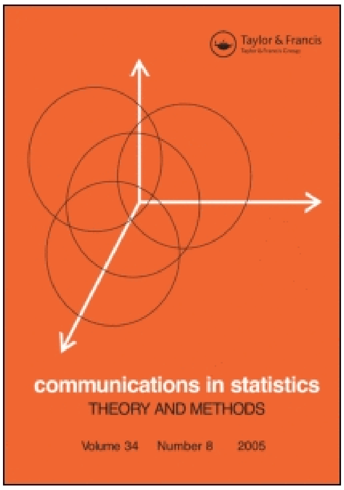

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Parameter Estimation in Minification Processes

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ABSTRACT

In this article it is proved that the stationary Markov sequences generated by minification models are ergodic and uniformly mixing. These results are used to establish the optimal properties of estimators for the parameters in the model. The problem of estimating the parameters in the exponential minification model is discussed in detail.

Key Words: Consistent and asymptotically normal estimators; Ergodicity; Exponential distribution; Minification models; Uniformly mixing sequences.

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1. INTRODUCTION

One of the basic assumptions in the classical analysis of the time series is that the sequence of observations is a realization from some Gaussian sequence. Further, most of the models employed in analyzing such series are linear in nature. However, in recent years it has been found that nonlinear models with nonGaussian marginal distributions are more suitable than linear Gaussian models in certain situations. See, for example, Lawrance (1991) and the references cited there. One of the important nonlinear models used to generate sequence $\{X_n\}$ of nonnegative random variables (r.v.s) is defined by

$$X_n = \begin{cases} X_0 & n = 0 \\ k \ \min(X_{n-1}, \varepsilon_n) & n = 1, 2 \ldots, k > 1 \end{cases} \tag{1.1}$$

where $\{\varepsilon_n\}$ is a sequence of independent and identically distributed (i.i.d) nonnegative nondegenerate r.v.s. called innovations and X_0 is independent of ε_1 . This model is referred to as a minification model. The Markov sequence $\{X_n\}$ defined by Eq. (1.1) has many properties of a first order autoregressive (AR(1)) sequence. In particular, the exponential minification process of Tavares (1980) is a time-reversed version of the first order exponential autoregressive (EAR(1)) process introduced by Gaver and Lewis (1980). This is an interesting result proved by Chernick et al. (1988). It is worth recalling that Gaver and Lewis (1980) proposed an estimation scheme based on the degeneracy property of the model, which determines the exact value of the autoregressive parameter and then estimates the scale parameter. In view of the time-reversibility relation cited above, this estimation scheme can be extended to determine the exact value of k in a minification process as we illustrated in Sec. 3.

Various aspects of the model (1.1) when X_n has a specified distribution have been studied by different researchers. For example, Tavares (1980) discuss the minification process with exponential marginal, Sim (1986) defined this model for Weibull r.v.s., Yeh et al. (1988) for Pareto r.v.s. and Pillai (1991) studied a model with semi-Pareto marginal distribution.

If $\bar{F}(x) = P(X_0 > x)$ and $\bar{G}(y) = P(\varepsilon_1 > y)$then $\{X_n\}$ defined by Eq. (1.1) is stationary if and only if

$$\bar{G}(x) = \frac{\bar{F}(kx)}{\bar{F}(x)}, \quad x \geq 0, \ k > 1 \tag{1.2}$$

(cf. Lewis and Mckenzie (1991)). Arnold and Hallett (1989) showed that if the distribution of X_0 is chosen as

$$\bar{F}(x) = \prod_{j=1}^{\infty} \bar{G}(x/k^j) \tag{1.3}$$

then Eq. (1.1) defines a stationary sequence with X_n having survival function $\bar{F}(.)$. It is assumed that the product (1.3) does not diverge to zero. Let us define

$$X_0 = \inf_{0 \le j \le \infty} k^j \varepsilon_{-j}, \tag{1.4}$$

where $\{\varepsilon_{-j},\ j = 0, 1, 2.....\}$ is a sequence of i.i.d. nonnegative r.v.s. with common survival function $\bar{G}(.)$. Now it follows that the survival function of X_0 is given by Eq. (1.3). The applications of these models in various areas such as geophysical science, reliability, etc., are discussed in the above mentioned references.

