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SIGNIFICANCE OF TIME SCALES IN NONLINEAR DYNAMICAL ANALYSIS OF ELECTROENCEPHALOGRAM SIGNALS

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We propose to show in this paper, that the time series obtained from biological systems such as human brain are invariably nonstationary because of different time scales involved in the dynamical process. This makes the invariant parameters time dependent. We made a global analysis of the EEG data obtained from the eight locations on the skull space and studied simultaneously the dynamical characteristics from various parts of the brain. We have proved that the dynamical parameters are sensitive to the time scales and hence in the study of brain one must identify all relevant time scales involved in the process to get an insight in the working of brain.

Keywords: EEG; chaotic dynamics; entropy; time scale

1. INTRODUCTION

During the last decade there has been a great interest in the characterization of several physical and biological systems using the methods in nonlinear dynamics and deterministic chaos (Ott, 1993). The methods of characterization depend mainly on the evaluation of ergodic measures such as dimensions, entropies, Lyapunov exponents *etc.* These parameters are evaluated from a

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scalar time series obtained either from experiments or simulation of models (Abarbanel *et al.*, 1993). In general, there are two approaches, metric and topological to characterize complex systems. In metric approach, the probability of finding a set of points within a hypersphere with a preassigned radius in a phase space is a measure, whereas in topological approach, the number of unstable periodic orbits are estimated to study the dynamics of a system (Badii, 1994). Biological systems are found to exhibit chaotic behavior. However, the evaluation of invariant parameters from observed time series have posed difficulties mainly due to the presence of noise, nonstationarity, finite data length *etc.* (Rapp, 1989).

Numerous authors (Jansen *et al.*, 1993) have reported the evidence of chaotic behavior in human brain, by evaluating some of the ergodic parameters from EEG signals. However, the existence of low dimensional chaos has recently questioned (Theiler and Rapp, 1996). Using the methods of surrogate data analysis (Theiler *et al.*, 1992), while the existence of nonlinearity has been confirmed in EEG signals, the low dimensionality is still in debate. The reliability of surrogate data analysis has recently been reexamined and it has been found that this test alone may not be sufficient to distinguish colored noise from low dimensional chaos (Pradhan and Sadasivan, 1997). It should further be emphasized that the ergodic parameters are indeed important, the ultimate goal should be an understanding of the dynamics of the system. It is therefore time that we should try to make models that would generate these parameters and which has predictive capabilities. We therefore propose to look at the problem *ab initio* from the dynamical point of view. Hence our approach is not to discuss the existence or otherwise of chaos in the human brain, but to discuss nonstationarity and its effect on information capacity in an EEG time series analysis. It has increasingly been realized that the real world data sets are nonstationary. Nonstationary nature can mainly be due to two reasons: it could be due to introduction of new time scales by new dynamical process that could be switched on during a process or it could be various distinctly different processes, dynamical processes, that could exist in the system. If these time scales are not widely separated, they could overlap in a nonlinear fashion thereby making it complex.

It should be pointed out that if the time series is nonstationary then the different measures would also be time dependent unlike the case of stationary time series where these measures are independent of time. This is precisely what we propose to discuss in this paper.

This paper is organized as follows: In Section 2 we derive a general time scale by performing a dimensional analysis of the various known parameters in neural systems. In Section 3 we give a brief discussion of data collection and the methods of analysis. The next section gives an account of the results and discussions. The final section gives the main conclusion as well as future direction of work.

2. TIME SCALES

EEG is recordings of electric potentials generated in the cerebral cortex by a large number of neurons, which are integrated at various stages of its propagation from the input to the central nervous system. These generate collective modes, which on the cortex are recorded in EEG. It has been pointed out that the amplitude of the action potential generated by the individual neuron is constant while the firing frequency is a linear function, as far as measurements go, of the applied stimulus strength. A simple addition of these amplitudes, however, cannot reproduce the observed characteristics of EEG. Hence the integration process should be an involved mechanism. Further, it has been found that the electrical characteristics of the action potential is quite different from those of EEG. The characteristic time scales of different waves of EEG is of the order of 0.1 second where as that of the action potential is ~ 1 millisecond. A factor of hundred between two time scales implies that the human brain has a wide range of time scales resulting in being highly nonlinear one and that it is thermodynamically open with feedback and feed forward process. Hence the construction of general time scale starts with identifying different characteristic parameters in neuron dynamics.

