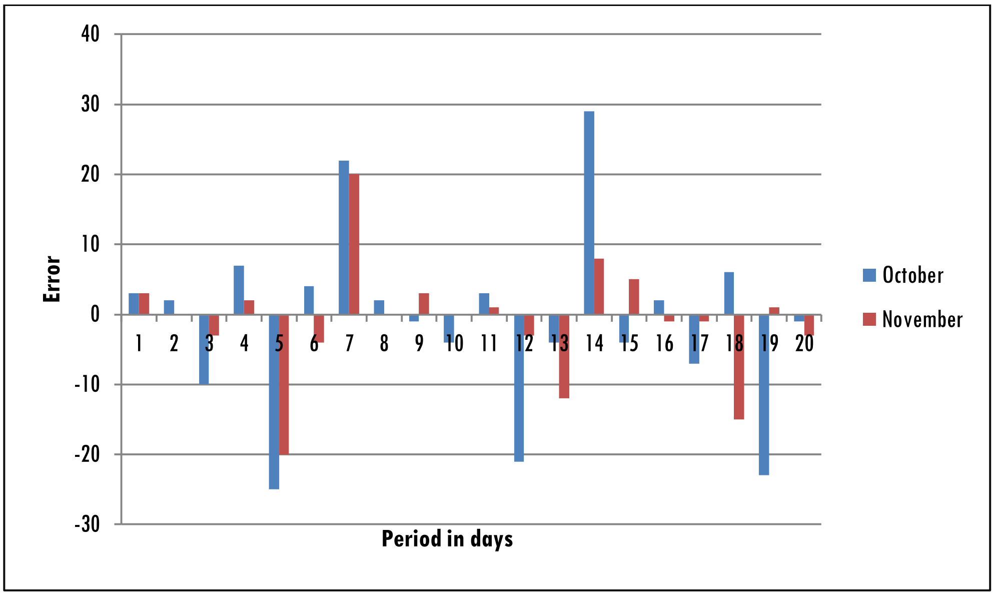

Figure 5.3 Error in demand for October and November

5.2.1 Control charts for errors based on naive method

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2*S* and +2*S*.

$$S = (MSE)^{.5} \qquad (5.1)$$

$$UCL = (0+2S) \qquad (5.2)$$

$$LCL = (0-2S) \qquad (5.3)$$

$$MSE = (\sum e^2/(n-1)) \qquad (5.4)$$

Where *e* is the error and *n* is the sample size.

For October, S = 12.75

2S = 25.5

UCL = (0 + 25.5)

LCL = (0 – 25.5)

Figure 5.4 represents the control chart of errors in demand for October

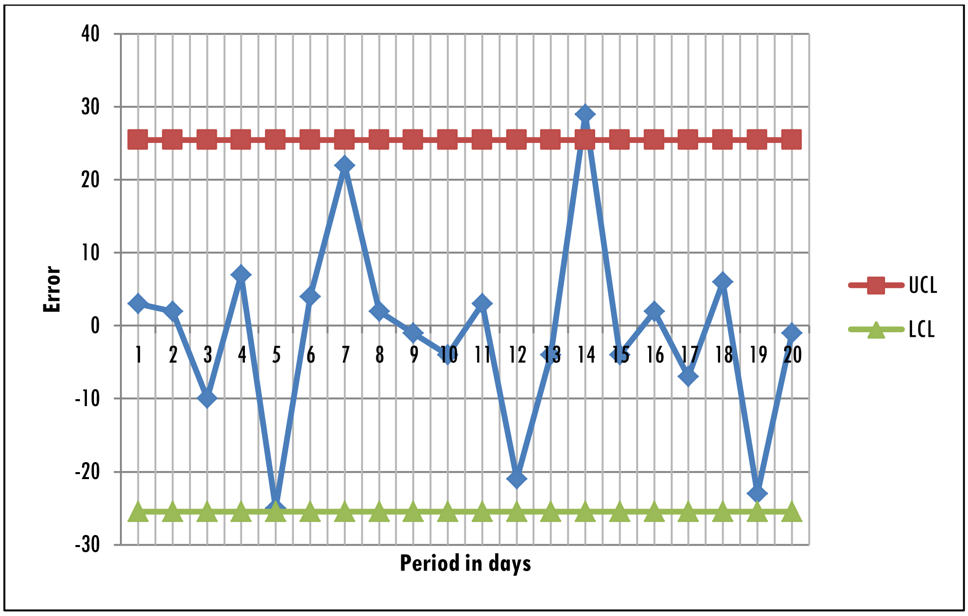

Figure 5.4 Control chart of error in demand for October

The error corresponding to 15th day is out of control.

For November, S = 8.15

2S = 16.3

UCL = (0 + 16.3)

LCL = (0 – 16.3)

Figure 5.5 represents the control chart of errors in demand for November.

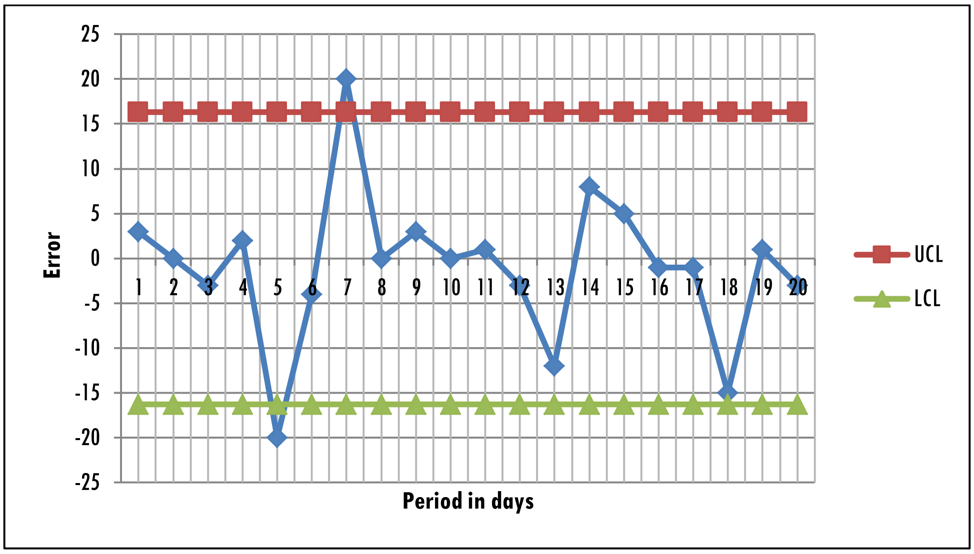

Figure 5.5 Control chart of error in demand for November

The errors corresponding to 6th day and 8th day are out of control.

5.2.2 Initial probability matrix based on naive method

The initial probability matrix is determined as per the step seven of the algorithm mentioned in section 3.5 and is furnished below (Table 5.3).

Table 5.3 Initial probability matrix

Class interval of error of demand	State	No of occurrence	Probability
-23,-22,-21,-20	X_1	2	0.117647
-19,-18,-17,-16	X_2	0	0
-15,-14,-13,-12	X_3	0	0
-11,-10, -9,-8	X_4	1	0.058824
-7,-6, -5,-4	X_5	4	0.235294
-3,-2-1,0	X_6	2	0.117647
1,2, 3,4	X_7	6	0.352941
5,6, 7,8	X_8	2	0.117647

The fourth column of the above table gives initial probability vector P^0 for the month October. This matrix gives the initial probability of all states say $X_{1,} X_{2\ldots} X_8$ in the month of October.

P^0 = [0.117647, 0, 0, 0.058824, 0.235294, 0.117647, 0.352941, 0.117647]

5.2.3 State transition table based on naive method

The state transition table is constructed as prescribed in the eighth step of algorithm given in section 3.5 and is furnished in the Table 5.4 below.

Table 5.4 State transition table

Day	Current state (October)	Subsequent state (November)
2	7	7
3	7	6
4	4	6
5	8	7
7	7	5
9	7	6
10	6	7
11	5	6
12	7	7
13	1	6
14	5	3
16	5	8
17	7	6
18	5	6
19	8	3
20	1	7
21	6	6

5.2.4 Transition probability matrix based on naive method

The obtained transition probability matrix based on the step nine of the algorithm discussed in section 3.5 is furnished below (Table 5.5).

