

Simple approach to determine resonant frequencies of microstrip antennas

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Abstract: A simple approach for accurate determination of the resonant frequencies of microstrip antennas of regular geometries is developed and presented. In this approach, a generalised empirical formula for the computation of effective dielectric permittivity is given which takes into account the ratio of the fringing area to the area of the patch. A correction to the equivalent side length of an equilateral triangular patch, previously published, is modified and a new formula is given. A correction to the effective dimensions of an elliptical microstrip antenna is also carried out. Numerical results obtained for the resonant frequencies of elliptical, circular, rectangular and equilateral-triangular microstrip antennas are in good agreement with the available theoretical and experimental results reported by others. The present approach is more efficient, simpler and more accurate.

1 Introduction

Accurate determination of resonant frequency is important in the design of microstrip antennas because of their narrow bandwidth. For an accurate estimation of resonant frequency, the fringing-field effects must be taken into account, and hence the effective dielectric permittivity and dimensions of the microstrip patch must be incorporated. Several correction methods have been reported in the literature to calculate accurately the resonant frequencies of rectangular, circular, equilateral triangular and elliptical microstrip antennas separately [1–5]. However, comparison of results of the above methods shows large differences between the computed and measured resonant frequencies. No generalised formula for the effective dielectric permittivity ϵ_{eff} for these geometries is reported in the literature. In this paper, a generalised empirical formula for the effective dielectric permittivity has been developed for the well known microstrip-antenna geometries. A formula for the effective side length of an equilateral triangular patch reported in [3] has been modified in this

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paper for more accurate results. Similarly, a new formula has been developed for elliptical microstrip patches. The computed values are compared with the results reported in [1–11] and found to be much closer to their experimental values.

2 Formulations

The effective dielectric permittivity and aperture dimensions are the two major parameters to be computed and incorporated in the resonant-frequency computations. Formulations of these parameters are given in the following sections.

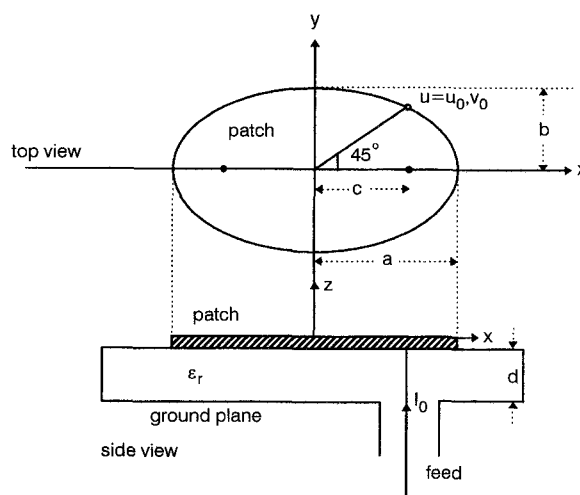


Fig. 1 Elliptical microstrip patch antenna: geometry and co-ordinate system

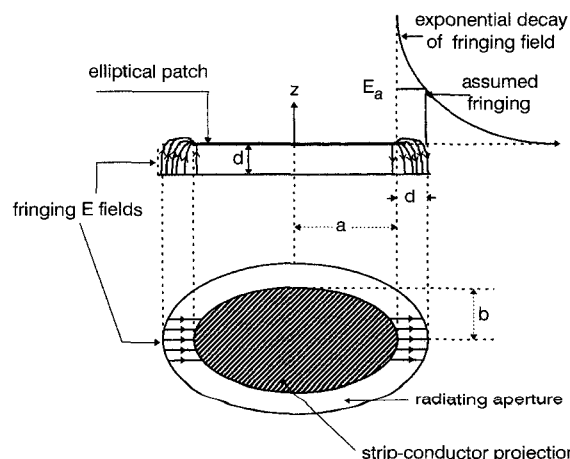


Fig. 2 Elliptical microstrip patch antenna: fringing field distribution near the boundary $u = u_0$

2.1 Effective dielectric permittivity

Figs. 1 and 2 show the geometry and the fringing-field distribution near the boundary of a typical elliptical microstrip antenna with a as semi-major- and b as semi-minor-axis lengths. The effective dielectric permittivity ϵ_{eff} is reported in terms of a function F which depends on the relative dielectric permittivity ϵ_r and the ratio of the aperture dimensions to substrate thickness of the microstrip antenna [12] as

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} F \quad 0 < F < 1 \quad (1)$$

In this paper, the function F is empirically derived from the ratio of the fringing area on the plane of the patch to the area of the patch and is given by

$$F = 1 - \frac{c_n \epsilon_r}{\epsilon_r - 1} \times \left(\frac{\text{fringing area on the plane of the patch}}{\text{area of the microstrip patch}} \right) \quad (2)$$

where $c_n (> 0)$ is a coefficient to be determined to find F . The fringing area is the area in the plane of the patch on which the fringing fields exist. It is assumed that the fringing fields extend uniformly over a distance equal to the substrate thickness d approximately on the plane of the patch, as shown in Fig. 2 [13]. Formulations for F and effective dielectric permittivity are established for different patch geometries having elliptical, circular, rectangular and equilateral triangular shapes.