The human brain consists of about 10^{10} neurons of which about 1 to 2% are active and they are connected in a certain pattern. These connections change with the input signals. Hence, to study the collective effects in the cortex we have to take into account the dynamics of the individual neuron, which contribute towards collective effects. The most significant feature in the neuron dynamics is the firing potential, ϕ which is about 100 millivolts above the resting potential, and is constant for all neurons. The next is the firing frequency, which is the inverse of the refractory time. It is usually about 1000 Hz and is linearly dependent on the stimulus strength. Since the firing potential is electrical in nature, they are generated by the various ions such as sodium, potassium, calcium *etc.*, and these are characterized by the electrical charge (e) and ion mass (m) as well as the number density (n). Furthermore, the chemical process also play a significant role in the signal transmission by changing the conductivity. We shall include this by a nondimensional parameter, ε_o representing the conductivity. These parameters along with the numerical values are given in Table I.

Using these parameters, we perform a dimensional analysis following Balescu (1975). We find a non dimensional parameter (details are given in Appendix I).

$$\Gamma = (\phi^2 n^{1/3}/m\nu^2)^x (V n^{1/3}/\nu)^y (\varepsilon_o e^2 n/m\nu^2)^z \tag{1}$$

And a general time scale T as

$$T = \Gamma\nu^{-1} = a^x b^y c^z \quad \text{(say)} \tag{2}$$

In (2)

a. ratio of electric frequency to firing frequency,
b. ratio of mechanical frequency to firing frequency,
c. ratio of plasma frequency to firing frequency.

The x, y and z are natural numbers. It could be real or complex, rational or irrational or integers. If they are complex, the process could have an amplitude and phase. If they are rational, the frequencies are compatible while if the frequencies are irrational, they are incommensurate. Again we can write $(\varepsilon_o e^2 n/m\nu^2)$ as $(\varepsilon_o \omega_p^2/\nu^2)$ with ω_p as plasma frequency. From (1) it is very clear there exists three fold infinite time scales in the system. This point is very important in the analysis of EEG signals. By proper choice of variables x, y and z some of the known relevant time scales can be obtained. Some of the relevant time scales are derived below.

(1) *Sodium Potassium pump.* The sodium potassium pump depends on the threshold potential and hence on ϕ. Since it is a chemical process depending on the number of sodium ions going in and potassium ions coming out during depolarization phases, the time scale should depend

TABLE I Different parameters used in the construction of general time scale

Name	*Symbol*	*Dimension*	*Order of magnitude*
Action potential	ϕ	$(MLT^{-2})^{1/2}$	10^{-1} volts
Propagation speed	V	LT^{-1}	100 m/seconds
Firing frequency	ν	T^{-1}	1000 Hz
Ionic charge	q	$(ML^3T^{-2})^{1/2}$	1.6×10^{-19} Coulomb
Ionic mass	m	M	10^{-27} kg
Ionic density	n	L^{-3}	–
Medium effect	ε_o	–	–
Ion plasma frequency square	ω_p^2	T^{-2}	$10^3 z \mu^{-1/2} n^{1/2}$ rad/sec

on mass and number density, but not on the firing frequency. To get relevant time scale corresponding to this process, one must choose $y = z = 0$ and $x = -1/2$. Thus relevant time scale for this process is

$$Tp = (m/\phi^2 n^{1/3}) \sim 10^{-7}$$

with $\phi \sim$ millivolts and $n \sim 100$

(2) *Synaptic transduction.* In the synaptic transduction, the process depends on the dielectric characteristics of the synaptic cleft as also on the speed at which the signal arrive at the presynapse as well as the threshold potential. However, there is no explicit dependence on the number density, since there is a randomness in the injection of neurotransmitters in the cleft due to the discharge of vesicles. We construct such a time scale by a proper choice of x, y, z *viz.*, $x = -y = z = -1/2$. Thus the time scale is obtained as