Table 5.5 Transition probability matrix

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0.5	0.5	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	0	0
5	0	0	0.25	0	0	0.5	0	0.25
6	0	0	0	0	0	0.5	0.5	0
7	0	0	0	0	0.167	0.5	0.333	0
8	0	0	0.5	0	0	0	0.5	0

5.2.5 Probability matrix for the succeeding periods based on naive method

The probability matrix for succeeding months of different states as discussed in the tenth step of algorithm in section 3.5 is given below (Table 5.6).

Table 5.6 Probability matrix for succeeding months

P^0	0.1176	0	0	0.0588	0.2353	0.1176	0.3529	0.1176
P^1	0	0	0.1176	0	0.0589	0.4706	0.2940	0.0588
P^2	0	0	0.0441	0	0.0491	0.4118	0.3626	0.0147
P^3	0	0	0.0196	0	0.0606	0.4117	0.3340	0.0123
P^4	0	0	0.0213	0	0.0558	0.4031	0.3232	0.0151
P^5	0	0	0.0215	0	0.0540	0.3911	0.3168	0.0139
P^6	0	0	0.0205	0	0.0529	0.3809	0.3080	.0135
P^7	0	0	0.0195	0	0.0501	0.3611	0.2919	0.0129
P^8	0	0	0.0189	0	0.0487	0.3515	0.2842	0.0125
P^9	0	0	0.0184	0	0.0475	0.3422	0.2766	0.0122
P^{10}	0	0	0.0180	0	0.0462	0.3331	0.2693	0.0119

The maximum probability is for state six and corresponding demands are 14 & 38 items.

Figure 5.6 represents the probability time graph for the sixth state.

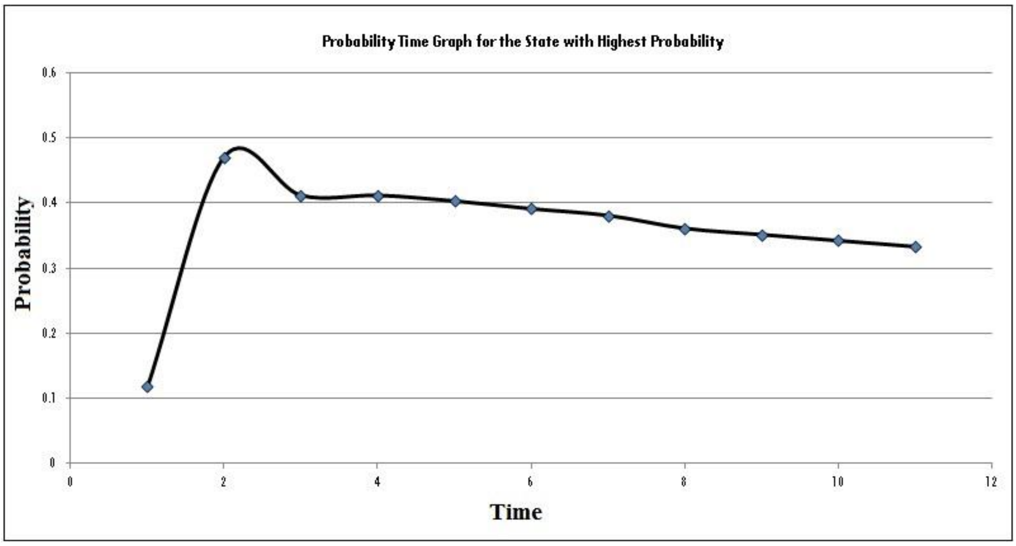

Figure 5.6 Probability time graph

5.2.6 Annual savings for various predicted demands based on naive

The profit per day by adopting the forecasted demand obtained as per the algorithm given in section 3.5 is furnished below

The parameters affecting the profit per day are, *S* the actual demands for a day, *P* the selling price, *D* the discarded items per day and *C* the cost of making unit quantity.

The term *(P-C)* gives the profit of a sold item and $(P - C) \times S$ provides the gross profit of a day. The term $D \times C$ gives the total cost of the discarded items of a day. Total cost of discarded items is deducted from gross profit to obtain the net profit per day.

The profit per day is estimated by using the generalized formulae

$$[S \times (P\text{-}C) - (D \times C)] \qquad (5.5)$$

Profits to the firm from existing forecast are listed in the Table 5.7 given below.

Table 5.7 Profits to the firm from existing forecast

Sl.No	Actual Sales (S)	Forecasted demand (F)	Discarded items D = (F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	45	11	300
2	37	45	8	390
3	39	45	6	450
4	29	45	16	150
5	36	46	10	344
6	11	25	14	-70
7	15	25	10	50
8	37	47	10	358
9	39	45	6	450
10	38	49	11	356
11	34	48	14	252
12	37	45	8	390
13	16	24	8	96
14	12	25	13	-40
15	41	46	5	494
16	37	48	11	342
17	39	48	9	402
18	32	48	16	192
19	38	50	12	340
20	15	24	9	66
21	14	22	8	68
				RS 5380

The annual profit for the product is Rs 5380 x 12 = Rs. 64560

Here, discarded items have positive values and hence there is no under stocking. The under stocking is not considered in further analysis and whenever a negative value appears in the cell of discarded item, it is taken as zero.

Profits to the firm for a predicted demand of 14 items are given in the Table 5.8.

Table 5.8 Profits to the firm when forecast is 14 items per day

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	14	0	196
2	37	14	0	196
3	39	14	0	196
4	29	14	0	196
5	36	14	0	196
6	11	14	3	106
7	15	14	0	196
8	37	14	0	196
9	39	14	0	196
10	38	14	0	196
11	34	14	0	196
12	37	14	0	196
13	16	14	0	196
14	12	14	2	136
15	41	14	0	196
16	37	14	0	196
17	39	14	0	196
18	32	14	0	196
19	38	14	0	196
20	15	14	0	196
21	14	14	0	196
				3966

The annual profit for the product is Rs 3966 x 12 = Rs. 47592

Profits to the firm for a forecasted demand of 38 items are furnished in the Table 5.9.

Table 5.9 Profits to the firm when forecast is 38 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	38	4	412
2	37	38	1	502
3	39	38	0	532
4	29	38	9	262
5	36	38	2	472
6	11	38	27	-278
7	15	38	23	-158
8	37	38	1	502
9	39	38	0	532
10	38	38	0	532
11	34	38	4	412
12	37	38	1	502
13	16	38	22	-128
14	12	38	26	-248
15	41	38	0	532
16	37	38	1	502
17	39	38	0	532
18	32	38	6	352
19	38	38	0	532
20	15	38	23	-158
21	14	38	24	-188
				5952

The annual profit for the product is Rs 5952 x 12 = Rs. 71424.

5.3 Forecasts based on basic exponential smoothing

The forecasts of the collected demand based on the basic exponential smoothing as discussed in section 3.2.2 and the corresponding errors of forecasts are given below (Table 5.10).

Table 5.10 Forecasts based on exponential smoothing.

Sl.No	Demand October	Demand November	Forecast October	Forecast November	Error October	Error November
1	34	35	30	30	4	5
2	37	38	31	30	6	8
3	39	38	31	30	8	8
4	29	35	32	30	-3	5
5	36	37	30	31	6	6
6	11	17	31	31	-20	-14
7	15	13	26	31	-11	-18
8	37	33	27	30	10	3
9	39	33	31	29	8	4
10	38	36	32	29	6	7
11	34	36	32	31	2	5
12	37	37	31	32	6	5
13	16	34	31	33	-15	1
14	12	22	27	33	-15	-11
15	41	30	26	31	15	-1
16	37	35	32	31	5	4
17	39	34	31	32	8	2
18	32	33	32	32	0	1
19	38	18	30	32	8	-14
20	15	19	32	29	-17	-10
21	14	16	27	27	-13	-11

Figure 5.7 provides actual demand for October and November whereas Figure 5.8 gives the forecasts of demand for October and November based on exponential smoothing method. Figure 5.9 gives the errors in forecasted demand for October and November.