2.1.1 Elliptic: From the geometry shown in Figs. 1 and 2 the fringing area of the elliptical antenna is computed as $\pi(a+d)(b+d) - \pi ab = \pi d(a+b+d)$. Substituting this into eqn. 2 we obtain

$$F = 1 - \frac{c_n \epsilon_r}{\epsilon_r - 1} \left(\frac{d}{a} + \frac{d}{b} + \frac{d^2}{ab} \right) \quad (3)$$

Substituting eqn. 3 into eqn. 1, the effective dielectric permittivity for the elliptical patch finally takes the form

$$\epsilon_{eff}(\text{elliptical}) = \epsilon_r - \frac{c_n \epsilon_r}{2} \left(\frac{d}{a} + \frac{d}{b} + \frac{d^2}{ab} \right) \quad (4)$$

A similar procedure is used to calculate the same for other geometries and these are given by

$$\epsilon_{eff}(\text{circular}) = \epsilon_r - \frac{c_n \epsilon_r}{2} \left\{ \frac{2d}{r} + \left(\frac{d}{r} \right)^2 \right\} \quad (5)$$

where r represents the radius of the circular microstrip antenna;

$$\epsilon_{eff}(\text{rectangular}) = \epsilon_r - c_n \epsilon_r \left\{ \frac{d}{w} + \frac{d}{l} + \frac{d^2}{wl} \right\} \quad (6)$$

Here l and w are the length and width of the rectangular microstrip antenna, respectively;

$$\epsilon_{eff}(\text{triangular}) = \epsilon_r - 2c_n \epsilon_r \left\{ \sqrt{3} \left(\frac{d}{s} \right) + 3 \left(\frac{d}{s} \right)^2 \right\} \quad (7)$$

Here s represents the sides of an equilateral triangular microstrip patch.

2.2 Effective aperture dimensions

Correction to the aperture dimensions of the rectangular microstrip antenna is given by Dearnley *et al.* [1]

and that of a circular antenna is given by [2, 14]. In this paper the ϵ_r in the correction formulas given by [1, 14] are replaced by ϵ_{eff} for the resonant frequency calculation. A correction to the aperture dimensions of an equilateral triangular and an elliptical microstrip antenna has been carried out using the capacitance formula as reported in [2, 3, 14]. For an equilateral triangular patch with side length s , the radius a_{eq} of the corresponding equivalent circular patch is obtained by using the capacitance formula of circular patch given by [14] and replacing ϵ_r by ϵ_{eff} , the effective radius of this equivalent circular disk due to fringing effects can be written as

$$a_{eff} = a_{eq} \left[1 + \frac{2d}{\pi \epsilon_{eff} a_{eq}} \left\{ \ln \left(\frac{a_{eq}}{2d} \right) + (1.41 \epsilon_{eff} + 1.77) + \frac{d}{a_{eq}} (0.268 \epsilon_{eff} + 1.65) \right\} \right]^{1/2} \quad (8)$$

where $a_{eq} = \sqrt{(A/\pi)}$ and A is the area of the equilateral triangular patch. The effective side length s_{eff} of the triangular microstrip patch antenna is calculated by an approach given by [7, 15] and can be written as $s_{eff} = s(a_{eff}/a_{eq})$ and hence the effective value of the side length of the equilateral triangular microstrip patch can be written as

$$s_{eff} = s \left[1 + \frac{2d}{\pi \epsilon_{eff} a_{eq}} \left\{ \ln \left(\frac{a_{eq}}{2d} \right) + (1.41 \epsilon_{eff} + 1.77) + \frac{d}{a_{eq}} (0.268 \epsilon_{eff} + 1.65) \right\} \right]^{1/2} \quad (9)$$