As far as statistical inference is concerned, little work has been done for these models. Adke and Balakrishna (1992) have estimated the parameters of exponential minification model. Balakrishna (1998) discussed the estimation problems in semi-Pareto and Pareto processes. In this article, we estimate the common mean of $\{X_n\}$ and the parameter k of the general stationary minification processes defined by Eq. (1.1).

In Sec. 2, we prove that a stationary minification process is ergodic and uniformly mixing. These results are used to prove the optimal properties of the estimators of k and the common mean of X_n in Sec. 3. As an illustration we discuss the details of estimation problems in exponential minification model in Sec. 4. The simulation results in Sec. 5 show that the proposed estimators perform well in the exponential case. The Sec. 6 gives some concluding remarks.

2. SOME PROBABILISTIC PROPERTIES OF THE MODEL

In this section we prove that the minification process is ergodic and uniformly mixing.

Lemma 2.1. *The stationary Markov sequence defined by Eq.* (1.1) *is ergodic.*

Proof. Let $\mathbf{F}_n = \sigma\{X_1, X_2, \ldots X_n\}$ and $\mathbf{G}_n = \sigma\{\ldots, \varepsilon_{-2}, \varepsilon_{-1}, \varepsilon_0, \varepsilon_1, \varepsilon_2, .., \varepsilon_n\}$ be the sigma fields induced by $(X_1, X_2, \ldots X_n)$ and $(\ldots, \varepsilon_{-1}, \varepsilon_0, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ respectively. Repeatedly using Eq. (1.1), we can write

$$X_n = \text{Min}\{k^n X_0, k^n \varepsilon_1, k^{n-1}\varepsilon_2, \ldots k\varepsilon_n\}, \quad n = 1, 2 \ldots. \tag{2.1}$$

Thus it now follows from Eq. (1.4) that

$$\mathbf{F}_n \subseteq \mathbf{G}_n \quad \text{for } n \geq 1 \tag{2.2}$$

and hence the tail sigma field τ of $\{X_n\}$ is contained in the tail sigma field τ^* of the i.i.d. r.v.s. $\ldots, \varepsilon_{-1}, \varepsilon_0, \varepsilon_1, \varepsilon_2 \ldots, \varepsilon_n$. It is well-known that each event of τ^* has probability 0 or 1. This implies by Eq. (2.2) that τ contains only events of probability 0 or 1, which is a sufficient condition for $\{X_n\}$ to be ergodic (see Stout, 1974, p. 182). Hence the lemma is proved.

Definition. A sequence $\{X_n\}$ of r.v.s. is said to be uniformly mixing (or ϕ-mixing) if

$$|P(A \cap B) - P(A)P(B)| \leq P(A)\phi(h),$$

where $A \in \sigma(X_0, X_1, \ldots X_n\}$, $B \in \sigma(X_{n+h}, X_{n+h+1} \ldots\}$ and $\phi(h) \to 0$ as $h \to \infty$.

Lemma 2.2. *A minification sequence* $\{X_n\}$ *generated by Eq.* (1.1) *and satisfying Eqs.* (1.2) *and* (1.4) *is uniformly mixing with mixing parameters*

$$\phi(h) = P[X_h = k^h X_0], \quad h = 0, 1, 2, \ldots \tag{2.3}$$

Proof. Suppose that $A \in \sigma(X_0, X_1, \ldots X_n\}$, $B \in \sigma(X_{n+h}, X_{n+h+1} \ldots\}$. A close inspection of the model (1.1) reveals that if at least one innovation occurs in the time interval $(n+1, n+h-1)$ then the events A and B are independent. Let N be the number of innovations occurring between X_n and X_{n+h}. Hence we have