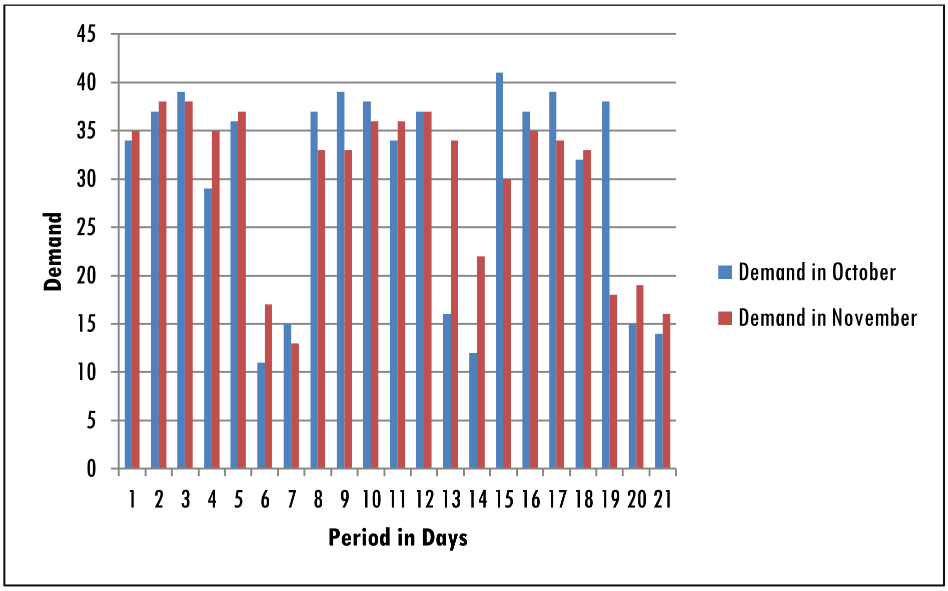

Figure 5.7 Demand chart for October & November

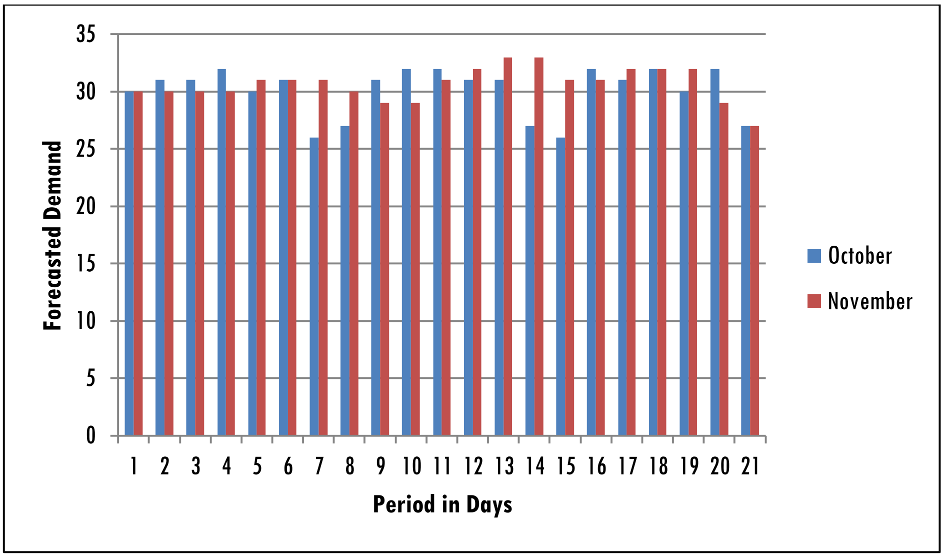

Figure 5.8 Forecast chart for October & November

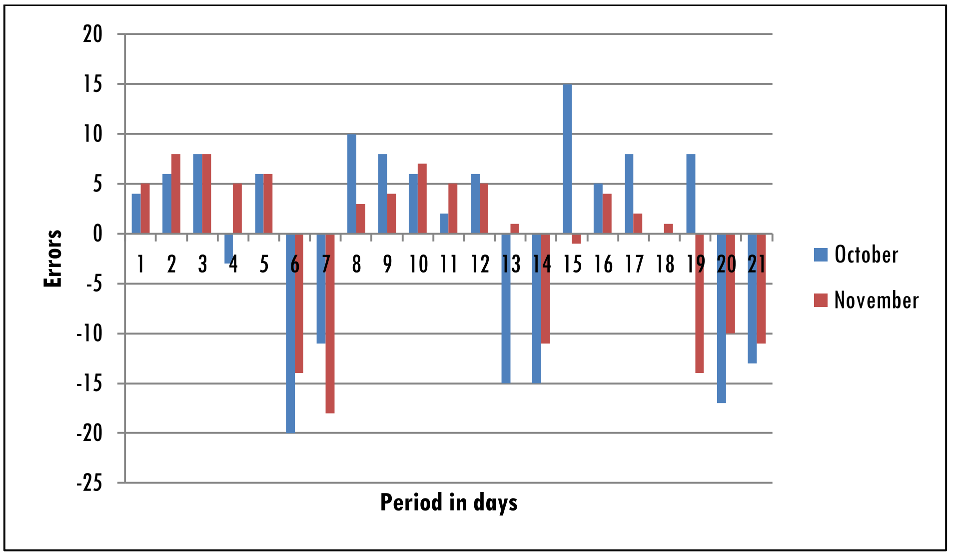

Figure 5.9 Error chart for October & November

5.3.1 Control charts for errors based on basic exponential smoothing

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2S and +2S. For October,

S = 10.25

2S = 20.5

UCL = (0 + 20.5)

LCL = (0 – 20.5)

Figure 5.10 represents the control chart of errors in demand for October.

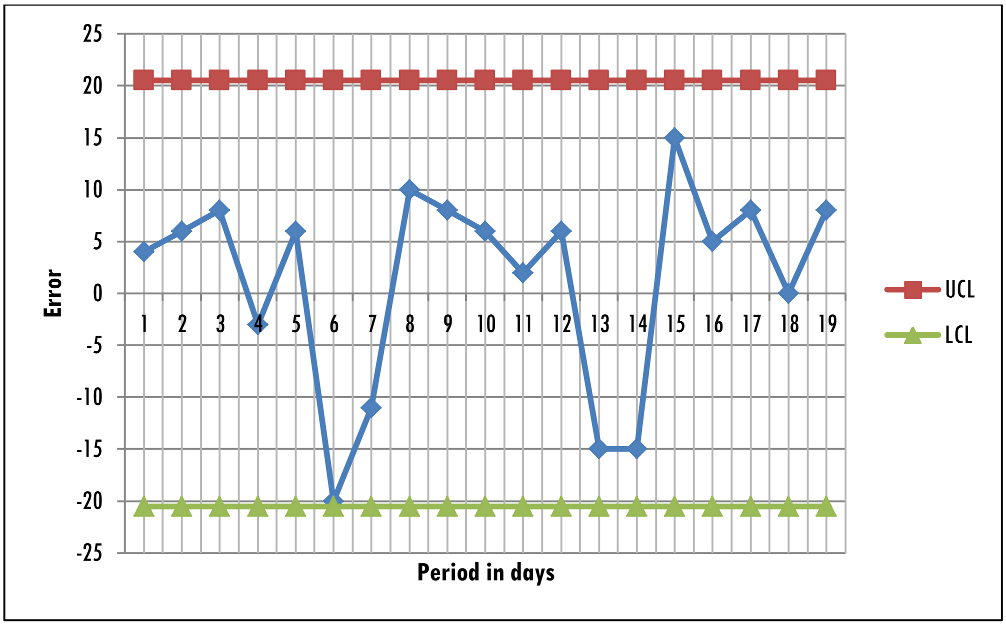

Figure 5.10 Control chart of error in demand based on exponential smoothing (October)

For November,

S	=	8.42
2S	=	16.84
UCL	=	(0 + 16.84)
LCL	=	(0 – 16.84)

Figure 5.11 represents the control chart of errors in demand for November.