$$Ts = (mV\,\nu/\varepsilon_o \phi^2 \omega_p^2)^{1/2} \tag{4}$$

(3) *Collective modes.* At various stages in a neural system, there are integration processes operative. The threshold potential is a consequence of the integration process. In a single neuron or in the case of synaptic transduction the process is always collective in nature. Also in the case of EEG signals, the process is indeed a collective one which does not exhibit all the apparent regularities seen in a single neuron firing sequences or the signal transmission from a presynapse to a postsynapse. If we consider this as a self-consistent process, which is in general responsible for the collective modes, then the relevant time scale is the interaction time scale,

$$Ti = (\varepsilon_o \omega_p^2)^{-1/2} \tag{5}$$

and this is obtained by setting $x = y = 0$ and $z = -1/2$. Thus different time scales for different mechanism could be derived from the general time scale. There may be other relevant parameters, however, we have considered only some of the known ones.

Various algorithms developed till now in the evaluation of ergodic parameters are suitable only for stationary data set (Grassberger and Procaccia, 1983). However for nonstationary data set, these parameters are slowly varying functions of time (Pradhan and Dutt, 1993). Hence, in choosing the data length, the number of data points must be sufficiently large in comparison with natural time scale in the system. This requires the identification of different time scales involved in the dynamical process.

Many authors in studying the dynamics of human brain have ignored the role of time scales. Parikh and Pratap (1983) developed an evolutionary equation in the framework of nonequilibrium statistical mechanics developed by Prigogine and his Brussel School in which the significance of time scales has been stressed. Further, Pratap (1999) formulated a theory of sensory transduction considering the interaction time scale. Our purpose is to study the relevance of time scales in the dynamics of brain observed in EEG signal, especially on nonstationarity and Kolmogorov entropy.

3. MATERIALS AND METHODS

3.1. Data Collection

Five subjects who were known epileptics with uncontrolled seizures in the age group of 20–25 years were recruited for this study. Despite anticonvulsant medication the subjects showed seizure discharges in EEGs. The eight channels of EEG data are collected using International 10–20 system with an EEG machine coupled to a PC using Analog to Digital Converter (DT-2841) and an array processor (DT-7020). The eight scalp loci are Fp1, Fp2, F7, F8, T3, T4, O1 and O2. The sampling is done at the rate of 256 samples/s/channel. The data are filtered using a bandpass (0.25–32Hz) fourth order Butterworth filter twice cascaded. The data were visually screened to remove artifacts as well as to identify epochs containing epileptic discharges. This was done by two experts who differed in their scoring by less than 5%. The differences were resolved by discussion and a consensus rating was assigned to the discrepantly rated epochs. The data from five normal volunteers without any history of neuropsychiatric illness or intake of any psychotropic drugs who were in the age group of 20–25 years were also collected.

3.2. Correlation Dimension and Kolmogorov Entropy

The scalar time series obtained is reconstructed in a phase space using the famous time delay embedding (Abarbanel, 1993). Thus a multivariate vector $X(i) = \{x(i), x(i+T_d), x(i+2T_d), \ldots\ldots\ldots\ldots\ldots\ldots, x(i+(d-1)T_d)\}$, in a d dimensional phase space, where $i = 1,2,\ldots, N-d$, is formed. The time lag $T_d = nT_s$, n is an integer and T_s is the sampling time, is arbitrary but is fixed either using the autocorrelation function (ACF) to decay to $1/e$ or the first minimum in the average mutual information (AMI) curve. The basic procedure for the evaluation of the correlation dimension, D2 and Kolmogorov entropy, K2 is by using Grassberger Proccasia algorithm (GPA) (Grassberger and Procaccia, 1983).

However, the accuracy of invariant parameters depends on the number of data points, time lag and several other factors. EEG time series are nonstationary, hence, it is not possible to consider a large number of data points. Further, because of high dimension nature of the signal and limited data set it is difficult to obtain a proper scaling region using GPA. So we applied a modified algorithm (Havstad and Ehlers, 1989). In this method instead of finding the number of data points less than a preassigned radius r, the number of data points are fixed and the radius r is varied to evaluate correlation integral.

This algorithm has been checked using the time series generated from Henon map and logistic equation with standard values.