$$P(A \cap B | N > 0) = P(A | N > 0) P(B | N > 0). \tag{2.4}$$

It also follows from Eq. (1.1) that $N = 0$ if and only if $X_{n+\text{h}} = k^h X_n$ and in this case the events A and B are dependent. These observations along with Markov property of $\{X_n\}$ show that A and $(N > 0)$ are independent events. That is,

$$\begin{aligned} P[A \cap (N > 0)] &= \int_0^\infty P\{[A \cap (X_{n+h} \neq k^h X_n)] | X_n = x\}\, dF(x) \\ &= \int_0^\infty P\{A | X_n = x) P\{N > 0)\, dF(x) \\ &= P(N > 0) P(A). \end{aligned}$$

Thus we have

$$P(A \cap B | N > 0) = P(A) P(B | N > 0). \tag{2.5}$$

On the similar lines, it can be shown that

$$\begin{aligned} P[A \cap B \cap (N = 0)] &= \int_0^\infty P\{B | X_{n+h} = k^h X_n\} P(N = 0) P(A | X_n = x)\, dF(x) \\ &\leq P(N = 0) P(A). \end{aligned} \tag{2.6}$$

Now using Eqs. (2.5) and (2.6) we can write

$$\begin{aligned} P(A \cap B) &= P[A \cap B \cap (N = 0)] + P[A \cap B \cap (N > 0)] \\ &\leq P(N = 0) P(A) + P(A) P(N > 0) P[B | N > 0] \end{aligned}$$

Hence

$$\begin{aligned} |P(A \cap B) - P(A) P(B)| &\leq P(A) \{P(N = 0) - P(B | N = 0) P(N = 0)\} \\ &\leq P(A) \phi(h), \end{aligned}$$

where $\phi(h) = P[N = 0] = P[X_{n+h} = k^h X_n]$.

Since the sequence defined by Eq. (1.1) is stationary, we have (cf. Lewis and Mckenzie, 1991),

$$\phi(h) = P[X_h = k^h X_0] = \int_0^\infty \frac{\bar{F}(k^h x)}{\bar{F}(x)}\, dF(x).$$

Note that $\bar{F}(k^h x)/\bar{F}(x)$is decreasing function of h and hence by the monotone convergence theorem it follows that $\phi(h) \to 0$ as $h \to \infty$. This completes the proof of the lemma.

3. ESTIMATION OF k AND THE COMMON MEAN

Let $\{X_n\}$ be a stationary sequence defined by Eq. (1.1) with common distribution function $F(.)$ and common mean $\mu = E(X_1)$. Assume further that $\mathrm{Var}(X_n) = \sigma^2 < \infty$ for all n. The ergodicity of $\{X_n\}$ implies that the sample mean $\bar{X}_n = (X_1 + X_2 + \cdots + X_n)/n$ is a natural estimator of μ. The asymptotic properties of $\bar{X}_n$ are proved in the following theorem.

Theorem 3.1. *The time average $\bar{X}_n$ is strongly consistent and asymptotically normal (CAN) estimator of μ. The asymptotic variance (AV) of $\bar{X}_n$ is given by*

$$AV(\bar{X}_n) = \sigma^2 A(k)/n, \tag{3.1}$$

where $A(.)$ is a continuous nonnegative function.

Proof. By Lemma 2.1 and the point wise ergodic theorem, it follows that $\bar{X}_n \to \mu$ almost surely (a.s) as $n \to \infty$. The uniform mixing property of $\{X_n\}$ implies that (cf. Billingsley, 1968, p. 174),

$$\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{L} Z_1, \tag{3.2}$$

where $\xrightarrow{L}$ stands for convergence in distribution and Z_1 is a normal r.v. with mean zero and variance

$$\sigma_1^2 = \sigma^2 + 2\sigma^2 \sum_{j=1}^{\infty} \rho(j). \tag{3.3}$$

In this case $\rho(j)$ is the autocorrelation between X_1 and X_{1+j}. Further $0 < \sigma_1^2 < \infty$. Thus $\bar{X}_n$ is a CAN estimator of μ and its asymptotic variance is given by

$$AV(\bar{X}_n) = \frac{\sigma^2}{n}\left\{1 + 2\sum_{j=1}^{\infty} \rho(j)\right\}. \tag{3.4}$$

