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Subcritical Hopf bifurcation in Ne–Nd hollow cathode discharge

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Abstract

We report the experimental observation of subcritical Hopf bifurcation and the existence of non-oscillating “windows” in the dynamics of a Ne–Nd hollow cathode discharge with discharge current as the control parameter.

Recent developments in the field of non-linear dynamics have brought to light various systems showing deterministic chaos [1]. One of the well studied systems which has wide applications in pure and applied sciences is the electrical discharge in gaseous media where problems associated with stability, turbulence, etc., are very important. Even though the oscillatory phenomenon in discharges is known for a long time [2], it is only recently that various non-linear properties leading to chaos in a discharge plasma have been identified. Experimental observation of period doubling [3], intermittent chaos [4,5] and quasi-periodic routes [6,7] to chaos have been reported in steady state plasma. Self-generated oscillation by an optogalvanic effect in a hollow cathode discharge [8], instabilities and chaos in laser systems have also been reported [9,10]. These universal characteristics of the chaotic behaviour can be observed by monitoring voltage, current or emitted light [2,11], and their features depend mainly on the nature of the discharge. Experimental observation of subcritical Hopf bifurcation in a NMR laser with an injected signal has been reported by Holzner et al. [12]. Recently Singh et al. [13] observed backward Hopf bifurcation in a hollow cathode discharge lamp in which vanadium is used as the cathode material

and neon as the buffer gas. By taking the dc discharge current as the control parameter to describe the dynamics of the plasma, they observed a metastable “window” in the control parameter (at 4.3 to 4.7 mA), in which discharge oscillation takes a frequency state of 0.6 kHz, which is a metastable state; and the oscillations ultimately goes over to 0.9 kHz for a current > 4.7 mA.

In this Letter we report the observation of subcritical (inverse) Hopf bifurcation and the existence of more than one “window” in the oscillating states in a Ne–Nd hollow cathode discharge. The concept of bifurcation can be described by a forced oscillator. Here when the control parameter is changed, the behavior of the system changes from one state to another at a critical value of the control parameter [14]. The point in the parameter space where this occurs is called bifurcation point. In a normal bifurcation, oscillations always start from zero amplitude. However, in subcritical Hopf bifurcation the oscillation starts with a finite amplitude. Another property of the subcritical Hopf bifurcation is hysteresis, i.e. if the bifurcation is studied with the control parameter being reduced, the threshold will be different from that observed with the control parameter being increased. Thus, oscillation which starts with a finite

amplitude and the existence of a hysteresis in the bifurcation diagram indicates subcritical Hopf bifurcation.

A schematic of the experimental setup is shown in Fig. 1. A commercial hollow cathode lamp (Cathodean UK) which contains a Nd cathode and a neon buffer gas is operated at low current. The discharge was excited by applying a stable ripple-free dc voltage using a highly regulated power supply (Thorn EMI, PM28B). The discharge current is limited using a resistance which is in series with the power supply. The output signal, across the load resistance was monitored using a 200 MHz digital storage oscilloscope (Iwatsu, DS-8621) through a blocking capacitor. Thus when we talk of a signal it is the ac signal and in the non-oscillating state the output is zero.

After striking the discharge in the hollow cathode tube, the current was decreased to the lowest value where the discharge is stable. When the current was gradually increased in the presence of the discharge it was found that pure sinusoidal oscillations start with a finite amplitude at the threshold current $i=i_1$. As the current is further increased, at $i=i_2$ the oscillations stop abruptly corresponding to the dc state which is continued till $i=i_3$. Above i_3 oscillations with a complex wave form (having more than one frequency) start again with a finite amplitude till $i=i_4$ and after that the oscillations stop and it goes to a dc state. We were not able to go beyond $i_4 \sim 5$ mA due to the limitations of the power supply. The various states in the dynamics of the plasma are designated as follows.

State I: $i \leq i_1$,

non-oscillating state.