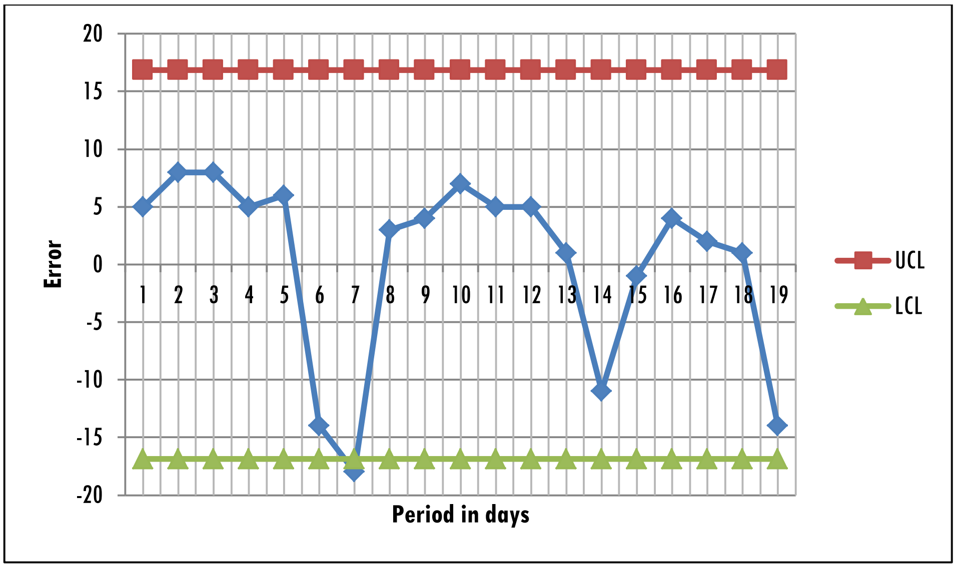

Figure 5.11 Control chart of error in demand based on exponential smoothing (November)

5.3.2 Initial probability matrix based on basic exponential smoothing

The initial probability matrix determined as per the step seven of the algorithm mentioned in section 3.5 is provided below (Table 5.11).

Table 5.11 Initial probability matrix

Class interval of error of demand	State	No of occurrence	Probability
-20,-19,-18,-17	X_1	2	0.1
-16,-15,-14,-13	X_2	3	0.15
-12, -11,-10, -9	X_3	0	0
-8, -7,-6, -5	X_4	0	0
-4-3,-2-1	X_5	1	0.05
0, 1,2, 3	X_6	2	0.1
4,5,6, 7	X_7	6	0.3
8,9,10,11	X_8	5	0.25
12,13,14,15	X_9	1	0.05

The fourth column of the above table gives initial probability vector P^0 for the month October. This matrix gives the initial probability of all states say $X_{1,} X_{2\ldots} X_{12}$ in the month of October.

P^0 = [0.1, 0.15, 0, 0, 0.05, 0.1, 0.3, 0.25, 0.05]

5.3.3 State transition table based on basic exponential smoothing

The state transition table is constructed as prescribed in the step eight of the algorithm in section 3.5 and is furnished below (Table 5.12).

Table 5.12 State transition table

Day	Current state (October)	Subsequent state (November)
1	7	7
2	7	8
3	8	8
4	5	7
5	7	7
6	1	2
8	8	6
9	8	7
10	7	7
11	6	7
12	7	7
13	2	6
14	2	3
15	9	5
16	7	7
17	8	6
18	6	6
19	8	2
20	1	3
21	2	3

5.3.4 Transition probability matrix based on basic exponential smoothing

The obtained transition probability matrix based on the step nine of the algorithm in section 3.5 is furnished below (Table 5.13).

Table 5.13 Transition probability matrix

	1	2	3	4	5	6	7	8	9
1	0	0	1	0	0	0	0	0	0
2	0	0	0.67	0	0	0.33	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0.5	0.5	0	0
7	0	0	0	0	0	0	0.833	0.167	0
8	0	0.2	0	0	0	0.4	0.2	0.2	0
9	0	0	0	0	1	0	0	0	0

5.3.5 Probability matrix for succeeding months based on basic exponential smoothing

The probability matrix for succeeding months of different states as discussed in step ten of algorithm in section 3.5 is given below.

Table 5.14 Probability matrix for succeeding months

P^0	0.1	0.15	0	0	0.05	0.1	0.30	.25	0.05
P^1	0	0.05	0.25	0	0.05	0.15	0.399	0.10	0
P^2	0	0.02	0.05	0	0	0.115	0.478	0.087	0
P^3	0	0.017	0.020	0	0	0.092	0.473	0.097	0
P^4	0	0.019	0.017	0	0	0.085	0.459	0.098	0
P^5	0	0.019	0.019	0	0	0.081	0.445	0.097	0
P^6	0	0.019	0.021	0	0	0.079	0.431	0.094	0

The maximum probability is for state seven and the corresponding demands are 34, 36, 37 & 38.

Figure 5.12 represents the probability time graph for the seventh state

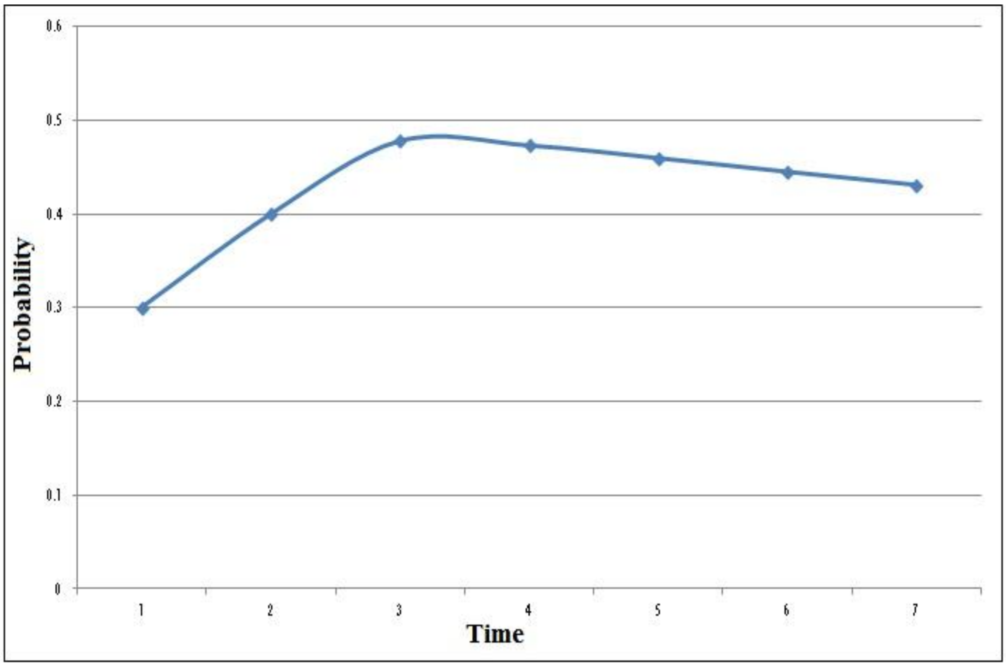

Figure 5.12 Probability time graph for the state with highest probability

5.3.6 Annual savings for various predicted demands based on basic exponential smoothing

The profit is estimated by using the generalized formulae stated as equation 5.5.

Profits to the firm from existing forecast are listed in the Table 5.15 given below. Here, since discarded items have positive values there is no under stocking. The under stocking is not considered in further analysis and whenever a negative value appears in the cell of discarded item, it is taken as zero.

Table 5.15 Profits to the firm from existing forecast

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	45	11	300
2	37	45	8	390
3	39	45	6	450
4	29	45	16	150
5	36	46	10	344
6	11	25	14	-70
7	15	25	10	50
8	37	47	10	358
9	39	45	6	450
10	38	49	11	356
11	34	48	14	252
12	37	45	8	390
13	16	24	8	96
14	12	25	13	-40
15	41	46	5	494
16	37	48	11	342
17	39	48	9	402
18	32	48	16	192
19	38	50	12	340
20	15	24	9	66
21	14	22	8	68
				RS 5380

The annual profit for the product is Rs 5380 x 12 = Rs 64560

Profits to the firm for a forecast of 34 items are listed in the Table 5.16 given below.

Table 5.16 Profits to the firm for a forecast of 34 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	34	0	476
2	37	34	-3	476
3	39	34	-5	476
4	29	34	5	326
5	36	34	-2	476
6	11	34	23	-214
7	15	34	19	-94
8	37	34	-3	476
9	39	34	-5	476
10	38	34	-4	476
11	34	34	0	476
12	37	34	-3	476
13	16	34	18	-64
14	12	34	22	-184
15	41	34	-7	476
16	37	34	-3	476
17	39	34	-5	546
18	32	34	2	416
19	38	34	-4	476
20	15	34	19	-94
21	14	34	20	-124
				6226

The annual profit for the product is Rs 6226 x 12 = Rs. 74712.

Profits to the firm for a forecast of 36 items are listed in the Table 5.17 given below.