Let us denote by $\rho(1) = \mathrm{Corr}(X_1, X_2)$ and assume that $\rho(1)$ is a continuous function of k, say $c(k)$. Then it is known (see Lewis and Mckenzie, 1991) that $\rho(j) = c(k^j)$. Thus we can write

$$AV(\bar{X}_n) = \sigma^2 A(k)/n, \tag{3.5}$$

where

$$A(k) = 1 + 2\sum_{j=1}^{\infty} c(k^j), \tag{3.6}$$

which is continuous in k. Proof of the theorem is complete.

Remark 3.1. The uniform mixing property of $\{X_n\}$ implies that the summations in Eqs. (3.3) and (3.6) are finite.

In the rest of this section, we discuss the problem of estimating k. For the model (1.1) let us define

$$W_n = \frac{X_n}{X_{n-1}} = \begin{cases} k & \text{if } X_{n-1} \le \varepsilon_n \\ k(\varepsilon_n / X_{n-1}) & \text{if } X_{n-1} > \varepsilon_n \end{cases} \tag{3.7}$$

so that $W_n \le k$ for all n. Let

$$\tilde{k}_n = \operatorname*{Max}_{1 \le i \le n} W_i. \tag{3.8}$$

Theorem 3.2. *The estimator $\tilde{k}_n$ is a strongly consistent estimator of k, which is not asymptotically normal.*

Proof. From Eqs. (3.7) and (3.8) it is clear that $\tilde{k}_n = k$ if and only if $X_{i-1} < \varepsilon_i$ for at least one i, $i = 1,2,\ldots,n$. Thus

$$\begin{aligned} P[\tilde{k}_n \ne k] &= P[X_{i-1} > \varepsilon_i \text{ for all } i = 1,2,\ldots,n] \\ &\le P[k\varepsilon_{i-1} > \varepsilon_i \text{ for all } i = 1,2,\ldots,n] \\ &\le P[k\varepsilon_{2i-1} > \varepsilon_{2i} \text{ for all } i = 1,2,\ldots,[n/2]-1]. \end{aligned}$$

This implies that

$$\sum_{n=1}^{\infty} P[\tilde{k}_n \ne k] \le \sum_{n=1}^{\infty} p^{[\frac{n}{2}]-1} < \infty, \tag{3.9}$$

where $p = P[k\varepsilon_1 > \varepsilon_2]$ and since ε_i's are i.i.d nondegenerate r.v.s. we have $0 < p < 1$. Now by the Borel-Cantelli lemma, we have

$$P[\tilde{k}_n \ne k, \text{ infinitely often}] = 0.$$

That is, $\tilde{k}_n = k$ infinitely often with probability 1 and hence $\tilde{k}_n \to k$ a.s. as $n \to \infty$.

However, $P[\sqrt{n}(\tilde{k}_n - k) \le x] = P[\tilde{k}_n \le k + x/\sqrt{n}] \equiv 1$and hence $\tilde{k}_n$ is not CAN. This completes the proof.

Remark 3.2. From Eq. (3.7) it is clear that $0 < W_i \leq k$ for all i and $W_i = k$ if and only if $\varepsilon_i > X_{i-1.}$ Thus the distribution function of W_i is concentrated on a finite interval with a positive jump at the right end point. From the study of extreme value theory, we know that there does not exist a non-degenerate limit distribution for $\{W_n\}$ for any choice of a norming sequence (cf. Leadbetter et al., 1983, p. 13 and p. 60).

