S.m.7. RAMACHANDRAN, P.T.–Some problems in set topology relating group of Homeomorphisms and order–1986–Dr. T. Thrivikraman

In this thesis, some problems in set topology, related to the concepts of group of homeomorphisms and order are investigated. Order theoretic methods are extensively used to investigate problems which are directly or indirectly related to the concepts of group of homeomorphisms of a topological space onto itself.

De Groot (1959) proved that any group is isomorphic to the group of homeomorphisms of a topological space. A related problem is to determine the subgroups of the group of permutations of a fixed set X, which can be represented as the group of homeomorphisms of a topological space (X,T) for some topology T on X. In chapter I of this thesis, some results along this direction are given. These include the result that no nontrivial proper normal subgroup of the group of permutations of a fixed set X can be represented as the group of homeomorphisms of a topological space (X,T) for some topology T on X.

Homogeneity and rigidity are two topological properties closely related to the group of homeomorphisms. Bankston, P defined (1979) an antiproperty for any topological property and discussed the antiproperties of compactness, Lindelofness, sequential compactness and so on. Reilly, I.L. and Vamanamurthy M.K. (1980) and (1981) obtained the antiproperties of separation axioms and compactness properties. In chapter III of this thesis, anti-homogeneous spaces are investigated and several characterisations are given. In particular it is proved that a space is antihomogeneous if and only if it is hereditarily rigid. The study is based on a pre-order associated with a topology (studied earlier by A.K. Steiner (1965), Lorran, F(1969), S.J. Andima and Thron W.J. (1978). The notions of homogeneity, anti-homogeneity and rigidity are introduced for preordered sets also. It is proved that a topological space is anti-homogeneous if and only if the associated preordered set is anti-homogeneous. A structure theorem for semi-well ordered sets (ie. linearly ordered sets in which every nonempty subset has either a first element or a last element) is the main order theoretic tool established and used.

Chapter 3 deals with the Cech closure spaces. Here an attempt is made to extend some results of the first two chapters to Cech closure spaces. These include the characterisation of completely homogeneous spaces and results related to the pre-ordeer associated with a topology.

In chapter 4, investigation is done on the lattice of closure operators on a fixed set X, with special attention to complementation. The atoms and the dual atoms of the lattice are determined first. The complementation problem is solved in the negative, using this. The lattice is dually atomic, but no element in it has more than one complement. Finally, some sublattices of this lattice and the fixed points of the automorphisms of this lattice are discussed.