Table 5.17 Profits to the firm for a forecast of 36 items

SL.NO	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	36	2	444
2	37	36	-1	504
3	39	36	-3	504
4	29	36	7	294
5	36	36	0	476
6	11	36	25	-246
7	15	36	21	-126
8	37	36	-1	504
9	39	36	-3	504
10	38	36	-2	504
11	34	36	2	444
12	37	36	-1	504
13	16	36	20	-96
14	12	36	24	-216
15	41	36	-5	504
16	37	36	-1	504
17	39	36	-3	546
18	32	36	4	384
19	38	36	-2	504
20	15	36	21	-126
21	14	36	22	-156
				6158

The annual profit for the product is Rs 6158 x 12 = Rs. 73896

Profits to the firm for a forecast of 37 items are listed in the Table 5.18 given below.

Table 5.18 Profits to the firm for a forecast of 37 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	37	3	428
2	37	37	0	518
3	39	37	-2	518
4	29	37	8	278
5	36	37	1	488
6	11	37	26	-262
7	15	37	22	-142
8	37	37	0	518
9	39	37	-2	518
10	38	37	-1	518
11	34	37	3	428
12	37	37	0	518
13	16	37	21	-112
14	12	37	25	-232
15	41	37	-4	518
16	37	37	0	518
17	39	37	-2	518
18	32	37	5	368
19	38	37	-1	518
20	15	37	22	-142
21	14	37	23	-172
				6108

The annual profit for the product is Rs 6108 x 12 = Rs. 73296

Profits to the firm for a forecast of 38 items are listed in the Table 5.19 given below.

Table 5.19 Profits to the firm for a forecast of 38 items.

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	38	4	412
2	37	38	1	502
3	39	38	-1	532
4	29	38	9	262
5	36	38	2	472
6	11	38	27	-278
7	15	38	23	-158
8	37	38	1	502
9	39	38	-1	532
10	38	38	0	532
11	34	38	4	412
12	37	38	1	502
13	16	38	22	-128
14	12	38	26	-248
15	41	38	-3	532
16	37	38	1	502
17	39	38	-1	532
18	32	38	6	352
19	38	38	0	532
20	15	38	23	-158
21	14	38	24	-188
				5952

The annual profit for the product is Rs 5952 x 12 = Rs. 71424

5.4 Forecasts based on moving average method

The forecasts based on moving average method as discussed in section 3.2.3 and the corresponding error of forecast is also listed below (Table 5.20).

Table 5.20 Forecasts for October and November based on moving average method.

Actual demand October	Actual demand November	Forecasted demand October	Forecasted demand November	Error in demand for each day of October	Error in demand for each day of November
34	35				
37	38				
39	38	37	37	2	1
29	35	35	37	-6	-2
36	37	35	37	1	0
11	17	25	30	-14	-13
15	13	21	22	-6	-9
37	33	21	21	16	12
39	33	30	26	9	7
38	36	38	34	0	2
34	36	37	35	-3	1
37	37	36	36	1	1
16	34	29	36	-13	-2
12	22	22	31	-10	-9
41	30	23	29	18	1
37	35	30	29	7	6
39	34	39	33	0	1
32	33	36	34	-4	-1
38	18	36	28	2	-10
15	19	28	23	-13	-4
14	16	22	18	-8	-2

Figure 5.13 provides actual demand for October and November whereas Figure 5.14 gives the forecasts of demand for October and November based on moving average method. Figure 5.15 gives the errors in forecasted demand for October and November

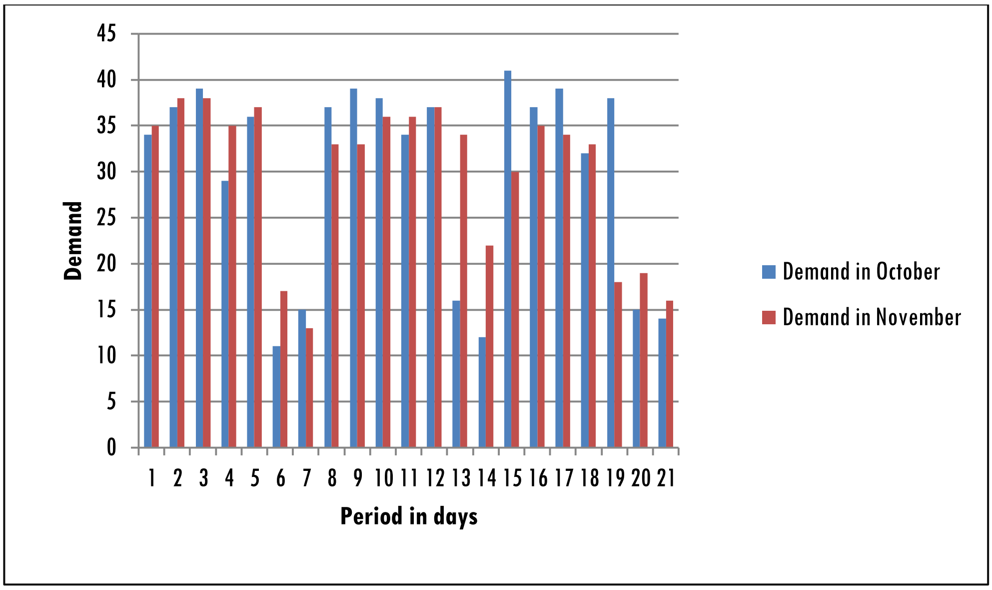

Figure 5.13 Demand chart for October & November

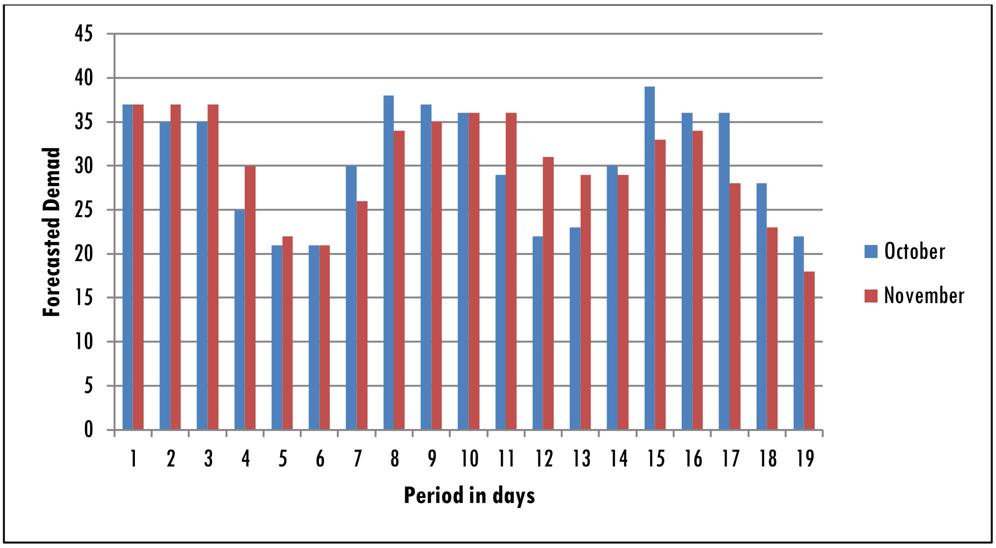

Figure 5.14 Forecast charts for October & November

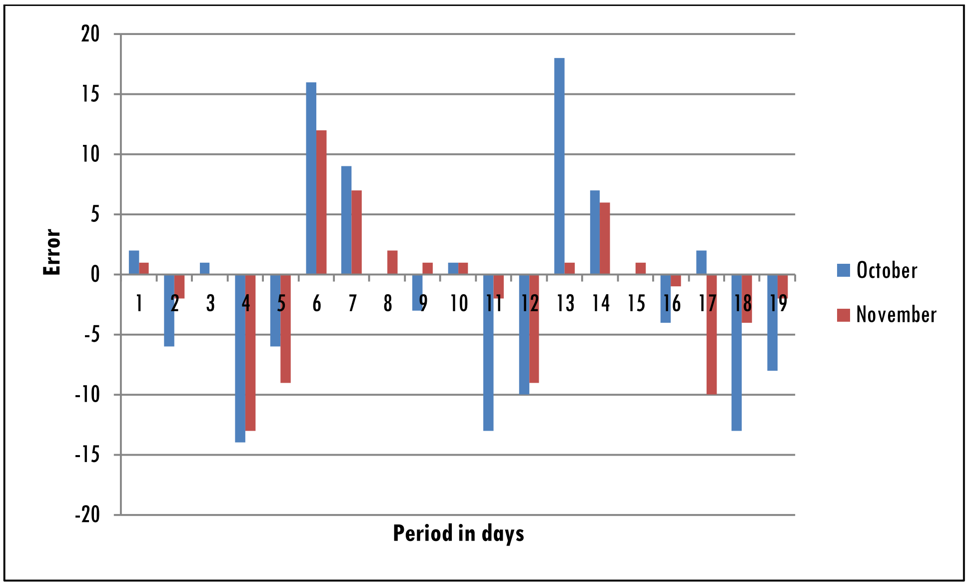

Figure 5.15 Error chart for October & November

5.4.1 Control charts based on moving average method

MSE is at first computed for plotting the control chart. The square root of MSE is used in practice as an estimate of the standard deviation of the distribution of errors. Control charts are designed on the assumption that when errors are random, they will be distributed according to a normal distribution around a mean of zero. For a normal distribution, approximately 95.5 percent of the values (errors in this case) can be expected to fall within limits -2S and +2S.