4. ESTIMATION FOR EXPONENTIAL MINIFICATION PROCESS

One of the well-known minification models is that defined by Tavares (1981) for exponential r.v.s. In this case X_0 has the distribution

$$F(x) = P(X \leq x) = 1 - e^{-x/\mu}, \quad \mu > 0,\ x \geq 0. \tag{4.1}$$

and the i.i.d. sequence $\{\varepsilon_n\}$ has the common distribution specified by

$$G(x) = P(\varepsilon_1 \leq x) = 1 - e^{-(k-1)x/\mu},\ x \geq 0. \tag{4.2}$$

Then X_n defined by

$$X_n = k\ \mathrm{Min}(X_{n-1}, \varepsilon_n),\ n = 1, 2 \ldots \tag{4.3}$$

has exponential distribution $F(x)$ for all $n \geq 0$. Here $\{X_n\}$ is referred to as an exponential minification sequence, which is uniformly mixing with (cf. Sec. 2)

$$\phi(h) = k^{-h},\ h = 1, 2, \ldots \tag{4.4}$$

The sample mean X_n is CAN estimator for μ and

$$AV(\bar{X}_n) = \frac{k+1}{k-1}\frac{\mu^2}{n}. \tag{4.5}$$

Hence we estimate μ by the sample mean $\bar{X}_n$.

In the following discussion we propose an estimator for k denoted by $\hat{k}$ which is different from $\tilde{k}_n$ defined by Eq. (3.8).

Let

$$U_j = \begin{cases} 1 & \text{if } X_j \geq X_{j-1} \\ 0 & \text{if } X_j < X_{j-1} \end{cases}, \quad j = 1, 2, \ldots \tag{4.6}$$

then

$$E(Uj) = k/(2k-1)$$

and

$$V(Uj) = k(k-1)/(2k-1)^2. \tag{4.7}$$

Let $\bar{U}_n = n^{-1}\sum_{j=1}^{n} U_j$ be the arithmetic mean of $U_1, U_2, \ldots, U_n$.

Theorem 4.1. *For the exponential minification process, the estimator $\hat{k}_n = \bar{U}_n/(2\bar{U}_n - 1)$ is strongly consistent and $\sqrt{n}(\hat{k}_n - k) \xrightarrow{L} Z_2$ as $n \to \infty$, where Z_2 is a normal r.v. with mean zero and variance*

$$\sigma_2^2 = k(k-1)(2k-1)^2 - 2(2k-1)^2(k-1)^3 \times \sum_{h=1}^{\infty} \frac{1}{\{k-1+k^{h-1}(2k-1)\}} \tag{4.8}$$

Proof. By the ergodicity of $\{X_n\}$ we have as $n \to \infty$, $\bar{U}_n \to k/(2k-1)$ a.s. and hence $\hat{k}_n \to k$ a.s.

As U_n is a function of X_n and X_{n-1}, by Lemma 2.2, it follows that $\{U_n\}$ is also stationary and mixing with coefficients (see Billingsley, 1968, p. 186)

$$\phi^*(h) = \phi(h-1) = k^{-(h-1)}, \quad h = 1, 2, \ldots \tag{4.9}$$

Now by applying the theorem 20.1 of Billingsley (1968), we get the result that

$$\sqrt{n}[\bar{U}_n - k/(2k-1)] \xrightarrow{L} Z, \tag{4.10}$$

where Z is normal r.v. with mean zero and variance

$$\mathrm{Var}(Z) = \mathrm{Var}(U_1) + 2\sum_{h=1}^{\infty}\mathrm{Cov}(U_l, U_{l+h})$$

$$\begin{aligned}\mathrm{Cov}(U_l, U_{l+h}) &= P[X_l > X_0, X_{l+h} > X_h] - P[X_1 > X_0]P[X_{1+h} > X_h] \\ &= 1 - P[X_0 > k\varepsilon_1]P[X_h > k\varepsilon_{h+1}] \\ &\quad + P[X_0 > k\varepsilon_1, X_h > k\varepsilon_{h+1}] - \{k/(2k-1)\}^2 \\ &= 1 - p_0 - p_h + p_{0h} - \{k/(2k-1)\}^2, \text{ say.}\end{aligned} \tag{4.11}$$