3.3. Selection of Time Lag

As mentioned earlier, the time lag T_d can be fixed either using the criteria of ACF to decay to $1/e$ or the first minimum in the AMI. The latter criterion is preferred, as it is a kind of generalization to the nonlinear world whereas the former is a correlation in the linear world (Abarbanel *et al.*, 1993). Using information theory, the average mutual information between the observations at n and $n+T$ is found for different values of T.

$$I(T) = {}_n\Sigma\, P(x(n), x(n+T)) \log\{P(x(n), x(n+T))/P(x(n))P(x(n+T))\} \tag{6}$$

where $P(x(k))$ is the probability of measuring $x(k)$ and $P(x(k), x(k+T))$ is the joint probability of measuring $x(k)$ and $x(k+T)$.

From (6) the first minimum of $I(T)$ is used as time lag T_d for phase space reconstruction. It should, however, be realized that this way of selecting the time lag could introduce a discrepancy since a finite time lag would imply an averaging process in time during the interval $0 - T_d$ and any scale in the range of $0 < T < T_d$ will not be taken into account. This point is very important in the evaluation of dynamical parameters.

3.4. Embedding Dimension

For a high dimensional system, evaluation of invariant parameters by increasing dimension of the phase space from 1 to a max value is tedious and time consuming. So embedding dimension d_E is found using the method of false nearest neighbors (Abarbanel *et al.*, 1993). In this method Euclidean distance between a point and its nearest neighbors as the point moves from d dimensional space to $d+1$ dimension is found and if the distance in high dimensional space is larger than a threshold R_t, false neighbors are identified. Since data set is limited, a second threshold criterion A_t based on the size of the attractor is also found.

The embedding dimension d_E is the dimension at which percentage of false nearest neighbor become zero. It has been found that for normal eyes closed data d_E is around 9 and for epileptic data d_E is around 7.

3.5. Analysis Procedure

The data set selected from each channel simultaneously consists of 12000 points and they are grouped into different nonoverlapping window of length 1000 points. Thus each window number corresponds to 1000 data points. The invariant parameters D2 and K2 are evaluated for different values of time lag for a single window corresponding to normal eyes closed as well as for epileptic condition. This is done to study the variation of these parameters with respect to time lag. Then we determined the time lag using average mutual information criteria and the embedding dimension with the threshold R_t fixed at 10 and A_t at 2 for each nonoverlapping sliding window. The correlation integral is evaluated for a few values of dimension greater than the embedding dimension. D2 is then evaluated with T_d from AMI whereas K2 is evaluated with T_d as well as with T_s.

4. RESULTS AND DISCUSSIONS

Any nonlinear dynamical system can be characterized only if all its invariant parameters are determined. This requires the identification of different time scales involved in the dynamical process. Since it has been theoretically established previously that human brain dynamics consist of a large number of time scales, a proper characterization is possible only if all its relevant time scales are identified. By calculating T_d from AMI, it has been found that the values of T_d are distinctly different for different windows, as shown in Figure 1, as well as for different channels. Since this time lag is a measure of time scale in the system, the variation of this proves the existence of different time scales in the dynamics. To further emphasize this aspect, using AMI, we calculated the average value of time lag, Tav for five adjacent windows as well as evaluated time lag, Tgr, grouping the same five windows together (5000

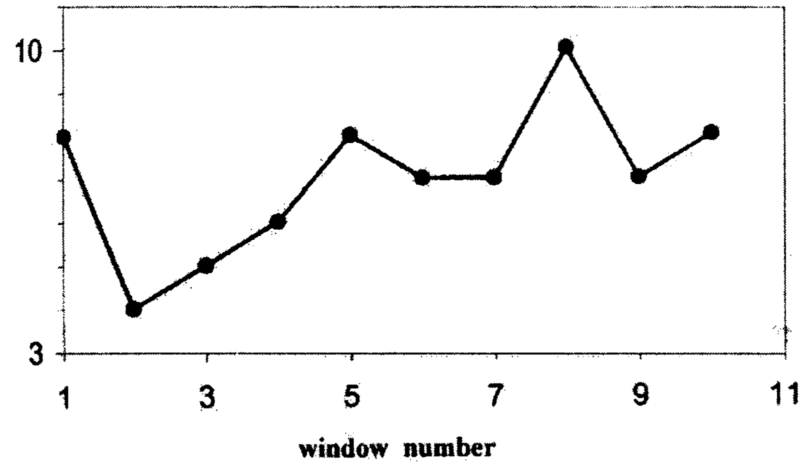

FIGURE 1 Variation of n (T_d/T_s) with respect to window numbers at Fp1 for normal eyes closed data.