State II: $i_1 \leq i \leq i_2$,

oscillating state – stable. 5.26–2.27 kHz.

State III: $i_2 \leq i \leq i_3$,

non-oscillating state.

State IV: $i_3 \leq i \leq i_4$,

oscillating state – metastable. 526–434 Hz.

State V: $i > i_4$,

non-oscillating state.

$i_1 = 0.83$ mA, $i_2 = 2.46$ mA.

$i_3 = 4.2$ mA, $i_4 = 4.96$ mA.

Fig. 2a shows these regions in the bifurcation diagram in which the amplitude of oscillations as a function of the discharge current in the dynamics of the plasma is given. CRO traces of the discharge oscillations at various states are shown in Fig. 2b. The observation of an intermediate oscillating state in the bifurcation has been reported previously [12] whereas in the present work we observed a non-oscillating intermediate state in the dynamics as the discharge current is varied. Oscillations in state II start at 0.83 mA with a frequency of 5.26 kHz and the frequency of oscillation in this state decreases with current to 2.27 kHz at 2.46 mA. For state IV it also decreases with current (Fig. 3) from 526 Hz (at 4.2 mA) to 434 Hz at (at $i_4 = 4.96$ mA).

The oscillating state II is found to be very stable, i.e. the oscillation amplitude remains constant with time if we fix the current between i_1 and i_2 . When the discharge current is in the vicinity of $i=i_4$, state IV decays to the non-oscillating state V. The relaxation of the state from IV to V is fast when i is close to i_4 while it is slow when i is near i_3 . Fig. 3 shows the variations of the amplitude of the oscillation as a function of time in the left edge of the second oscillating regime. This indicates that state IV is a metastable state.

When we go in the reverse direction, i.e. when the discharge current is decreased, the critical currents are shifted to lower values demonstrating the existence of hysteresis (Fig. 4). This hysteresis is an indication of subcritical Hopf bifurcation in which the dc state is a fixed point and the oscillating state is a limit cycle. In the reverse direction the frequency of oscillation is less than that in the forward direction of i (Fig. 5). From the presence of two oscillating states (II and IV) and three non-oscillating states (I, III and V) we

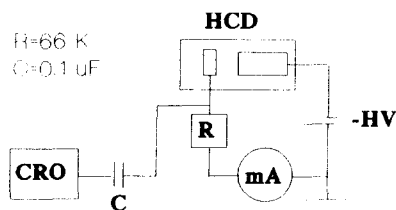


Fig. 1. Experimental setup.

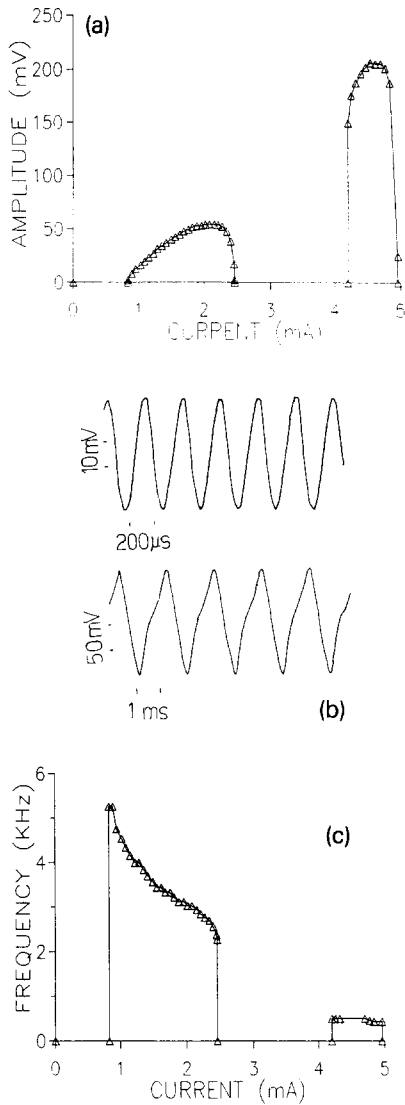


Fig. 2. Bifurcation diagram of the plasma dynamics with discharge current as the control parameter. (a) Variation of the signal amplitude with discharge current. (b) Time evolution of the oscillation for (I) state II and (II) state IV. (c) Variation of the frequency with discharge current.

can think of a series of subcritical Hopf bifurcations in the dynamics of the discharge plasma with windows of a non-oscillating state in between them. Switching between a fixed stable point to a stable limit cycle takes place as the discharge current is increased.