Now simplify the term p_{0h},

$$p_{0h} = P[X_0 > k\varepsilon_1, k^{h-1}X_0 > \varepsilon_{h+1}, k^{h-1}\varepsilon_1 > \varepsilon_{h+1}, \\ k^{h-2}\varepsilon_2 > \varepsilon_{h+1}, \ldots \varepsilon_h > \varepsilon_{h+1}].$$

By conditioning on $(\varepsilon_1, \varepsilon_{h+1})$., it becomes,

$$p_{0h} = \int_0^\infty \frac{\lambda(k-1)^2}{(2k-1)} \exp\left[\{-\lambda(2k-1)\varepsilon_{h+1}/k^{h-1}\} \right. \\ \left. -\lambda(k-1)\left\{2 + \sum_{j=1}^{h-2} k^{-j}\right\}\varepsilon_{h+1}\right] d\varepsilon_{h+1}.$$

After simplifying the integral we get

$$p_{0h} = (k-1)^2 k^{h-1}(2k-1)^{-1}\{k-1+2k^h - k^{h-1}\}^{-1}$$

From the definition of the model (4.3), it immediately follows that

$$p_o = p_h = (k-1)/(2k-1).$$

Substituting these values in Eq. (4.11), we get

$$\mathrm{Cov}(U_1, U_{h+1}) = -(k-1)^3(2k-1)^{-2}\{k-1+k^{h-1}(2k-1)\}^{-1}. \tag{4.12}$$

An application of the ratio test for convergence of series implies that

$$\sum_{h=1}^{\infty} |\mathrm{Cov}(U_1, U_{h+1})| < \infty.$$

Further, it is readily verified that

$$\{k(k-1)\}/(2k-1)^2 + \sum_{h=1}^{\infty} \mathrm{Cov}(U_1, U_{h+1})$$

is strictly positive. Thus we have the result specified by Eq. (4.10) with $0 < \mathrm{Var}(Z) < \infty$.

Let us write

$$\sqrt{n}\left(\hat{k}_n - k\right) = -\frac{(2k-1)}{(2\bar{U}_n - 1)}\sqrt{n}\left(\bar{U}_n - \frac{k}{2k-1}\right).$$

Now, since $(2k-1)/(2\bar{U}_n - 1) \to (2k-1)^2$ a.s. as $n \to \infty$, by Slutsky's theorem, we have

$$\sqrt{n}(\hat{k}_n - k) \xrightarrow{L} Z_2,$$

where Z_2 is a normal r.v. with mean 0 and variance σ_2^2 given by Eq. (4.8). This completes the proof.

5. SIMULATION STUDY

In this section we study the performance of the estimators proposed for the parameters of an exponential minification process discussed in Sec. 4. We estimate μ by $\bar{X}_n$ and k by $\hat{k}_n$ defined in Theorem 4.1. For specified values of k and μ we generated samples of 1100 observations and computed $\hat{\mu}$ and $\hat{k}_n$ after skipping the first 100 values. We repeated the experiment 100 times and then evaluated the averages of $\hat{\mu}$ and $\hat{k}_n$

Table 5.1. Averages of $\hat{\mu}$ and $\hat{k}$ from 100 simulations based on the exponential minification model for specified values of μ and k, where the observations in each sample is 1000.