Correction to the aperture dimensions of an elliptical microstrip patch antenna is also carried out by considering an equivalent circular patch having metallised area equal to that of the elliptical patch. The area of the elliptical patch can therefore be resolved as a product of the area of two circular patches with radius a and b , respectively. Therefore

$$\text{area}_{\text{ellipse}} = \sqrt{\text{area}_{\text{circle1}} \text{area}_{\text{circle2}}} = \pi ab \quad (10)$$

Thus from eqn. 10, if the radii of circles 1 and 2 are replaced by their equivalent values given by eqn. 8, eqn. 10 becomes $\pi a_{eq} b_{eq}$. This directly gives the formula for the area of an equivalent elliptical microstrip antenna with equivalent semimajor- and minor-axis lengths as a_{eq} and b_{eq} , respectively. The expression for a_{eq} can be written in a way similar to that given in eqn. 8 as

$$a_{eq} = \left[a^2 + \frac{2da}{\pi \epsilon_{eff}} \left\{ \ln \left(\frac{a}{2d} \right) + (1.41 \epsilon_{eff} + 1.77) + \frac{d}{a} (0.268 \epsilon_{eff} + 1.65) \right\} \right]^{1/2} \quad (11)$$

Similarly b_{eq} is obtained by replacing a and a_{eq} by b and b_{eq} , respectively in eqn. 11.

Resonant frequencies of the above microstrip antennas are calculated accurately by replacing the aperture dimensions and the relative dielectric permittivities with their effective values in their respective resonant-frequency formulas [2-4, 6].

3 Results and discussions

The value of coefficient c_n was varied from zero upwards and the corresponding resonant frequencies were calculated and compared with the experimental values reported in [1–11]. When c_n was equal to 0.7, the resonant frequencies computed from the resonant-frequency formulas [2–4, 6] matches the experimental resonant frequencies with minimum percentage error. Hence c_n is taken as 0.7 for the computation of resonant frequencies for the above antenna geometries.

Theoretical resonant frequencies obtained for elliptical, circular, rectangular and equilateral triangular microstrip antennas are given in Tables 1–4, respec-

tively. From Table 1 it is observed that the resonant frequencies of an elliptical microstrip antenna computed by the present approach are much closer to the experimental values of [4, 5]. In Table 2, the resonant frequencies obtained by the present approach are compared with the results of [2, 9]. The comparison shows that the resonant frequencies computed by the present method are in good agreement with the measured data for electrically thin dielectric substrate (i.e. $d/\lambda_d \leq 0.02$, where λ_d is the wavelength in the dielectric substrate). For the cases where $d/\lambda_d \geq 0.02$, the results obtained by the present method are not better than those in [2] but better than the other methods [6, 10, 11] and hence the data with $d/\lambda_d \geq 0.02$ are not given in Table 2. Since,

Table 1: Theoretical and experimental results of resonance frequencies of odd and even modes of an elliptical microstrip antenna

Mode	Eccentricity e	Experimental f_{res} [4] (GHz)	Rengarajan [5] from variational method		Rengarajan [5] from Hammerstad formula		Damiano [4]		Present method	
			Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)
Even	0.3	1.872	1.883	0.6	1.862	-0.5	1.869	-0.2	1.875	0.16
	0.5	1.882	1.891	0.5	1.871	-0.6	1.876	-0.3	1.883	0.05
	0.62	1.890	1.900	0.5	1.879	-0.6	1.886	-0.2	1.8905	0.03
	0.7	1.892	1.907	0.8	1.887	-0.3	1.889	-0.1	1.896	0.21
Odd	0.3	1.948	1.933	-0.8	1.912	-1.9	1.941	-0.4	1.953	0.26
	0.5	2.139	2.103	-1.7	2.084	-2.8	2.133	-0.3	2.135	-0.19
	0.62	2.332	2.296	-1.5	2.271	-2.7	2.318	-0.6	2.3325	0.02
	0.7	2.535	2.492	-1.7	2.466	-2.7	2.524	-0.4	2.5335	-0.06

$$a = 30.65 \text{ mm}, c = ae, c = \sqrt{a^2 - b^2}, d = 1.524 \text{ mm}, \epsilon_r = 2.2, \% \text{error} = (f_{theo} - f_{expt})/f_{expt} \times 100$$

Table 2: Theoretical and experimental results of resonance frequencies of a circular microstrip antenna

Patch radius (cm)	Substrate thickness (cm)	ϵ_r	Experimental f_{res} (GHz)	Calculated [2]	Error (%)	Present theoretical f_{res} (GHz)	Error (%)
3.493	0.1588	2.50	1.570	1.555	-0.96	1.560	-0.63
1.270	0.0794	2.59	4.070	4.175	2.58	4.150	1.96
4.950	0.2350	4.55	0.825	0.827	-0.24	0.822	-0.3
3.975	0.2350	4.55	1.030	1.027	-0.30	1.028	-0.19
				Calculated [9]			
4.85	0.318	2.52	1.099	1.095	-0.39	1.101	0.18