TABLE II Comparison of Tav and Tgr during normal eyes closed an epileptic conditions

Channel No.	*(Normal eyes closed)* *Tav*	*Tgr*	*(Epileptic discharge)* *Tav*	*Tgr*
1	7.0	7.0	13.4	20
2	7.1	8.0	20	28
3	7.0	7.0	15	15
4	7.7	8.0	13	13
5	8.0	8.0	11.6	23
6	8.0	8.0	11.6	20
7	6.8	7.0	15.4	22
8	7.0	8.0	17.2	20

data points) for epileptic as well as for normal eyes closed case. For the epileptic condition these windows are selected as one with predominant epileptic discharge and two on either side. In the case of normal subjects with eyes closed, Tav and Tgr are very close and very often coincides for most of the channels whereas for epileptic data the values are quite apart in many of the channels as given in Table II. Even though the variation of time lag is an indication of time scale in the system, the difference in Tgr and Tav, especially when Tgr greater than Tav indicates that during the epileptic condition the time scales involved in the dynamics are more compared to normal eyes closed condition. Further it has been found that for those windows with epileptic discharge, the value of T_d is comparatively larger than that of other windows. This is because during epileptic discharge many relevant time scales are introduced in the dynamics which are reflected in T_d. As time scales are very much related to nonstationary nature of EEG, a proper study of human brain dynamics is possible only if all the relevant time scales are identified and incorporated in the analysis.

In spite of these limitations several attempts are made to study the dynamics of human brain from the nonlinear standpoint. But the fact that many evaluate the invariant parameters D2 and K2 at the same value of time lag (Popivanov *et al.*, 1998). The evaluation of invariant parameters, especially dynamical parameters, for pathological conditions need caution as they involve interaction of different time scales. It has been found that for normal eyes closed condition K2 with T_d as well as K2 with T_s shows a certain degree of 'parallelism' in variation with respect to windows as shown in Figure 2. However K2 with T_d and K2 with T_s bears *no resemblance*

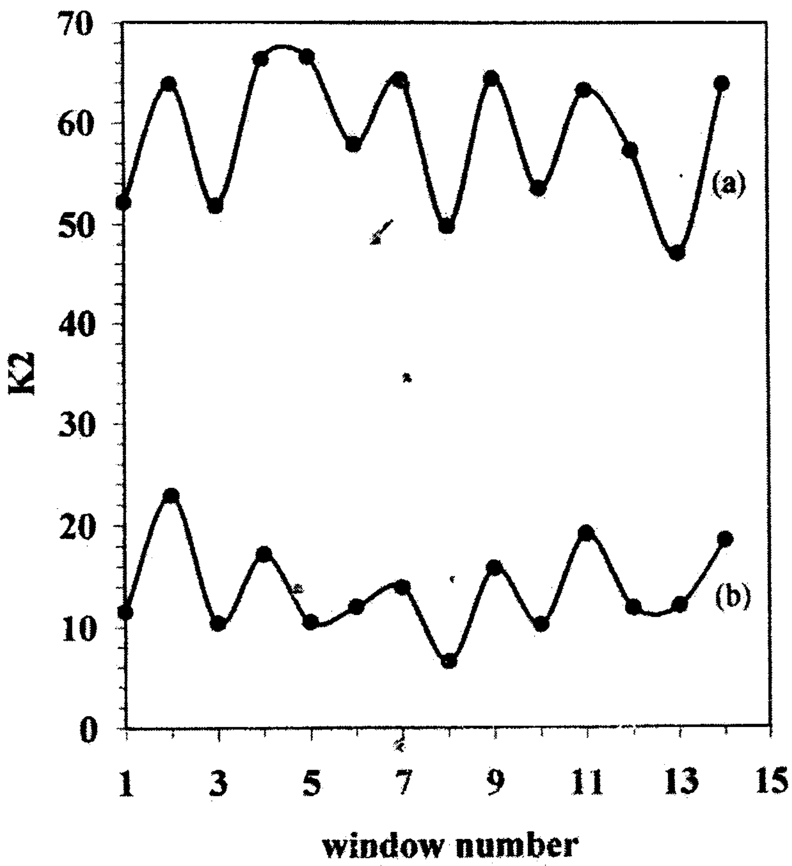

FIGURE 2 K2 for resting state (eyes closed) against window numbers at Fp1. Curve (a) corresponds to T_s. Curve (b) corresponds to T_d.

whatsoever for epileptic condition as has shown in Figure 3. The variation of K2 with respect to different values of time lag for the case of epileptic as well as for normal eyes closed condition are identical as shown in Figure 4.