For October, S = 9.17

S = 2MSE = 18.34

UCL = (0 + 18.34)

LCL = (0 – 18.34)

Figure 5.16 represents the control chart of errors in demand for October.

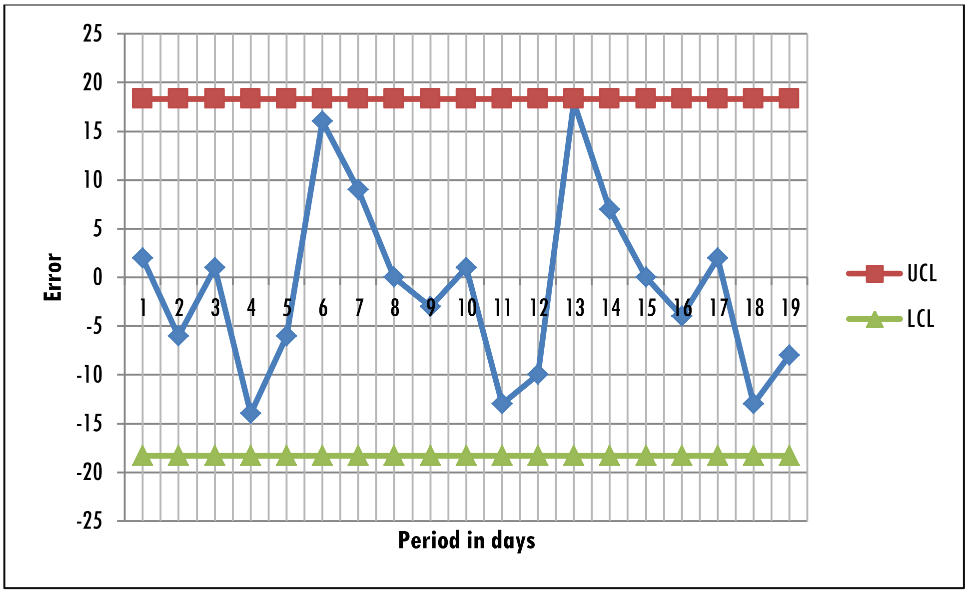

Figure 5.16 Control chart for error in demand for October

For November,

S = 6.22

2S = 12.44

UCL = (0 + 12.44)

LCL = (0 – 12.44)

Figure 5.17 represents the control chart of errors in demand for November

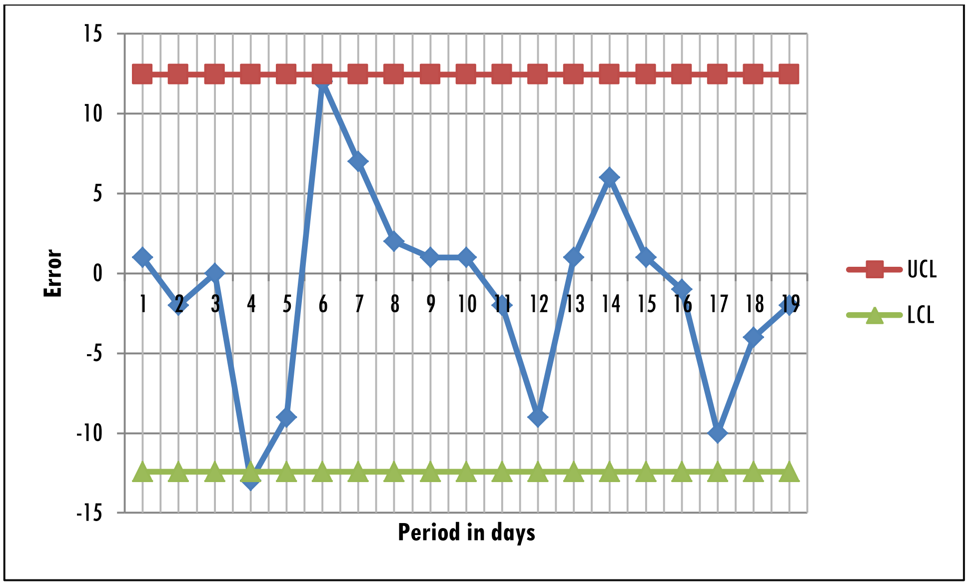

Figure 5.17 Control chart for error in demand for November

The demand corresponding to 4th day is out of control.

5.4.2 Initial probability matrix based on moving average

The initial probability matrix obtained as per the step seven of algorithm in 3.5 is given below (Table 5.21).

Table 5.21 Initial probability matrix

Class interval of error of demand	State	No of occurrence	Probability
-13,-12,-11,-10	X_1	3	0.166667
-9,-8,-7,-6	X_2	3	0.166667
-5,-4,-3,-2	X_3	2	0.111111
-1,0,1, 2	X_4	6	0.333333
3,4,5,6	X_5	0	0
7,8,9,10	X_6	2	0.111111
11,12,13,14	X_7	0	0
15,16,17,18	X_8	2	0.111111

The fourth column of the above table gives initial probability vector P^0 for the month October. This matrix gives the initial probability of all states say $X_1, X_2 \ldots X_{12}$ in the month of October.

$$P^0 = [0.166667, 0.166667, 0.111111, 0.333333, 0, 0.111111, 0, 0.111111]$$

5.4.3 State transition table based on moving average

The state transition table is constructed as prescribed in the step eight of the algorithm in 3.5 and is furnished below.

Table 5.22 State transition table

Day	Current state (October)	Subsequent state (November)
3	4	4
4	2	3
5	4	4
7	2	2
8	8	7
9	6	6
10	4	4
11	3	4
12	4	4
13	1	3
14	1	2
15	8	4
16	6	5
17	4	4
18	3	4
19	4	1
20	1	3
21	2	3

5.4.4 Transition probability matrix based on moving average

The obtained transition probability matrix based on the step nine of the algorithm in section 3.5 is furnished below.

Table 5.23 Transition probability matrix

	1	2	3	4	5	6	7	8
1	0	0.3333	0.6667	0	0	0	0	0
2	0	0.3333	0.6667	0	0	0	0	0
3	0	0	0	1	0	0	0	0
4	0.1667	0	0	0.8333	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0.5	0.5	0	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0.5	0	0	0.5	0

5.4.5 Probability matrix for succeeding months based on moving average

The probability matrix for succeeding months of different states as discussed in tenth step of the algorithm in section 3.5 is given below in Table 5.24.

Table 5.24 probability matrix for succeeding months

P^0	0.1667	0.1667	0.1111	0.3333	0	0.1111	0	0.1111
P^1	0.0556	0.1111	0.2222	0.4444	0.0556	0.0556	0.0556	0
P^2	0.0741	0.0556	0.1111	0.5926	0.0278	0.0278	0	0
P^3	0.0988	0.0432	0.0864	0.6049	0.0139	0.0139	0	0
P^4	0.0988	0.0473	0.0947	0.5905	0.0069	0.0069	0	0
P^5	0.0984	0.0494	0.0988	0.5867	0.0035	0.0035	0	0
P^6	0.0984	0.0494	0.0988	0.5867	0.0035	0.0035	0	0
P^7	0.0978	0.0493	0.0986	0.5877	0.0017	0.0017	0	0
P^8	0.0980	0.0490	0.0981	0.5883	0.0009	0.0009	0	0
P^9	0.0981	0.0490	0.0980	0.5883	0.0004	0.0004	0	0
P^{10}	0.0981	0.0490	0.0980	0.5882	0.0002	0.0002	0	0
P^{11}	0.0981	0.0490	0.0981	0.5882	0.0001	0.0001	0	0

The maximum probability is for state four and the corresponding demands are 36, 37, 38 & 39.