Table 3: Theoretical and experimental results of resonance frequencies of rectangular microstrip antennas

Dimensions (mm × mm)	Modes								
	10			20			30		
	Experimental f_{res} [1] (GHz)	Present theoretical f_{res} (GHz)	Error (%)	Experimental f_{res} [1] (GHz)	Present theoretical f_{res} (GHz)	Error (%)	Experimental f_{res} [1] (GHz)	Present theoretical f_{res} (GHz)	Error (%)
38.15 × 31.40	—	—	—	4.950	4.952	0.05	—	—	—
38.70 × 31.50	2.450	2.440	-0.45	—	—	—	7.290	7.320	0.37
38.50 × 31.80	2.420	2.450	1.29	4.755	4.900	3.10	7.085	7.350	3.79
38.25 × 31.30	—	—	—	4.875	4.920	0.90	—	—	—
38.35 × 31.25	2.470	2.457	-0.51	—	—	—	7.295	7.372	1.10
38.30 × 31.45	2.450	2.451	0.05	4.850	4.900	1.10	7.250	7.350	1.40
37.95 × 31.35	—	—	—	4.825	4.960	2.90	—	—	—
38.40 × 31.55	2.470	2.460	-0.39	—	—	—	7.320	7.380	0.84
38.00 × 31.80	2.430	2.473	1.77	4.835	4.950	2.30	7.215	7.420	2.84

Table 4: Theoretical and experimental results of resonance frequencies of an equilateral triangular microstrip antenna

Mode	Experimental f_{res} [3] (GHz)	Reported [7]		Reported [8]		Reported [3]		Present method	
		Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)	Theoretical f_{res} (GHz)	Error (%)
TM ₁₀	1.280	1.273	-0.55	1.299	1.48	1.289	0.70	1.282	0.16
TM ₁₁	2.242	2.205	-1.65	2.252	0.45	2.233	-0.40	2.222	-0.89
TM ₂₀	2.550	2.546	-0.16	2.599	1.92	2.579	1.14	2.565	0.58
TM ₂₁	3.40	3.369	-0.91	3.439	1.15	3.411	0.32	3.394	-0.12
TM ₃₀	3.824	3.820	-0.11	3.899	1.96	3.868	1.15	3.848	0.63

$s = 10\text{cm}$, $d = 0.159\text{cm}$, $\epsilon_r = 2.32$

for most manufactured dielectric materials, the relative permittivity does not strictly follow its estimated value, a tolerance of 1% has been introduced in ϵ_r and its effect in the resonant frequencies has been observed. This feature is shown in Fig. 3. From Fig. 3 it is seen that the resonant frequency varies linearly with $\Delta\epsilon_r$. This variation has been studied for all the geometries and is only presented for the elliptical geometry. It was found that, for a variation of $\pm 1\%$ in ϵ_r , a variation of $\sim \pm 0.4\%$ was observed in the resonant frequency for all the above geometries. From Table 4 it can be seen that the results obtained by the present modified formula are more accurate than those in [3]. The advantages of this method are accuracy and simplicity.

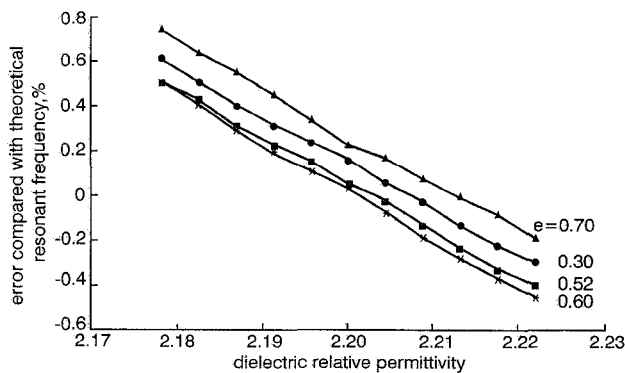


Fig. 3 Variation of resonance frequency against ϵ_r for an elliptical microstrip antenna

4 Conclusions

A simple approach for the accurate determination of resonant frequencies of elliptical, circular, rectangular and equilateral triangular microstrip antennas are presented. A generalised expression for effective dielectric permittivity ϵ_{eff} is formulated. A new formula for the effective dimensions of elliptical and equilateral triangular microstrip antennas are also formulated.

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