The observation of “windows” may be due to the presence of noise in the discharge system. A quantitative analysis is not possible because most under-

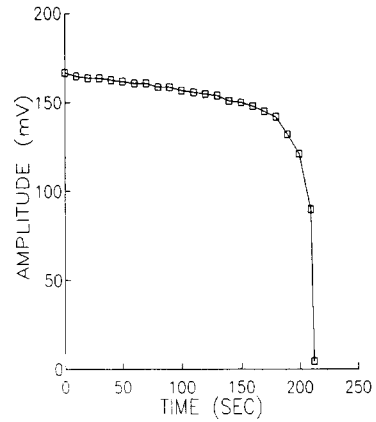


Fig. 3. Relaxation of oscillating state IV to the non-oscillating state V.

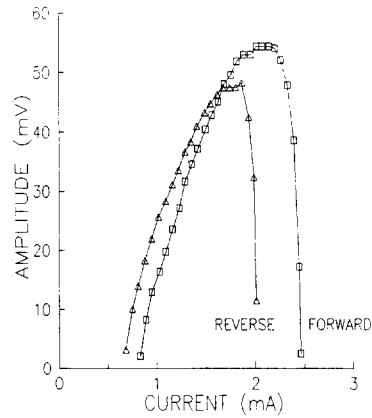


Fig. 4. Hysteresis in the amplitude of oscillation.

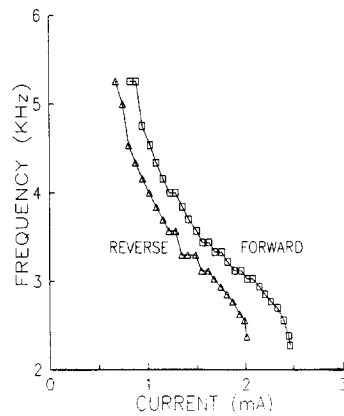


Fig. 5. Hysteresis in the frequency of oscillation.

standing of the processes in a hollow cathode is qualitative in nature. However a general mechanism that leads to chaotic behaviour in discharge and modelling of the plasma dynamics showing the windows in the bifurcation diagram has been discussed in a few papers [11,13]. Dynamics of a gas discharge can be studied by considering only the macroscopic properties based on the fluid equations obtained from average values such as number density, velocity and energy. The plasma acts as a conducting fluid and most of the basic properties of the gas discharge can be explained on the basis of the fluid equations. The rate of the plasma formation is determined by the rate of neutral ionization by primary electrons and the plasma decay time. The amplitude of oscillation depends on the density of electrons n and is controlled by [15]

$$\frac{\partial n}{\partial t} + \nabla n v = \alpha n - \beta n^2, \quad (1)$$

where v is the drift velocity, while α and β are the ionization coefficients, respectively. The drift velocity depends on the electric field according to

$$nv = -\frac{ne}{m} \gamma E - \frac{kT}{m} \gamma \nabla n, \quad (2)$$

γ being the electron–atom collision frequency and e , m , k and T having their usual meaning. The above equations coupled to Maxwell's equations constitute a set of non-linear partial differential equations. The investigation of partial differential equations for possible chaotic behaviour is still in a beginning stage [3].

In summary, subcritical Hopf bifurcation has been observed in the dynamics of a Ne–Nd hollow cathode discharge with discharge current as the control parameter. The existence of the non-oscillating state windows in the bifurcation diagram, the amplitude and the frequency of oscillations, hysteresis, etc., are investigated.

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