Figure 5.18 represents the probability time graph for the state four.

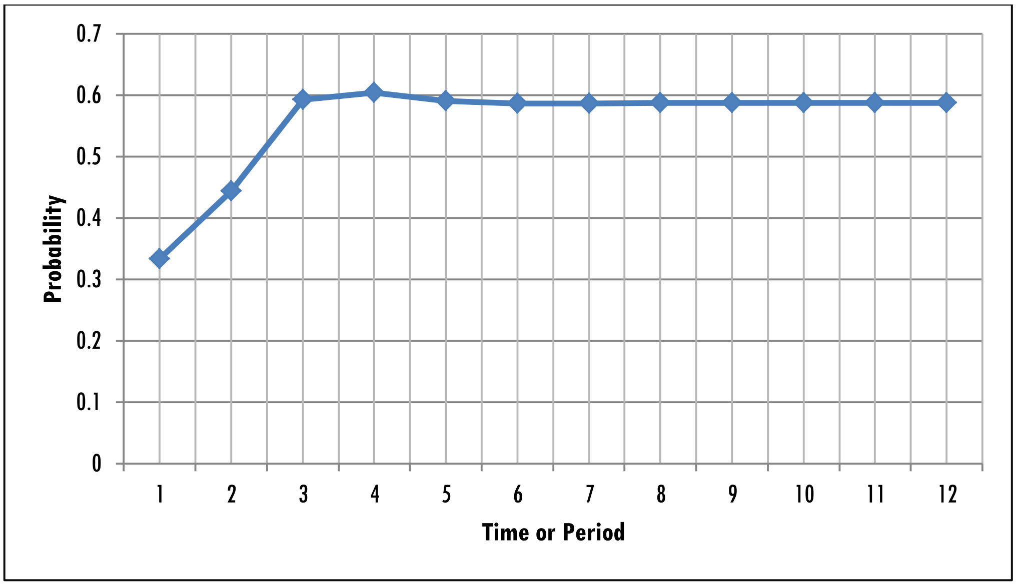

Figure 5.18 Probability time graph for the state with highest probability

5.4.6 Annual savings for various predicted demands based on moving average

The profit is estimated by using the generalized formulae stated as equation 5.5.

Profits to the firm from existing forecast are listed in the Table 5.25 given below. Here since discarded items have positive values there is no under stocking. The under stocking is not considered in further analysis and whenever a negative value appears in the cell of discarded item, it is taken as zero.

Table 5.25 Profits to the firm from existing forecast

SL.NO	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	45	11	300
2	37	45	8	390
3	39	45	6	450
4	29	45	16	150
5	36	46	10	344
6	11	25	14	-70
7	15	25	10	50
8	37	47	10	358
9	39	45	6	450
10	38	49	11	356
11	34	48	14	252
12	37	45	8	390
13	16	24	8	96
14	12	25	13	-40
15	41	46	5	494
16	37	48	11	342
17	39	48	9	402
18	32	48	16	192
19	38	50	12	340
20	15	24	9	66
21	14	22	8	68
				RS 5380

The annual profit for the product is Rs. 5380 x 12 = Rs. 64560

For the above product maximum probability is for state four and the corresponding demand are 36, 37, 38 and 39.

Profits to the firm for a forecast of 36 items are listed in the Table 5.26 given below

Table 5.26 Profits to the firm for a forecast of 36 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	36	2	444
2	37	36	-1	504
3	39	36	-3	504
4	29	36	7	294
5	36	36	0	476
6	11	36	25	-246
7	15	36	21	-126
8	37	36	-1	504
9	39	36	-3	504
10	38	36	-2	504
11	34	36	2	444
12	37	36	-1	504
13	16	36	20	-96
14	12	36	24	-216
15	41	36	-5	504
16	37	36	-1	504
17	39	36	-3	546
18	32	36	4	384
19	38	36	-2	504
20	15	36	21	-126
21	14	36	22	-156
				6158

The annual profit for the product is Rs. 6158 x 12 = Rs. 73896.

Profits to the firm for a forecast of 37 items are listed in the Table 5.27 given below.

Table 5.27 Profits to the firm for a forecast of 37 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	37	3	428
2	37	37	0	518
3	39	37	-2	518
4	29	37	8	278
5	36	37	1	488
6	11	37	26	-262
7	15	37	22	-142
8	37	37	0	518
9	39	37	-2	518
10	38	37	-1	518
11	34	37	3	428
12	37	37	0	518
13	16	37	21	-112
14	12	37	25	-232
15	41	37	-4	518
16	37	37	0	518
17	39	37	-2	518
18	32	37	5	368
19	38	37	-1	518
20	15	37	22	-142
21	14	37	23	-172
				6108

The annual profit for the product is Rs. 6108 x 12 = Rs. 73296.

Profits to the firm for a forecast of 38 items are listed in the Table 5.28 given below

Table 5.28 Profits to the firm for a forecast of 38 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	38	4	412
2	37	38	1	502
3	39	38	-1	532
4	29	38	9	262
5	36	38	2	472
6	11	38	27	-278
7	15	38	23	-158
8	37	38	1	502
9	39	38	-1	532
10	38	38	0	532
11	34	38	4	412
12	37	38	1	502
13	16	38	22	-128
14	12	38	26	-248
15	41	38	-3	532
16	37	38	1	502
17	39	38	-1	532
18	32	38	6	352
19	38	38	0	532
20	15	38	23	-158
21	14	38	24	-188
				5952

The annual profit for the product is Rs. 5952 x 12 = Rs. 69216.

Profits to the firm for a forecast of 39 items are listed in the Table 5.29 given below.

Table 5.29 Profits to the firm for a forecast of 39 items

Sl.No	Actual sales (S)	Forecasted demand (F)	Discarded items (D=F-S)	Profit (RS) [S*(P-C)-D*C]
1	34	39	5	396
2	37	39	2	486
3	39	39	0	546
	29	39	10	246
5	36	39	3	456
6	11	39	28	-294
7	15	39	24	-174
8	37	39	2	486
9	39	39	0	546
10	38	39	1	532
11	34	39	5	396
12	37	39	2	486
13	16	39	23	-144
14	12	39	27	-264
15	41	39	-2	546
16	37	39	2	486
17	39	39	0	546
18	32	39	7	336
19	38	39	1	532
20	15	39	24	-174
21	14	39	25	-204
				5768

The annual profit for the product is Rs. 5768 x 12 = Rs. 69216.

5.5 Results

The single point estimate of demand is determined for the combination of naive and Markov method, exponential and Markov method and moving average with Markov method. For each combination the state with maximum probability is identified and the profits corresponding to various demand of this state are determined. The obtained results are furnished below.

5.5.1 Naive and Markov method

The maximum probability is for state 6 with errors -3, -2,-1and 0.The corresponding demands are 14 and 38. To determine the single point estimate of demand, the annual profit by adopting each of the above demand is determined. The predicted demand with maximum profit is selected. Predicted demand and annual savings for naive and Markov method are shown in Table 5.30

Table 5.30 Predicted demand and annual savings for naive and Markov method

Forecasted demand	Annual savings compared to existing method
14	Rs. -16968
38	Rs. 6864

The predicted demand with maximum savings is 38 items.

5.5.2 Exponential and Markov method

The maximum probability is for state 7 with errors 4, 5, 6 and 7.The corresponding demands are 34, 36, 37 and 38. To determine the single point estimate of demand, the annual profit by adopting each of the above demand is determined. The predicted demand with maximum profit is selected. Predicted demand and annual savings for exponential smoothing and Markov method are shown in Table 5.31.

Table 5.31 Predicted demand and annual savings for exponential & Markov method

Forecasted demand	Annual savings compared to existing method
34	Rs. 10152
36	Rs. 9336
37	Rs. 8736
38	Rs. 6864

The predicted demand with maximum savings is 34 items.

5.5.3 Moving average and Markov method

The maximum probability is for state 4 with errors -1, 0,1and 2.The corresponding demands are 36, 37, 38 and 39. To determine the single point estimate of demand, the annual profit by adopting each of the above demand is determined. The demand with maximum savings is selected. Predicted demand and annual savings for moving average and Markov method are shown in Table 5.32.