k	μ	$\hat{k}$	SE($\hat{k}$)	ASE($\hat{k}$)	$\hat{\mu}$	SE($\hat{\mu}$)	ASE($\hat{\mu}$)	COR($\hat{k}, \hat{\mu}$)
1.1	0.5	1.099	0.012	0.011	0.486	0.072	0.071	0.723
	1	1.102	0.011	0.012	1.006	0.135	0.144	0.618
	2	1.103	0.010	0.012	2.007	0.299	0.287	0.641
	5	1.100	0.011	0.012	5.096	0.682	0.738	0.649
	10	1.099	0.013	0.011	10.024	1.577	1.460	0.701
1.25	0.5	1.251	0.025	0.022	0.506	0.048	0.048	0.654
	1	1.252	0.022	0.022	1.005	0.086	0.095	0.565
	2	1.249	0.022	0.022	1.983	0.169	0.189	0.607
	5	1.249	0.021	0.022	4.956	0.432	0.471	0.517
	10	1.250	0.023	0.022	9.943	0.938	0.943	0.595
2.5	0.5	2.514	0.149	0.157	0.502	0.026	0.024	0.506
	1	2.493	0.142	0.154	1.006	0.046	0.049	0.228
	2	2.510	0.178	0.157	1.985	0.097	0.096	0.268
	5	2.508	0.150	0.156	5.033	0.227	0.243	0.211
	10	2.532	0.168	0.160	10.027	0.479	0.481	0.221
5	0.5	5.108	0.899	0.780	0.498	0.021	0.019	0.101
	1	5.270	1.000	0.835	1.000	0.044	0.038	0.044
	2	5.178	0.851	0.803	1.989	0.074	0.076	0.177
	5	5.239	0.971	0.824	4.972	0.190	0.191	0.175
	10	5.151	0.910	0.794	10.016	0.373	0.386	0.148

Table 5.2. Averages of $\hat{\mu}$ from 100 simulations based on the exponential minification model for known values of k, where the observations in each sample is 100.

	k								
	1.1			1.25			2.5		
μ	$\hat{\mu}$	SE($\hat{\mu}$)	ASE($\hat{\mu}$)	$\hat{\mu}$	SE($\hat{\mu}$)	ASE($\hat{\mu}$)	$\hat{\mu}$	SE($\hat{\mu}$)	ASE($\hat{\mu}$)
0.5	0.520	0.217	0.238	0.486	0.137	0.146	0.516	0.072	0.079
1	1.041	0.430	0.477	1.001	0.314	0.301	0.990	0.164	0.151
2	2.013	0.789	0.923	2.088	0.528	0.624	1.969	0.305	0.301
5	5.177	2.295	2.373	5.151	1.556	1.545	5.030	0.683	0.768
10	9.996	4.481	4.581	10.21	3.378	3.063	10.07	1.306	1.538

over the repetitions. The Table 5.1 presents the specified values of the parameters μ, k, the averages of the estimates based on 100 trials, along with the standard errors. The columns of ASE ($\hat{\mu}$) and ASE ($\hat{k}$) provide the asymptotic standard deviations evaluated at $(\hat{\mu}, \hat{k})$, using Eq. (4.5) and (4.8) respectively. The column with COR gives the estimated coefficient of correlation between $\hat{\mu}$ and $\hat{k}_n$. Observe that the estimates $\hat{\mu}$ and $\hat{k}_n$ are very close to the respective parameter values with their standard errors close to the corresponding asymptotic standard errors for all combinations of the parameters.

However, the correlation between the estimators decreases when k increases. The simulated values of the estimator $\tilde{k}$ defined by Eq. (3.8) coincides with the theoretical value of k in each trial, but such situations may not occur in practice.

When k is known or it is replaced by $\tilde{k}$, the estimates of μ coincide with $\hat{\mu}$ with its standard error equal to SE($\hat{\mu}$) for $n = 1000$. However, in this case we get a good estimate for μ for relatively smaller sample size. The Table 5.2 summarizes the computation of the estimates along with the standard errors for known values of k when $n = 100$.

6. CONCLUDING REMARKS

There are several minification models available in the literature for generating variety of nonGaussian sequences. If we want to use these models in the practical situations, a valid estimation procedure is necessary. In this article we proposed some estimators for the common mean and k of a stationary minification model. The asymptotic properties of the estimators are also studied. The simulation study shows that the estimators perform well.

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