However, K2 with T_s has larger value and variation compared to K2 with T_d. This is because during, epileptic condition the time scales involved in the process are more, which is turn show a larger value of T_d in AMI criteria. As time lag increases in the phase space reconstruction, many interactions are lost which in turn masks the dynamics corresponding to time scales within the time lag T_d. This effects the evaluation of entropy. Hence K2 with T_s shows more variation compared to K2 with T_d in the case of epileptic data. This indicates that the criteria of selecting time lag for the evaluation of invariant parameters from average mutual information or any of the similar criteria should be used only for D2 not for K2. K2 being a dynamic

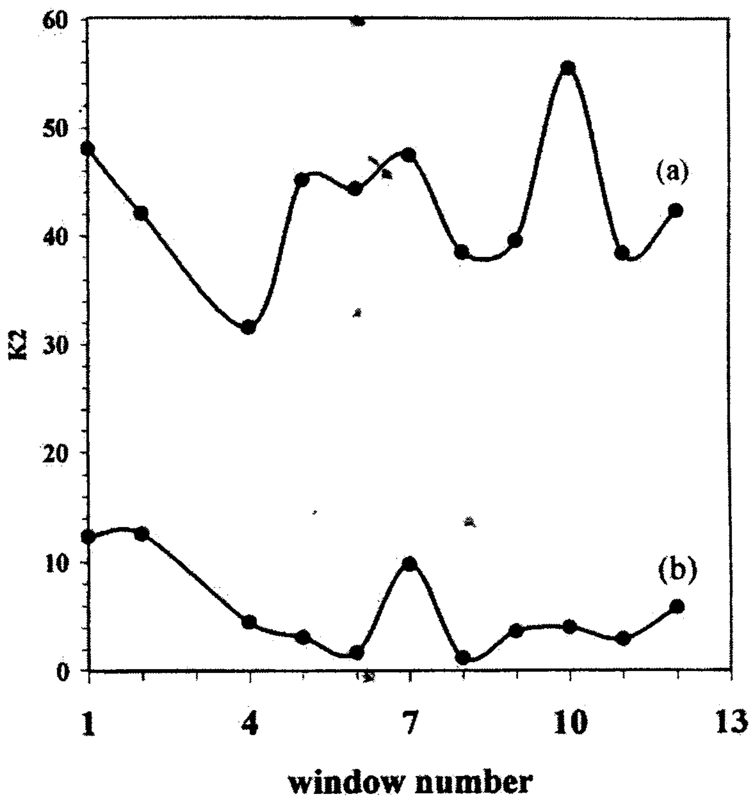

FIGURE 3 K2 for epileptic condition against window numbers at Fp1. Curve (a) corresponds to T_s. Curve (b) corresponds to T_d.