Table 5.32 Predicted demand and annual savings table for moving average & Markov method

Forecasted demand	Annual savings compared to existing method
36	Rs. 9336
37	Rs. 8736
38	Rs. 6864
39	Rs. 4656

The predicted demand with maximum savings is 36 items

5.6 Conclusion

The single point estimate of demand is determined for the combination of naive and Markov method, exponential and Markov method and moving average with Markov method for a baked product B of relatively small quantity. For each combination the state with maximum probability is identified and the profits corresponding to various demand of this state are determined, and are compared and maximum profit is for exponential smoothing and Markov combination which predicts a demand of 34 items for which annual savings when compared to existing method is Rs 10152. The above concept can be extended to more products and huge annual savings can be obtained.

Chapter 6

CONCLUSION AND FUTURE RESEARCH

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6.1 Introduction

6.2 Summary of the results

6.3 Research findings.

6.4 Limitations of the study and future scope for research

6.5 Conclusion

6.1 Introduction

The objective of this work was to develop a model or algorithm to predict the demand of relatively novel and short life products. The developed model or algorithm was applied for two novel baked products of a shelf life of only one day and was validated by the excess annual savings to the firm when compared to existing practice. Big enterprises can afford costly forecasting tools. Developed model can help small scale and medium scale enterprises to predict the demand of their novel short life products with minimum expenses. Little literature exists for predicting demand of novel products and hence this model can act as the start up for predicting demand of novel and short life products.

6.2 Summary of the results

The predicted demand as well as annual savings by adopting different combinations when compared to existing practice for both the products are furnished below.

6.2.1 Baked product-A

Naive and Markov method

The maximum probability is for state 3 and 5 with errors -2,-1 and 2, 3. The corresponding demands are 1248, 1255, 1263 and 1265. Table 6.1 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.1 Predicted demand and annual savings for naive and Markov method

Forecasted demand	Annual savings compared to existing method
1248	Rs. 46128
1255	Rs. 42972
1263	Rs. 37716
1265	Rs. 34512

Exponential and Markov method

The maximum probability is for state 3 with errors 0, 1 and 2. The corresponding demands are 1260, 1262 and 1265. Table 6.2 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.2 Predicted demand and annual savings for exponential and Markov method

Forecasted demand	Annual savings compared to existing method
1260	Rs. 41496
1262	Rs. 39048
1265	Rs. 34512

Moving average and Markov method

The maximum probability is for state 3 with errors-2,-1, 0, and 1. The corresponding demands are 1260, 1262, 1263 and 1265. Table 6.3 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.3 Predicted demand and annual savings for moving average and Markov method

Forecasted demand	Annual savings compared to existing method
1260	Rs. 41496
1262	Rs. 39048
1263	Rs. 37716
1265	Rs. 34512

6.2.2 Baked product-B

Naive and Markov method

The maximum probability is for state 6 with errors -3, -2,-1and 0.The corresponding demands are 14 and 38. Table 6.4 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.4 Predicted demand and annual savings for naive and Markov method

Forecasted demand	Annual savings compared to existing method
14	Rs. -16968
38	Rs. 6864

Exponential and Markov method

The maximum probability is for state 7 with errors 4, 5,6and 7.The corresponding demands are 34, 36, 37 and 38. Table 6.5 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.5 Predicted demand and annual savings for exponential and Markov method

Forecasted demand	Annual savings compared to existing method
34	Rs. 10152
36	Rs. 9336
37	Rs. 8736
38	Rs. 6864

Moving average and Markov method

The maximum probability is for state 4 with errors -1, 0,1and 2.The corresponding demands are 36, 37, 38 and 39. Table 6.6 gives the excess annual profit to the firm by adopting the demands furnished above when compared to the existing method.

Table 6.6 Predicted demand and annual savings table for moving average and Markov method

Forecasted demand	Annual savings compared to existing method
36	Rs. 9336
37	Rs. 8736
38	Rs. 6864
39	Rs. 4656

The summary of the results obtained for baked products A & B are furnished below.

For baked product-A

Table 6.7 gives the predicted demand corresponding to maximum annual savings of each combination for the product A of relatively large quantity.

Table 6.7 Predicted demand and annual savings for different methods

Method	Predicted demand	Annual savings (Rs.)
Exponential smoothing with Markov based algorithm	1260	41496
Naive forecasting with Markov based algorithm	1248	46128
Moving average with Markov based algorithm	1260	41496

The maximum profit is for the combination of naive with Markov based algorithm.

For baked product-B

Table 6.8 gives the predicted demand corresponding to maximum annual savings of each combination for the product B of relatively small quantity.

Table 6.8 Predicted demand and annual savings for different methods

Method	Predicted demand	Annual savings (Rs.)
Exponential smoothing with Markov based algorithm	34	10154
Naive forecasting with Markov based algorithm	38	6864
Moving average with Markov based algorithm	36	9336

The maximum profit is for the combination of exponential smoothing with Markov based algorithm.

6.3 Research findings

1. Percentage increase in profit by adopting a combination of naive and Markov method for product-A when compared to existing practice was 30 %.
2. Percentage increase in profit by adopting a combination of exponential smoothing and Markov method for product-A when compared to existing practice was 27 %.
3. Percentage increase in profit by adopting a combination of moving average and Markov method for product-A when compared to existing practice was 27 %.
4. Percentage increase in profit by adopting a combination of naive and Markov method for product-B when compared to existing practice was 11 %.

5. Percentage increase in profit by adopting a combination of exponential smoothing and Markov method for product-B when compared to existing practice was 11 %.
6. Percentage increase in profit by adopting a combination of moving average and Markov method for product-B when compared to existing practice was 15 %.
7. Return on investment for naive and Markov method for product-A is 75 %.
8. Return on investment for exponential smoothing and Markov method for product-A is 74 %.
9. Return on investment for moving average and Markov method for product-A is 74 %.
10. Return on investment for naive and Markov method for product-B is 47 %.
11. Return on investment for exponential smoothing and Markov method for product-B is 55 %.
12. Return on investment for moving average and Markov method for product-B is 51 %.
13. Return on investment for existing practice for product-A is 70 %.
14. Return on investment for existing practice for product-B is 42 %.

Percentage increase in the profit for Products A and B when compared to existing method is given below in Table 6.9.

Table 6.9 Percentage increase in profit for products A and B

Method	% Increase in profit for product-A	% Increase in profit for product-B
Naive & Markov method	30	11
Exponential smoothing & Markov method	27	11
Moving average & Markov method	27	15

Return on investments by adopting various methods for products A and B are given below in Table 6.10.

Table 6.10 Return on investments for different methods

Method	Product-A	Product-B
ROI for naive & Markov method	75%	47%
ROI for exponential smoothing &Markov method	74%	55%
ROI for moving average & Markov method	74%	51%
ROI for existing practice	70%	42%

6.4 Limitations of the study and future scope for research

The data was collected from two reputed baking firms in Kerala. The geographic and organizational changes are applicable to collected data. The model can be applied to more types of novel and short life products. Further the feasibility of incorporating seasonality and cyclic component into this model can be analyzed. Model was applied for products of shelf life of one day or very short life products and future research can be conducted for its modification for applying it to products with shelf life of one or two weeks or other short life products. The model can be applied to macro short life products (with a life span more than one day) after suitable modification. Macro short life products include bread, biscuits, cakes, fruits, vegetables, magazines etc., whose shelf life varies from one day to two weeks. Meat, poultry and fish are some other examples for macro short life products. Model can be also applied for determining demand forecasts of newspapers as its shelf life is limited to only one day.

This algorithm can be further combined with other existing methods and can be compared by using the corresponding profits obtained. Further these combined algorithms can be coded and commercialized as user friendly software.

6.5 Conclusion

The research started with the objective of developing a model for demand prediction of novel and short life products was achieved and was successfully implemented for two novel baked products with a single day shelf life. This was accomplished by conducting literature survey, data collection and implementation in accordance with research model and structure.

The suitability of the model can be validated by the fact that its implementation on product-A fetches an annual gain of Rs 46000 and on product-B it gives an annual savings of Rs 10000 when compared to existing practice. Return on investment increases by 5% for product-A and by 13% for product-B when compared to existing methods. Percentage increase in profit for product-A when compared to existing practice is 30% while that for product-B is 15%. Thus we can conclude that a firm can further enhance its profits by implementing this model or algorithm for more number of products. Further the model can be generalized by applying it to more types of novel products with short shelf life.

The forecasting of novel and short life products was not much explored in previous research works. This model can act as the benchmark for future researches in forecasting of novel and short life products.

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