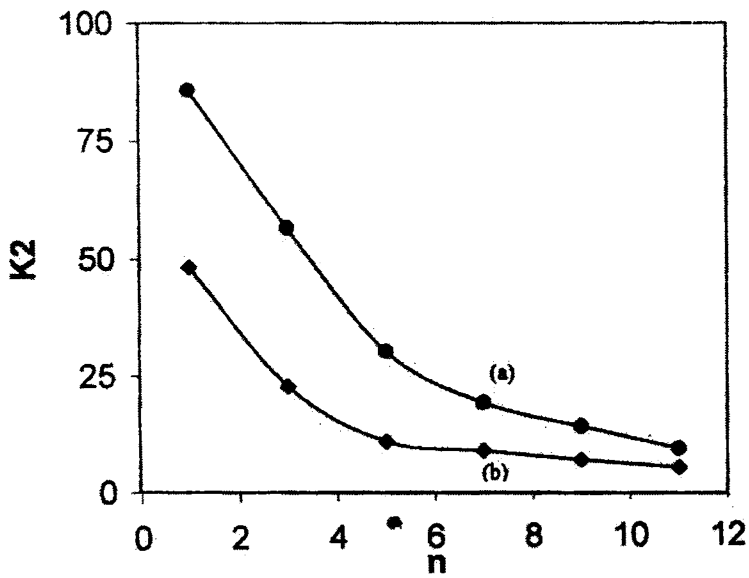

FIGURE 4 Variation of K2 against different values of time lag $T_d = nT_r$. Curve (a) corresponds to normal eyes closed condition. Curve (b) during epileptic discharge.

parameter should be evaluated with time lags as small as possible to incorporate as many time scales as possible.

5. CONCLUSIONS

This paper has two significant results. In the first place we have shown the real importance of time scales as they are indeed crucial in the study of brain functioning. We have shown how to obtain a relevant time scale pertaining to a particular state of the brain. Since we have performed a global analysis by taking an eight channel output, we have shown that these characteristics time scales vary with the different locations and during an experience, a large number of attractors are present in the brain and they obviously must be interacting amongst each other. These time scales are either a single scale or as a group of scales can be a set of synergic time scales and are not fundamental ones. Hence each time scale is a consequence of combination of various fundamental scales which combine in a nonlinear manner. It is indeed difficult or rather impossible to break down this synergic time scale into fundamental components. One method is to formulate a nonlinear theory for the process and evaluate a synergic frequency as was done by Pratap

(1999) for the case of synaptic transduction. Thus we have an "attractor gas" in the brain some or many of them being strange ones with fractal dimension. It may also be realized that these attractors change their characteristic parameters, resulting in nonstationary nature in the time series. Hence, one has a system of grand canonical ensemble of strange attractors with its characteristics including its number ever changing. It is also true that the system is thermodynamically open. We shall be working out the thermodynamics of this attractor – strange attractor gas in near future.

The second result is of clinical importance. It has been shown that the information capacity K2, which is a dynamical parameter, is significantly sensitive to the choice of time scale. In determining D2 and K2, the same time lag determined from AMI is generally used. While this time lag indeed gives a good measure of D2, any time scale shorter than this is being averaged out which in turn affects the evaluation of information capacity. Thus in the study of various dynamical conditions of brain one must choose time lag as small as possible to incorporate many time scales involved in the dynamical process. Nevertheless, this prescription is more useful to get a qualitative insight in the functioning of such a complex, nonlinear, nonequilibrium system such as human brain.

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I. APPENDIX

From Table I, using the parameters we construct a dimensionless constant as

$$\begin{aligned}\Gamma &= \phi^i V^j e^k \nu^p m^q n^r \\ &= (\mathrm{MLT}^{-2})^{i/2}(\mathrm{LT}^{-1})^j(\mathrm{ML}^3\mathrm{T}^{-2})^{k/2}(\mathrm{T}^{-p})(\mathrm{M}^q)(\mathrm{L}^{-3r}). \qquad (1)\end{aligned}$$

Equating powers of M, L and T on both sides we have

$$i/2 + k/2 + q = 0 \qquad (2)$$

$$i/2 + j + 3/2k - 3r = 0 \qquad (3)$$

$$-i - j - k - p = 0 \qquad (4)$$

We have only three equations with six unknowns, so expressing p, q and r in terms of i, j and k, we get

$$p = -i - j - k \qquad (5)$$

$$q = -i/2 - k/2 \qquad (6)$$

$$r = 3/2i + 1/3j + 1/2k \qquad (7)$$

Substituting (5), (6) and (7) in (1) we obtain,

$$\begin{aligned}\Gamma &= (\phi^2 n^{1/3}/m\nu^2)^{i/2}(Vn^{1/3}/\nu)j(\varepsilon_o\, e^2 n/m\nu^2)^{k/2} \\ &= (\phi^2 n^{1/3}/m\nu^2)^x(Vn^{1/3}/\nu)^y(\varepsilon_o\, e^2 n/m\nu^2)^z\end{aligned}$$

The variables x, y and z are natural numbers and can take different values depending on the dynamics.